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Mathematical reasoning through a broad range of communicational resources

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Mathematical reasoning is examined in this paper that investigates student – teacher communication at the front of the classroom where students gave account of their solutions to a mathematical problem. Adopting a multimodal approach we have discerned how students communicate reasoning through a broad range of communicational resources, such as speech, drawing, hand gestures and the like. We adopted Toulmin's (2003) model of argumentation as means to capture different elements of the reasoning given account for in the communication.

Keywords: Reasoning, argument, multimodality, communication.

INTRODUCTION

The importance for students to participate in mathematical reasoning has been highlighted in international frameworks (e.g., Niss & Jensen, 2002). In the ongoing PhD study, of which this paper is a part, the interest of exploring and understanding reasoning concerns reasoning in public, as a communicative act, not as a thinking process. Specifically, we, in this paper, investigate reasoning when students explain solutions to problems while positioned at the front of the classroom. The teacher takes part in the communication as well.

Reasoning is a collective and human transaction, in which we present ideas or claims to particular sets of people within particular situations or contexts and offer the appropriate “reasons” in their support. (Toulmin, Rieke, & Janik, 1979, p. 9)

An argument is described by Toulmin and colleagues (1979) as a “train” of reasoning. Various works have studied observable reasoning or argumentation in mathematics education, some focusing on collective

or collaborative argumentation/reasoning with a focus on peer interaction (e.g., Bjuland, Cestari, & Borgersen, 2008; Mueller, 2009). When analysing students’ collaborative reasoning, Bjuland, Cestari and Borgersen (2008) highlighted the need for paying attention to more than writing and speech when analysing arguments. Another example is Meaney (2007), who considered the role of gestures in strengthening arguments when analysing levels of mathematical literacy demonstrated by students.

Within research on mathematical communication, there is a trend to recognise the multimodal nature of communication even though it continues to privilege language as the primary mode (Morgan & Alswaikh, 2012). In order to analyse and understand communication in mathematics education, a multimodal approach has been adopted (e.g., Björklund Boistrup, 2015; Morgan & Alswaikh, 2009). While taking on a multimodal approach, Morgan and Alswaikh (2009) drew attention to the duality of a mode, the drawing mode: as the process of drawing and as the outcome, the picture itself. By using a multimodal approach when analysing reasoning expressed in student – teacher communications we hope to contribute to the understanding of reasoning as a sequence of communicative acts. In this study we investigate how students display reasoning in student – teacher communications through a variety of communicational resources when giving account of solutions at the front of the classroom.

ANALYTICAL FRAMEWORK

Drawing on Toulmin and colleagues (1979), we view an argumentation as consisting of ways of giving reasons and hence we understand argumentation and reasoning as strongly connected (Krummheuer, 1995). In order to examine reasoning we adopt Toulmin’s model of argumentation (2003). When analysing com-

munication, as constituted through a wide range of resources (such as speech, symbols, pictures and the like), we added to the framework a multimodal social semiotic perspective.

TOULMIN'S MODEL OF ARGUMENTATION

In mathematics education, Toulmin's model of argumentation has been used, sometimes in a reduced form (e.g., Krummheuer, 1995, 2007; Meaney, 2007), consisting of four of the six original elements of the model, where three of them represent the "core" of an argument. The elements are the data/ground, the warrant and the claim/conclusion, as well as the fourth element, backing. The model that serves as the basis for our analysis of reasoning is presented in Figure 1.

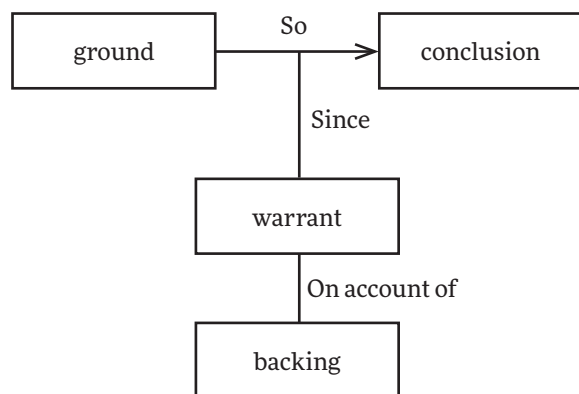


Figure 1: Toulmin's reduced model of argumentation

In a mathematical argument, a *conclusion* (Figure 1 top right), is presented, often in the form of a solution of a solved problem. In order to support a conclusion, some underlying foundation for the conclusion needs to be produced, which is to be seen as the *grounds* (Figure 1 top left), often consisting of facts or information. In order to justify the step between the presented grounds and the asserted conclusion, the *warrant*, can be provided (Figure 1, middle). The warrant is provided to show that starting from the grounds, the step to the given conclusion is appropriate and legitimate. The *backing* (Figure 1, bottom), gives authority to the warrant and can, like the grounds, often be expressed in the form of facts. Toulmin (2003) described the process of argumentation as G because W so C. Participants in a communication do not necessarily structure their contribution clearly according to the elements of the model but they may still be identified through a detailed analysis of interaction (Krummheuer, 1995). We adopted this model as an analytical tool in order to reconstruct reasoning in a

situation where students presented solutions to problems while positioned at the front of the class and with the teacher participating in the communication.

Multimodal social semiotics

Focusing on communication, we adopted a multimodal social semiotic perspective (Kress, 2010) in order to describe and understand the mathematical reasoning in classroom communication. Kress (2010) stressed that resources for communication are to be understood as more than writing and speech. They include images, facial expressions, gestures, and the like. The various communicational resources form multimodal ensembles which constitute the communication. In our study we focused on the semiotic resources that communicated mathematical reasoning identified in classroom interaction. Through this perspective we have analysed how the participants used various semiotic resources in their interaction to present reasons in support of a conclusion, which is regarded as the central activity of reasoning when forming an argument (Toulmin, et al., 1979). The multimodal approach affected transcripts as well as analysis and findings, which will be described further on.

METHODOLOGY

The context of this paper is a case study (in the sense of Hammersley & Gomm, 2009) including four classes from grade three to five. In this particular paper we draw on data from two episodes from a grade four class. The episodes are from two presentations at the front of the classroom about solutions to a problem. In order to identify potential reasoning in the presentations we structured and analysed data from the two videorecorded episodes in the following way. The episodes were transcribed taking on a multimodal approach by using the software Videograph. Videograph made it possible to note the semiotic resources the participants were using in terms of our interest in communicated reasoning. With a focus on elements of an argument, the following semiotic resources were identified as being relevant for this study: *speech*, *written text* (including symbolic notions), *drawings*, and *hand gestures* including the use of manipulatives (physical resources). This provided an overview of the communication (see Excerpt 1 as an example), making it possible to identify different elements of an argument according to Toulmin's model and a multimodal approach.

ANALYSIS

The teacher’s aim with the lesson which we give account for in this paper, was to involve the students in a problem solving activity, in pairs or small groups, as well as a whole class activity where selected solutions were presented. These presentations incorporated students explaining and justifying their reasoning. The mathematical content of the lesson was fractions and the task was formulated as follows:

It is a sport/field day and it is sunny and warm. The school will provide food and drinks. Each student is given $\frac{1}{4}$ of a liter of juice to drink. There are 16 students. How much juice will be needed?

At the end of the lesson the teacher asked some of the groups to present their solutions to the class. We present two episodes from the presentations, including analysis and findings. We chose these episodes since they represent two different solution strategies and, as will be shown, different aspects of student reasoning.

Description of Episode 1: Stina starts with the bottles

In this first episode, Stina and her friend are standing in front of the class in order to present their solution. The teacher asks Stina, who is doing the presentation of her group’s work, how she initially was “thinking”. Stina tells the teacher that she drew four bottles and then she starts to draw one bottle on the board. Being asked by the teacher regarding the number of bottles, Stina clarifies that they started off with drawing one bottle. Stina continues to talk but is interrupted by

the teacher who wants to know what they did with the bottle. Stina starts to explain that they counted (inaudible continuation) and is prompted by the teacher to do that with the bottle on the board. Stina divides the bottle into four parts and clarifies in words what she did. Asked for a clarification as to why they divided the bottle into four parts, Stina responds verbally “For it to be...since everybody could drink one quarter”. The follow up by the teacher to Stina’s response is yet another question, concerning the meaning of a quarter.

In Excerpt 1 (see Table 1) we give account for the continuation of the interaction. Actions taking place at the same time are beside each other horizontally.

Stina continues to draw a third and a fourth bottle, each time dividing each bottle into four parts, while explaining verbally how many they were enough for, ending up with “and then they were enough for sixteen” with a picture of four bottles each divided into four parts. The student – teacher interaction continues for a while and includes another student as well, ending with a verbal clarification that there are four liters needed for sixteen students.

Analysis of the first part of Episode 1

In Episode 1 we could identify in the analysis the elements of Toulmin’s model. Here we give account of our analysis of the first part of Episode 1. Stina’s argumentation here is made in relation to the teacher’s question regarding how many can drink from the first bottle. The image of the four parts of the bottle visualized how many that could drink from one bottle (Excerpt 1, row 5) and this was also expressed by speech, “four” (row 6). This utterance (picture+speech)

	Speech	Writing/Drawing	Hand gestures	
1	Teacher (T): How many parts should one divide it into then?		T takes a magnetic circle divided into four quarters and puts it on the white board.	Stina (S) points at the parts in the bottle
2	S: Four			
3	T: Four. Ok		T points at the circle	
4	S: It was not enough. We had to do one more	S starts to draw one more bottle next to the first one.		
5	T: How many could drink from the first bottle?	S divides the second bottle into four parts.		
6	S: Four			
7	T: Four. Ok			

Table 1: A multimodal transcript from Episode 1

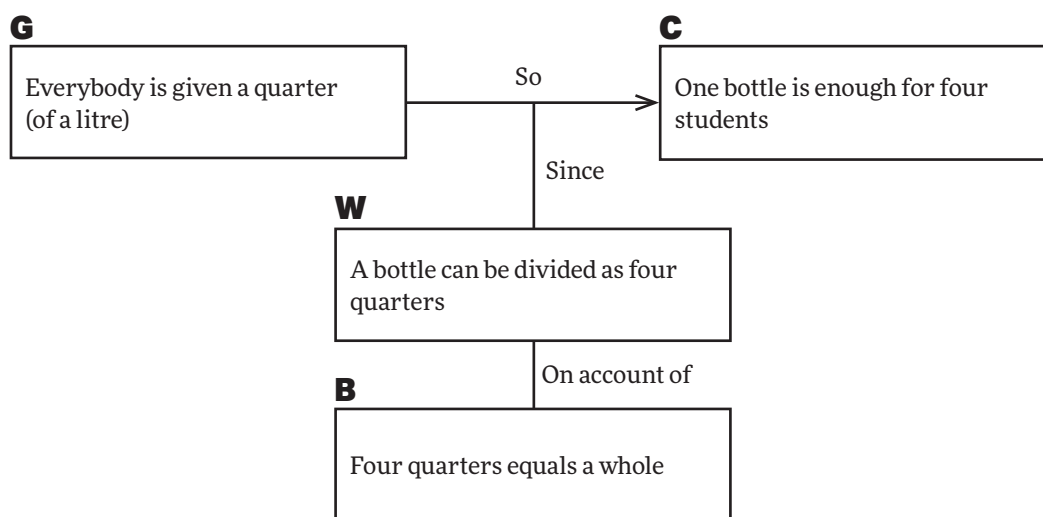


Figure 2: Elements of Stina's argument in Episode 1

has been identified as the first conclusion. In Figure 2 we summarise our analysis according to Toulmin's model.

As we wrote earlier, we could identify a conclusion from Stina's picture of bottle divided into four (row 5) and speech, "four" (row 6). This utterance from Stina was seen as claiming that four students could drink from one bottle. The fact Stina appealed to as the foundation for claiming that four students can drink from one bottle was categorised as the grounds and was identified from her verbal response to why the bottle was divided into four parts "because everybody can drink a quarter".

The warrant in this specific situation has been identified in this argument by taking into account the use of different semiotic resources. The warrant in this case should answer to why one can claim a bottle to be enough to four students (the conclusion) if each person is given a quarter (the grounds). The warrant, which we construed here, was: a whole can be divided as four quarters, which was communicated by Stina through drawing, speech, and hand gesture. More specifically, we construed that she communicated that a whole can be divided into four in her drawing where she divided the bottle (as a whole) into four parts together with her saying "we divided it into four parts". That each part is to be seen as a quarter was stated in her answer to the teacher's question on why they divided the bottle into four parts. Stina also indicated, according to our analysis, that one of the drawn parts of the bottle was to be considered as a quarter when pointing (row 1), a bit vaguely, towards the parts of the bottle when asked by the teacher what a quarter

means. The teacher also contributed to the argument by stating, "Four" (row 3) as a verbal reinforcement to Stina's verbal response "Four" (row 2) to her own question regarding how many parts it (the bottle as a whole) can be divided into.

Further evidence, authorizing the warrant, was categorised as backing: a whole equals four quarters, and in this episode was identified in the image of the magnetic circle consisting of four quarters of a circle put at the white board by the teacher (row 1).

Description of Episode 2: Frida starts with the mugs

After two groups had presented their solution, it was time for Frida to present her and her friend's solution. With both Frida and the teacher standing at the whiteboard, the teacher asks Frida to start with how she began to solve the problem. Frida begins to explain how she started off solving the problem by stating "I started to draw each student's mug". While expressing this Frida starts to draw a rectangle on the white board, emphasizing it by also pointing at it. While beginning to draw a second rectangle/mug (from now on referred to as "mug") the teacher asks her how much each mug contains. Frida completes drawing the second mug and writes $\frac{1}{4}$ in the first mug saying, "There was"... without continuation. The teacher expresses the content in each mug verbally "There was a quarter in each mug" and puts a quarter of a magnetic circle over the first mug drawn by Frida. Frida writes $\frac{1}{4}$ in the second mug and continues to draw six more mugs on the same row and eight mugs on a row below ending up having drawn sixteen mugs in total (Figure 4)

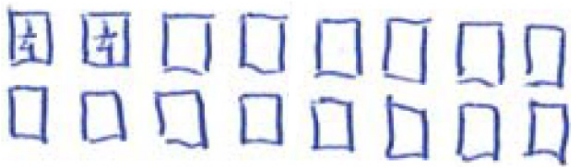


Figure 4: A picture of the way Frida drew the mugs

The teacher asks Frida how many quarters she had but Frida does not respond to that question and continues “I did like this” and shows on her picture of the sixteen mugs how she divided the sixteen mugs into four groups with four mugs in each group by drawing a line after four mugs and after eight mugs in the first row, and after four mugs in the second row and partly after eight mugs in the second row (Figure 5).

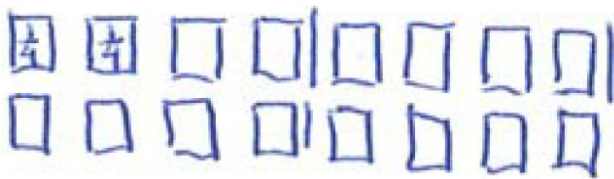


Figure 5: A picture of how Frida divided the mugs

Here Frida is interrupted by the teacher who wants to return to her question regarding the number of quarters. After some elaboration by the teacher regarding the number of quarters, she asks another student how many litres there are and receives the answer “Four litres”. As a final contribution in this presentation the teacher is stating. “We do know that in one litre there are four quarters” at the same time as she is putting the four quarters of the magnetic circle into a full circle on the white board.

Analysis of Episode 2

Also in Episode 2 we could, in the analyses, identify the elements of Toulmin’s model. We summarise our analysis in Figure 6.

The presentation in Episode 2 ended up with the conclusion that four liters were needed. The conclusion was identified in Frida’s final image reflecting the outcome as the four groups of four mugs/quarters in each liter (Figure 5) and was identified in the other student’s verbal answer “Four liters” towards the end of the presentation.

In these episodes two grounds, supporting the conclusion that four liters were needed, were identified. These grounds were identified as 1) there are sixteen students, and 2) each student is given a quarter each. The use of the first ground was identified in Frida’s drawing of the sixteen rectangles. Each rectangle was explained to illustrate a mug for each student, which is clear in Frida’s speech “I draw all students’ mugs” when she started to draw the mugs. The number of the mugs/students, sixteen, was expressed in the drawing and the picture of sixteen mugs (as well as by the teacher when elaborating verbally on what Frida was drawing). The second ground – each student is given a quarter – was identified when Frida wrote $\frac{1}{4}$ in two of the mugs as a response to a question from the teacher regarding how much the mug contained. After Frida had written $\frac{1}{4}$ in one of the mugs the teacher made this clearer by saying “It was a quarter in each mug”. A visual representation of the content was seen in the form of the magnetic quarter of a circle placed over the first mug by the teacher.

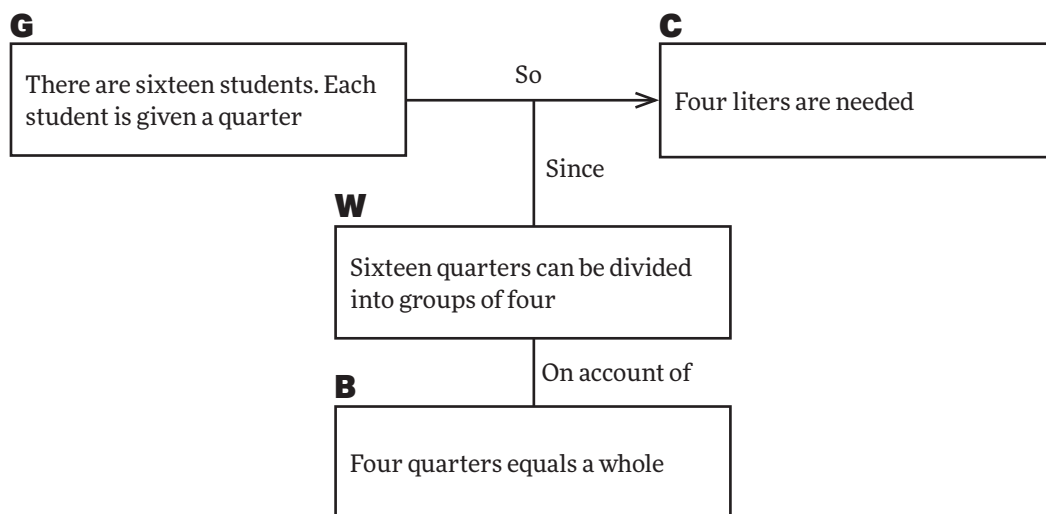


Figure 6: Elements of Frida’s argument in Episode 2

Our understanding is that Frida's method, to divide the sixteen quarters into groups of four, leading to the conclusion of four litres, indicates the warrant, justifying the step from grounds to conclusion. We construed the warrant to be: sixteen quarters can be divided into groups of four. The grouping of quarters was indicated by the process of drawing a line after the fourth mug in the first row, after the eighth mug in the first row and after the fourth mug in the second row.

In our analysis we identified that the teacher provided the backing by putting the four quarters of a circle, into a whole circle, showing: four quarters equals a whole. This was also identified in the teacher's, less formal, verbal expression "We do know that in one liter there are four quarters".

CONCLUDING DISCUSSION

The aim of the study was to identify reasoning in student – teacher communication, interpreted by us as presenting support, (that is, reasons) for a conclusion, showing how these reasons can be seen as giving strength to the claim (Krummheuer, 1995; Toulmin, 2003). By using Toulmin's model as an analytical tool together with a multimodal approach we have been able to discern aspects of reasoning.

We investigated how students display reasoning through a variety of communicational resources when giving account of solutions at the front of the classroom. In our findings we discerned how both students in the episodes presented here displayed reasoning through several communicational resources but in different ways. An example of this is how the girls displayed a ground they both referred their argument to: each student is given a quarter. In the first episode we could see Stina displaying the use of this ground by picture and speech whereas in Frida's case we could see how three different resources were identified as being used, interplaying with each other. In the second episode Frida displayed the ground by writing $\frac{1}{4}$ in a drawn rectangle (she had previously in speech communicated it to represent a mug to one of the students). Another example was when they were justifying the step from ground to conclusion, the warrant in Toulmin's model. Stina communicated this both through drawing, speech and gesture and Frida only through drawing. This illuminates how essential it is for teachers, as well as researchers, to pay attention to what is displayed in various commu-

nicative resources in order to capture and illuminate the reasons, mathematical justifications, and to make them accessible for all students in classroom communication.

By taking a broad range of communicative resources into account, this study opened up for capturing students' silent and non-symbolic display of reasoning which would have passed unnoticed if we had only been looking for verbal or written expressions. One example was our identification of Frida's process of drawing lines, which we construed as her justification, providing warrant for the conclusion considering the grounds she provided. Expressed verbally, such as in "A number of quarters can be divided into groups of four in order to get the number of wholes", it would be likely to receive a response from a teacher as a "proper" justification. If it was only expressed in drawing it might go unnoticed. If the warrants, such as Frida's in this case, are not noticed and elaborated upon, they might pass as unnoticed by other students in the classroom as well. Hence, an opportunity for the teacher to highlight and generalize mathematical ideas may be lost.

We want to clarify that as researchers adopting a multimodal approach, we always need to make choices of what resources to pay attention to in the analysis. In this paper we focused on reasoning and the resources identified as being relevant to this. If we had chosen to focus more broadly on the interaction itself, for example on feedback, other resources, such as voice, facial expression etcetera, would have been part of the transcripts and analysis as well.

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