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# Constructing mathematical competence in interaction: Whose mathematics is it?

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*Interactions in the mathematics classroom affect both the mathematical learning and the identities of those involved. In this paper, we draw upon Discursive Psychology to examine how identities can be developed and altered in whole class interactions. In this sense, identity is not an attribute of a person but is something that is co-constructed through and in interaction. We demonstrate how these identities can shift moment-to-moment within an interaction. Importantly, these identities shift within the same interaction. These changes in identity development have important consequences for mathematical learning and continuing participation and contribute to our understanding of the variance in identities that students self-report.*

**Keywords:** Identity, discourse, classroom interaction, positioning theory.

## INTRODUCTION

Identity has become increasingly prominent in mathematics education research and in this paper we build on recent research that focuses on the discursive construction of identity through classroom interaction. We consider understanding and learning mathematics to be an aspect of participating in discourse practices. Furthermore, participating in discourse practices influences, and is influenced by, participants' identities (Esmonde, 2009). In other words, students' mathematical identities are discursively constructed through their interactions and experiences in mathematics classrooms (Grootenboer, Smith, & Lowrie, 2006).

Research has revealed important relationships between students' mathematical identities and their experiences of mathematical practices. For instance, early work by Boaler (1997) showed that students had

qualitatively different forms of mathematical knowledge and beliefs about mathematics and the learning of mathematics depending upon the teaching methods they experienced. Her later research then examined how these different teaching methods influenced students developing mathematical identities and their decisions about continuing to study mathematics (Boaler & Greeno, 2000). Cobb, Gresalfi and Hodge (2009) developed this research further by developing an interpretive scheme to explore the relationships between particular classroom norms and sociomathematical norms and the developing identities of the students in those classrooms.

Different conceptions of identity in mathematics education research have arisen since then such as Sfard and Prusak's (2005) narrative work, Solomon's work with figured worlds (2007) and work by Cobb and colleagues (2009) with normative and personal identities. All of these approaches have largely drawn upon data from interviews with students describing their experiences with mathematics. Heyd-Metzuyanim and Sfard (2012) and Wood (2013) have focused on the construction of identity in the moment-to-moment interactions in classrooms. This paper contributes to this body of work.

## METHODOLOGY

The conception of identity developed in this paper arises from discursive psychology (DP) (Edwards & Potter, 1992). Discursive psychology is based on the principles of Ethnomethodology (Garfinkel, 1967) and Conversation Analysis (Sacks, 1992). It examines the practical ways in which identity is managed in interaction, which may differ from the narratives or stories individuals may offer in interview situations. The focus here is on how teachers and students discursively co-construct what it means to be a learner of mathe-

matics. From this perspective, identity is something that we ‘do’, rather than something we ‘are’. This is a micro view of identity where identity is conceived as dynamic, complex and situated in the context of the interaction itself. Identities are constructed through these interactions by how the participants orient to each other. Teachers and students construct identities for themselves and for each other through participation in interaction, and also for all the participants and observers of the interactions that occur in the mathematics classroom. This micro approach complements the approaches taken by researchers such as Boaler and Cobb, and those developed from positioning theory (Wood, 2013) which themselves draw from discursive psychology, by giving us a glimpse of *how* these identities are constructed as well as how they shift and change in interaction.

Studies taking this micro approach have illustrated how interactions with different participants within the classroom result in different identities being adopted as the consequences of some of these different identities. For example, Wood (2013) demonstrates how one student’s mathematical identity shifts in different classroom interactions with his teacher and his peers within one lesson. Two of the identities constructed, one in interaction with the teacher and one in interaction with a peer, relate to mathematic competence whilst a third, constructed in interaction with another peer, is one of a menial worker needing to be told each step. Yoon’s (2008) study also showed that teachers’ discursive positioning of non-native speakers affected their participation in lessons but also affected how their peers treated them in the classroom and Turner, Dominguez, Maldonado and Empson’s work (2013) offers insight into how teachers can position non-native speakers as mathematically and interactionally competent in a way that supports their peers in positioning them in a similar way. Yet each of these studies examines patterns over time and consistencies in how the different participants orient to each other. In contrast, this study examines moment-to-moment differences in the way identities are constructed in order to contribute to our understanding of the variety of identities that students develop in the mathematics classroom.

The analysis below focuses on three aspects of the talk: the structure of the turns taken by the teacher and his students; and the authorship and ownership of the mathematics (also referred to as the epistemic

agency (Ruthven & Hofmann, 2015)). One key feature of talk that is used by teachers related to ownership is revoicing (O’Connor & Michaels, 1993). Revoicing involves repeating students’ contributions in a way that attributes the ideas involved to the student. The student is the owner of the ideas involved and capable of offering these ideas. Enyedy and colleagues (2008) emphasise the importance of this attribution in classroom interaction as it “shares the intellectual authority with the students and helps establish their role as one of contributing to the construction of knowledge” (p. 137).

The data discussed in this paper comes from a larger study involving eight mathematics teachers from seven schools in the UK, all working with students aged between 11 and 14 years. Some of the schools serve areas with high levels of social deprivation, whilst others are fee paying independent schools. The teachers all volunteered to be video recorded and the data collected is naturally occurring in that no instructions were given to the teachers about what or how to teach. The transcripts included in this paper are from one of these teachers’ lesson with an all-attainment class of 12–13 year old students. The two extracts have been chosen to illustrate how the discursive construction of identity can change within one topic segment within a single lesson. The extracts come from one particular whole class interaction towards the beginning of the lesson where the students are reporting on work they have completed in a previous lesson and at home. The students are preparing for end of year exams and have been working on a worksheet with problems designed by the teacher to support them in their revision. Whilst the majority of studies have focused on small group work in mathematics lessons, whole class interactions are being considered here because the interaction not only positions the students who contribute but also the rest of the class observing and listening to the interaction. Whilst students are generally more agentic in small group discussions than in large whole-class discussions (Turner et al., 2013), the first extract below illustrates an example of where students can be agentic in teacher-led whole class discussions.

### **IS A MICROCENTURY LONGER THAN A MATHEMATICS LESSON?**

In the first extract the students are reporting their work on the question of whether a microcentury is

longer than a mathematics lesson. Immediately before this extract there has been some discussion on the meaning of microcentury and the notation associated with the prefix micro.

- 27 George: well (.) erm (.) I (.) worked it out (.) on erm the calculator and (0.3) it came up as one times (.) ten (1.6) to the power four. and er um (0.9) I times'd it by three hundred and sixty five, to simpl- to make it simpler (.) and um (1.1) and um it still wasn't (0.5) what I wanted so (0.4) I times'd it by twenty four which (0.4) um (1.2) um gave me that nought point eight seven six so it times'd it again by sixty (.) um minutes and it came as fifty two minutes and (1.3) so that's longer than forty minutes so (the answer is it's longer than the lesson) ((teacher is writing down the calculation while it is being said on the whiteboard))
- 28 Teacher: oh wonderful answer? thank you very much indeed. um you've said it all really haven't you. um this first bit comes up a bit funny on some of the calculators depends what sort of calculator you've got. sometimes when there are numbers that (.) don't fit easily on the display or have lots of noughts in. we represent them in a different way which we'll look at in year nine but um (.) it's basically (.) a ten thousandth. if you um (0.7) press the right button on your calculator and you get ten thousandth. and then with all these timesing what was what was George doing with all this? what was she doing here (1.7) ((turns round to point at a specific part of the calculation on the whiteboard)) yes Lauren
- 29 Lauren: um she was trying to (re do it) into minutes?
- 30 Teacher: she was making it into minutes eventually wasn't she I should have written that. (2.8) ((teacher is changing the colour of his whiteboard pen)) she was. so what where- what where the stages she went through. why was she- (0.5) why was she doing each of these steps. Hannah?
- 31 Hannah: because times three hundred and sixty five is like (.) three hundred and sixty five days in the year, times twenty four because there's twenty four hours in a day, times sixty because there's sixty minutes in an hour,

- 32 Teacher: but why do you think she stopped when she got to minutes. um I mean she said she got the answer here didn't she but (.) then she carried on and times by sixty Sam?
- 33 Sam: because it's the same units as what you're comparing it to
- 34 Teacher: perhaps yes. if you know the lesson is (.) thirty five or forty minutes, then that's what you compare it to...

The extract begins with George explaining her answer to a problem on the worksheet the class had been working on. Her explanation contains multiple pauses and hesitations but she completes her explanation without the teacher or any other student speaking. The teacher, by writing down her explanation on the whiteboard, makes the explanation available to the rest of the class in another representation and also gives weight to the validity of the explanation. This is reinforced further by the teacher's positive evaluation in the next turn.

Several of the pauses in George's turn are longer than the 'standard maximum tolerance' (Jefferson, 1988). Long pauses are often interpreted by the other participants as indicating that there is some trouble in the turn, such as difficulties with the mathematics or difficulties in expressing what the speaker wants to say. It is rare to see pauses of this length in students' turns in teacher-led discussions as the teacher will often step in to speak (Ingram & Elliott, 2014). By the teacher not speaking during these pauses, the student completes the explanation and as such both demonstrates their competence in producing this explanation and their competence in communicating this explanation. This competence is co-constructed with the teacher who neither adds to, rephrases, repeats or revoices the explanation in the following turn. The student is displaying their mathematical knowledge which is structured through the original task and the interaction.

Near the beginning of turn 28 the teacher makes reference to George's comment that the calculator said "one times ten to the power four", which he describes as "a bit funny". He then refers to the different displays produced by different calculators. Whilst George interpreted the calculator display without difficulty, the teacher's comment identifies this as a possible source of difficulty for the other students. This pre-emptive assessment by the teacher constructs the students as not knowing standard form or how calculators deal

with large numbers (Barwell, 2013). The difficulty is initially attributed to the way calculators display numbers. The teacher then offers an account for why calculators might do that. The teacher uses 'we' here twice. This emphasises that it's a representation that is used by the general mathematical community (Dooley, 2015) but also accounts for the students not being able to interpret the representation by referring to this idea as something that the students have not yet met, rather than as a deficit in the students themselves. He then minimises the importance of the issue using 'basically' and talking about it as just needing to "press the right button on the calculator". Whilst on the one hand the teacher is constructing the class as lacking the knowledge to work with this representation, the teacher is also treating the difficulty as being outside the responsibility of the students themselves.

The teacher then shifts the focus to the calculation that George was describing in the previous turn. This calculation is attributed to George both by the teacher and by the student that takes the next turn. Lauren phrases her turn hesitantly and answers the question of what George was doing with the calculation as a whole. The teacher partially repeats her response, thereby accepting it, but the addition of the word "eventually" indicates that there was a problem with the response and this is followed by a question that focuses on the stages of the calculation rather than the calculation as a whole. Hannah's subsequent response includes an explanation of what each of the numbers in George's calculation represent. The teacher makes no explicit evaluation or assessment of this explanation but follows Hannah's turn with 'but why' and a reformulation of the question which is answered by Sam. This is then positively evaluated by the teacher, but not in strong terms. "Perhaps yes" indicates that whilst Sam's answer is correct, it is not the answer that the teacher was looking for and the teacher adds additional information in his turn.

In this extract the authority for the mathematics is often given to the students, and they are invited to supply thinking about their peer's strategy for answering the question on the research. The teacher directs each turn to a new student, each of which builds on the explanation offered before and focuses on the calculation initially offered by George. The students are co-constructed as being both mathematically capable and capable of making sense of others' ideas. This also treats the contributions and explanations from

the students as important and worthy of consideration. Maintaining the authorship of the explanation with George also implies that she has a mathematical justification for her solution, and therefore evaluates her explanation as valid and treats her as a competent problem solver.

### DIVIDING 3.05 BY 2.5

The second extract follows the question 3.05 divided by 2.5 (written as 3.05 as the numerator of a fraction and 2.5 as the denominator) which occurs later on the same worksheet as the earlier question. A student has suggested multiplying the numerator and the denominator by a hundred and the teacher has written 305 over 250 on the board, but the student reported that they had not yet got further with the calculation.

- 40 Teacher: ... I think there's probably an easier number maybe can you see anything to do there Sarah?
- 41 Sarah: um
- 42 Teacher: could you use the idea of [a hundred]
- 43 Sarah: [times by four] yeah
- 44 Teacher: times by four yep so (0.6) whatever that makes times by four (.) you could probably do that. and then (.) how would you get it finally as a decimal
- 45 Sarah: um
- 46 (1.9)
- 47 Teacher: any ideas. (1.4) if you've got this number here Bella (.) if you got that number d- do you see what's happened here to go from this fraction to this fraction. (1.8) can you see what: (.) um (.) what I've done to convert this into this. what have I done (1.3) I've put a hundred on the bottom and I had twenty five a moment ago
- 48 Bella: you times'd it by fou[r]
- 49 Teacher: [yes I times'd that by four so I'd have to times the top by four as well and I can do that I just don't know what it is. But whatever it is, how would I get my final answer (0.5) if I wanted it as a decimal. (0.8) this is quite hard actually isn't it. We haven't done much of this ...

In this extract the authorship of the calculation shifts and the student contributions are considerably shorter than in the previous extract and often overlap with the teacher's turns. It is a student who first suggests multiplying the denominator by four, though this does follow hints from the teacher in turn 42 and in the turn immediately preceding the extract. The original question invites a range of possible strategies but the hints narrow this range down, so whilst the student has given an appropriate answer of "times by four" (as indicated by the teacher's acceptance of this answer in turn 44) this may not have been a strategy that the student used themselves or would think of themselves without the prompt of "a hundred".

There are several noticeable pauses in this extract in turns 46 and 47. The first two of these pauses offer Sarah the opportunity to answer the teacher's question in turn 44. When no answer is forthcoming, the teacher adapts that questioning towards the calculation that has already been performed. These first two pauses treat Sarah as being able to offer an explanation. However, when these opportunities are not taken up by Sarah, the trouble is treated by the teacher as being with the calculation that has already occurred. The teacher then checks that another student in the class has understood this calculation in turn 47.

The authorship shifts in turn 47 where initially the teacher refers to the numbers involved as belonging to the students, "you've got this number" but then the teacher begins to use 'I' to talk about the calculation that has been performed. The 'you' is used when talking about the strategies that could be used next, the strategies that could be used on the fraction with a denominator of 100. 'I' is used to refer to the calculation that has already been performed when converting between equivalent fractions. This shift from you to I is a further indication that the teacher is treating this calculation as a source of trouble. The teacher is now responsible for the calculation that has been performed and this treats the students as needing to follow his reasoning. The question asked in turn 47 asks a student to explain how the teacher converted the denominator of twenty five to a denominator of a hundred. In Bella's response she also positions the calculation as being the teacher's. In the final turn of the extract, there is another shift in who is doing the calculation that comes next. In turn 44 the teacher asks the students how they would get the final answer

and it shifts in turn 49 to how the teacher could get the final answer.

The emphasis in this extract and in the turns that came immediately before has been on the strategies that students can use rather than the final result of the calculation. The teacher is asking for what the students did in turn 40, not what answer they got. In turn 44, the teacher does not perform the calculation involved in changing the numerator once the denominator has been multiplied by four, reducing the importance of the answer to the calculation by referring to it as "whatever this makes". The teacher also positions the students as capable of performing this calculation in this turn before asking for strategies for what could be done next. In turn 49 again the teacher places the emphasis on the strategy but as a response from the students is not forthcoming he then states that "this is quite hard actually" to account for difficulties the students are having in responding to his question. This is then followed by an explanation of "we haven't done much of this" which also accounts for the difficulty by a lack of experiences of working with calculations like this, which locates the issue as outside of the students' competencies.

## DISCUSSION

In both extracts the interactional roles of teacher and students are clear. It is the teacher who controls the topic, asks the questions, and decides who can speak when. It is also clear that the teacher has control over assessing the appropriateness of the students' turns. In both extracts the students are treated as capable of explaining and communicating their ideas. They are given opportunities to do so

In this classroom there is a clear focus on the process of doing mathematics. The extracts presented here both focus on calculations but the attention is on the choices made in order to solve the problems and the justification for these choices. Difficulties with the mathematics are also treated in similar ways in both extracts, with the problem being located in the teaching rather than as a deficit in the students. In the first extract the difficulty is attributed to a representation that the students have not yet met and in the second extract the difficulty is attributed to the students not having had enough experience of this type of calculation and to the teacher posing a difficult problem in the first place.

The differences in the treatment of the students in the two extracts arise from the differences in the students' contributions to the discussions. In the first extract the students' responses do not always match what the teacher is looking for but they are accepted as answers to his questions, whilst in the second extract the students' responses are limited in terms of content, length and frequency.

The ownership of the mathematics shifts during the interactions in reaction to the students' contributions. In the first extract the mathematics is attributed to George throughout and this is indicated by both the teacher and the other students. In the second extract the ownership shifts progressively from the students to the teacher. The students position themselves in the first extract as mathematically capable and as capable of inferring George's mathematical reasoning. The teacher could have easily clarified George's answer himself and could also have stepped in when George was hesitating in her turn. By not doing so he enables the students to maintain a position of capability, which is supported further by the teacher in his turns. In the second extract, the students are more hesitant and do not take up the position of being able to suggest a strategy. The teacher initially continues to position the students as capable of suggesting a strategy but as the students do not take up this position the teacher shifts both the ownership of the mathematics and also positions the students as needing to understand the mathematics he is doing rather than as capable of doing it themselves. These different positionings could have consequence for how students come to see themselves as mathematical thinkers (Cobb et al., 2009).

As is evident from the extracts this is a classroom where students are given opportunities to engage in mathematical communication. They are frequently invited to contribute ideas and explanations and to build on other students' contributions. The students also work collaboratively throughout the lessons and for this teacher the majority of time in lessons is spent with the students working in small groups on a variety of tasks. We know from previous research (Boaler & Greeno, 2000; Cobb et al., 2009) that students in classrooms 'like' this are more likely to develop identities associated with mathematical capability and are more likely to continue with their study of mathematics. However, this research does not explain *how* these identities are developed and also it does not explain the variation in students' identities who have similar

experiences in their mathematics lessons. The micro approach such as the one taken in this paper and by Wood (2013) begins to give some insight into these two aspects of identity construction. Students are not either positioned as mathematically competent or not according to which teacher they have, but these positionings change and develop moment-to-moment in interaction with all teachers. Students consequently experience different mathematical positionings within each lesson, some of which may be positive but some of which may not. Whilst Wood focuses on the different identities co-constructed in interactions with different participants, teacher and peer, in this paper we have examined extracts from the same topic segment and between the teacher and the whole class.

This paper provides evidence for, and analysis of, the identities enacted by the students and developed by a teacher. The students enact positions where they are mathematically capable and the teacher ratifies and supports these positions. It also demonstrates that identities and positions can change across a short interaction. Paying attention to the influence of minor changes in context may help explain why students may take up different identities within mathematics.

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