Emotional interactions of high achieving students in mathematical argumentation: The case of Jasmine

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The study of emotional aspects of interaction may tell us much about the social norms and the meta-mathematical rules according to which participants act during their mathematical activity. To illustrate this idea, we present the case of Jasmine, a 16-year-old student participating in a summer camp intended for exceptionally high achieving students in mathematics. Through a commognitive analysis of the interaction between Jasmine and her instructor (the first author), we examine the implicit meta-rules of discourse according to which the two participants acted and that, at a certain point, led to Jasmine's frustrated disengagement from the discussion. Adding the concept of "framing" shows that this episode could be characterized by the student and instructor's "misaligned frames".

Keywords: Emotion, interaction, students, mathematical argumentation.

INTRODUCTION

In past decades, a growing amount of research has been dedicated to the examination of student affect and emotions in mathematical learning (Hannula, 2012). However, most of this research has been concentrated on students' subjective experience, as elicited by self-reports. More recently, researchers have started looking at the effects of emotion on student-student and student-teacher *interactions* (Heyd-Metzuyanim & Sfard, 2012). These studies have been inspired by a sociocultural lens that sees learning as a form of participation in a discourse. Emotional expressions are an important part of any human communication and thus make up an important and indispensable piece of the puzzle when one wishes to understand how learning takes place in real-life situations.

The socio-cultural view has been used in research that attempts to show the marginalization of students who get disengaged from mathematics (Boaler & Greeno, 2000; Heyd-Metzuyanim, 2013). However, rarely has this lens been turned to the learning of students identified as 'mathematically gifted'. In the present study, we employ such a lens to examine the ways in which emotions, social interactions, and mathematical cognition interact in the activity of high-level mathematical problem solving practiced in a summer camp for mathematically gifted students. We do so by using the *communicational* (commognitive) method for examining mathematical discourse as it intertwines with identity construction in mathematical learning (Heyd-Metzuyanim & Sfard, 2012; Sfard, 2008). Our goal is to explore the analytical tools that may shed light on the affective side of teaching-learning interactions in settings where students engage in high level mathematical argumentation.

IDENTITY, GIFTEDNESS, AND MATHEMATICAL ARGUMENTATION

Studies about mathematical giftedness have mainly examined cognitive aspects of learning (Leikin, Berman, & Koichu, 2009). Whenever studies about giftedness examine affective aspects of learning, they do so from an individual perspective, using concepts like self-concept and self-esteem (Zeidner & Schleyer, 1999). Despite the fact that emotions and self-perceptions have been acknowledged as important for understanding gifted students' learning, rarely have they been studied as they take place in these students' mathematical learning. Similarly, the literature about mathematical argumentation rarely deals with the emotional side of argumentation.

In the present study, we focus on *communication*, including its emotional and non-verbal aspects, to integrate the study of mathematical cognition, social interaction and emotions in the activity of mathematical argumentation. We do this by using the communicational (commognitive) framework (Sfard, 2008), whose main tenet is that thinking can be viewed as an intra-personal type of communication, not qualitatively different from inter-personal communication. Within this sociocultural framework, learning is conceptualized as participation in a specific type of discourse (here, the mathematical discourse). Discourses, claims Sfard, are defined by four characteristics: word use, routines, visual mediators and endorsed narratives. In mathematics, all these are used to create discursive objects such as "2", or "prime numbers". Sfard (ibid) defines learning as a *change in discourse* and differentiates between two types of such change: object-level learning, where students learn new routines for dealing with familiar objects, and meta-level learning, in which the *meta-rules* of the discourse change. Meta-rules define patterns in the activity of the discursants trying to produce and substantiate object-level narratives.

Heyd-Metzuyanim and Sfard (2012) pointed to the fact that while learning mathematics, students do not just participate in the mathematical discourse (or mathematize), they also participate in an *identifying* discourse whose main goal is to produce narratives about oneself and others. In line with Sfard and Prusak (2005) they defined identity as a collection of stories that are reifying and significant, told by a person about herself (1st person identity) or by others about her (3rd person identity). Identifying discourse, or discourse that constructs identities, is made up of *subjectifying* utterances, whose object is people (rather than mathematical objects). However, not all subjectifying utterances can be categorized as identifying. Only those utterances that relate to stable, significant attributes of the person (such as "she is mathematically gifted") are considered to be identifying. Subjectifying (and to certain degrees identifying too) can be, and often is, communicated via non-verbal or indirect means. For instance, emotional expressions almost always communicate some sort of subjectifying message (such as "this is embarrassing for me"). Heyd-Metzuyanim (2013) has shown that focusing on the *disruption* of routines can provide useful insights into the meta-discursive rules that often go unnoticed in teaching-learning situations, in particular to the meta-rules of identification. These are implicit rules governing who is supposed to refer to certain actions (e.g. a student giving a correct answer, a teacher challenging a student's claim), what certain actions convey about participants, and what emotional responses are appropriate for certain situations. We incorporate the concept of "framing" to better articulate the discrepancy that can occur between sets of meta-discursive rules (Goffman, 1974; Tannen, 1993). Tannen (1993) explains framing as the participants' sense of "what is going on" in the interaction. Within a communicational framework, we conceptualize this as participants' sense of the meta-rules that are governing the discourse in which they participate. Disruption in routines, and the emotional reactions that accompany such disruptions (Giddens, 1984) may point to *misaligned frames* (Sande & Greeno, 2012) of the interlocutors. In the present study, we ask: what may emotional reactions of participants in a mathematical discussion tell us about misaligned frames, or different meta-discursive rules according to which the interlocutors are acting?

THE STUDY: SETTING AND METHODS

The research was conducted in a mathematical camp for "mathematically gifted" youth (20 participants aged 15–18) that took place during the summer vacation where the first researcher acted both as a researcher and an instructor. It lasted two weeks and included both mathematical and social activities. Throughout the course of the camp, participants engaged in both group and individual study sessions. The group sessions took place with the guidance of an instructor who is a mathematician (holding at least an M.A. in mathematics). The first author was one of these instructors. Every day the students were given a worksheet of problems that progressively increased in their difficulty. Twice a week students were given the opportunity to present problems on the board that they had previously solved and get their solution 'peer reviewed' by their fellow students.

Lessons and social activities were videotaped, in addition to interviews with students during the camp and recordings of episodic happenings such as casual conversations that took place after the study sessions. The first author held a research diary in which she documented the events taking place during the day including her feelings about these events. Based on this diary, we chose for close examination several

events, in which it was clear that some emotional interaction was taking place, along with a rich mathematical discussion. These events were transcribed, including interviews and casual conversations that were relevant for understanding the event. The excerpts presented in this report were translated from Hebrew by the authors.

In this report we focus on one event that happened during a "peer-review" session. In it, one of the most active students in the group, Jasmine, volunteered to present her solution to an advanced problem that was given the previous day. The data for this analysis consisted of the recording and transcription of the lesson, as well as recordings of spontaneous conversations about the incident that took place after the lesson and a semi-structured interview with Jasmine at the end of the camp.

The analysis was based on the communicational framework, as presented above. In particular, we focused on three aspects of the communication between participants: a. the mathematical objects being talked about b. the subjectifying/identifying messages of the interlocutors about themselves and about each other; and c. the meta-discursive rules, or framing implied by these subjectifying messages. For instance, Jasmine's expressions of embarrassment or dismay were interpreted as indicative of an incident in which the happenings were misaligned with what she expected to happen or thought should have been appropriate for the situation. The interpretation of emotional expressions was based on the first and second author's view of the videos, in addition to Jasmine's description of the episode later in a casual conversation with other students. In particular, the interpretation of emotional signals and implicit identifying messages was based on our familiarity with the culture where this incident was taking place. Moreover, though we use Jasmine's report as a source of information, we should clarify that our method for analyzing emotional expressions does not necessitate 1st person reports of their emotions (or feelings). Rather, our method focuses on the communicational work that is done by emotional expressions, that is, how they are interpreted by others (see Heyd-Metzuyanim & Sfard, 2012).

FINDINGS – AN EPISODE OF ARGUMENTATION THAT WENT ASTRAY

The given problem was: *Prove that for each number n! + 2, n! + 3, ..., n! + n there is a prime divisor that does not divide any other number from this set.* None of the other students had solved this problem before. Jasmine was the only one who claimed to have solved it. Whilst starting to write her proof on the board, she noticed one of her peers starting to copy it. Alerted, she said "wait, why are you copying it?... what if I do something wrong?"

Figure 1: One of the drawn diagrams

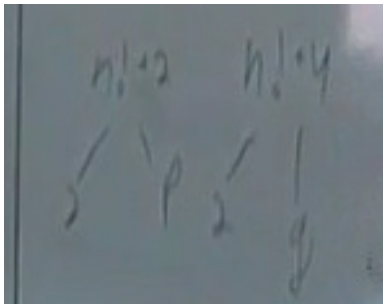

Figure 2: Another drawn diagram

One of her peers responded "no worries, it happens". This instance of subjectifying already indicated that Jasmine was somewhat concerned with her identity as told by the other participants when walking to the board. Yet right after that she went straight to mathematizing. She began her proof by drawing the diagram seen in Figure 1 explaining it by: "So there is a number N! and then one takes it and adds to it all sorts of things, that's my famous diagram" [13–14]. Again, this rather casual remark about her "famous diagram" indicated that Jasmine was thinking about the way her solution is perceived by others. However, this indirect identifying remark went unnoticed within the rest of the proof, the main argument of which was as follows:

Jasmine's proof

"So that... I added 2, so the number won't be divisible by any number that is smaller than N except 2. OK? Good. And then if I added 3, then it (points to N!+3) won't be

divisible by any number smaller than N except 3. And then, once we reach four, that's a multiple of two, then this number (points to N!+4) can also be divided by two. Then, wait, but it *must* be divided also by another prime factor that is greater than N" [19–25].

As she was talking, Jasmine was accompanying her explanation by drawing a diagram of N!+2 and N!+4, visually mediating their divisibility by 2 and another prime factor p or q as seen in Figure 2. At that point, the instructor interrupted, asking "why did you say that?"[27]. Here occurred the first failure in communication. While the instructor referred with the "that" to the claim that there are only two primary divisors of N!+4, Jasmine understood it as referring to the signification of the second prime number with q (and not p). She explained "'cause otherwise it would be the same number" [28]. However, since the misunderstanding of the teacher seemed to be related (at least in Jasmine's eyes) to mere signification of numbers by different letters, this interruption did not produce much stress and Jasmine went on in explaining her solution to the group. At this point, other students started getting involved and asking questions. Some asked only clarifying questions. Others' were more challenging, such as the instance below:

52 Yoav: Why do you say that this is prime (Points to p)?
53 Instructor: Wait, are you saying that it (N!+2) is divisible by 2? OK, but why is the other factor a prime?
54 Jasmine: There must be another prime factor

In the above interchange we see more evidence of a communicational breach. While the instructor and Yoav talked about "the other factor" being "a prime", Jasmine talked about the mere existence of "another prime". Therefore, it is not clear that she saw the necessity of the other factor being itself a prime, it might be that she was content with the factor being made up of several primes.

The instructor, together with some of the students, tried to find counter examples to Jasmine's claim that N!+2 is divisible by 2 and another prime. However, the first few numbers that were used for substitution actually confirmed Jasmine's conjecture. Thus, N=4 produced (24+2)/2=13 and N=5 produced (120+2)/2 =61, both prime numbers. The students and instructor did not get to the next N=6 which would have produced a counter-example ((720+2)/2 = 361, 361 is not a prime). Instead, the following interchange occurred:

The instructor's challenge

84 Instructor: So you say that it's always like that
85 Jasmine: I proved it! (Smiling, in a high, anxious voice). I just don't want to start with all the... (mumbles in a high voice, seems embarrassed)
86 Instructor: Are you sure?
87 Jasmine: Yes! I think so.. (tone of voice moves from assertion to a hint of doubt) I proved it. I just don't remember how.
88 Instructor: No, 'cause it's very interesting. 'Cause if you say this, then you are saying it's very easy to find prime numbers. That means I'll take something factorial, I'll divide it by 2, add 2, um.. no, add 2 divide by 2 and then I get a prime number. That means I found an algorithm for finding a prime number. That's why it seems strange to me.

Several points are worth mentioning in this short interchange. The first regards [85–86]. The instructor started by making a claim that combined subjectifying and mathematizing ("*you say* that it always turns out that way") (*subjectifying,* mathematizing). While she expected Jasmine to respond to the mathematical part (for instance by "yes, it always turns out that way") possibly with another justification, Jasmine responded to the subjectifying part. She referred to her *actions* ("I proved it") rather than to her mathematical claims. This divergence from mathematizing to subjectifying could also be seen in her emotional expressions at this point. Giggling and raising her voice were not going to help Jasmine justify her mathematical claims. Rather, they communicated she was mainly interested at this point in her identity (or the 3rd P identity of hers' as told by her classmates).

Another point regarding the above interaction refers to the meta-mathematical rules according to which the two interlocutors were acting, specifically, what "proving" entails and what may be considered as sufficient justification. For Jasmine, the fact that she did something in the past ("proved" the claim or solved the problem) was relevant and even sufficient for putting forward a mathematical claim [85]. Obviously, the instructor, who was an experienced participant in

the mathematical community, did not abide by this meta-rule. However, she did give some credit for Jasmine's hypothetical proof by asserting it is "very interesting" and communicating genuine doubt with regard to her own understanding (see also later in [122]).

Another important point in the above excerpt regards the counter argument that the instructor uses [88]. While up to this point, the counter arguments concentrated on finding a numerical counter example that would disprove Jasmine's claim, this counter argument was of a different sort. It drew on some common knowledge of the mathematical community about the feasibility (or unfeasibility) of finding any prime number. However, this knowledge was not necessarily shared by the students. Jasmine showed she did not share it by replying "It's possible to find prime numbers in this way, it's just that for a calculator it's really difficult to do factorials even with small numbers" [98]. The type of meta-mathematical rule for proving that the instructor was following here is a distinct, rather intuitive meta-rule of the mathematical community, not a formal logical routine. According to it, in order to refute a claim, one can draw on the *implications* of this claim to other problems known to be difficult or unsolved in the history of the mathematical community. Any claim that would imply a simple solution for such a difficult problem (such as providing an algorithm for finding prime numbers) would be regarded with high suspicion. Jasmine did not seem to be familiar with this meta-rule. Rather, she concentrated on checking "very big numbers" (or factorials) on her calculator thus resorting to a familiar routine -proving by giving examples.

At this point in the lesson, the instructor moved to explaining the disagreement between Jasmine and herself to the other students. Once the instructor clarified her suspicion about (N!+2)/2 being a prime, Jasmine started backing off:

Jasmine's retreat

117 Jasmine No, no. It can be 2 times many things but at the end there's a prime number there.

118 Instructor Of course at the end you have prime factors, but why are they 2, that's what's bothering me.

119 Jasmine No, you don't have only 2.

120 Instructor But this is what you drew here... (points to Figure 2) ...OK, but if you have more (than 2) then you do have a problem.

121 Jasmine Why?

122 Instructor Because you assumed you have 2 divisors and then you said that because there are 2, then that's exactly N! + 2 divided by 2, if I understood what you said... (Jasmine looks puzzled). If you get to here (implying N! +N) you'll get to (that fact that) the divisor of this is necessarily different than this denominator (pointing to N!+2). That's how I understood your proof, and maybe I didn't understand anything and that's alright.

123 Jasmine I didn't understand what you understood from my proof (smiles).

The excerpt above marks a transition in the argumentation of Jasmine and the instructor. Instead of checking Jasmine's claims, the instructor moved to an elaborate explanation of the "holes" or gaps in Jasmine's proof. One should notice that Jasmine's difficulty to follow the instructor [123] is understandable. Despite the fact that the instructor claimed to be revoicing Jasmine's proof, she used quite a different language for its expression. Jasmine talked about N!+2, N!+3,... being divisible into a number (2, 3) and a P (where P referred to a prime factor or simply a factor interchangeably). The instructor talked about P and (N!+2)/2 as interchangeable. In Sfard's (2008) words, she "samed" these two realizations into one object. It is probable that Jasmine did not go through the same "saming" process and therefore had difficulty in following the instructor's argument. Indeed, she remained puzzled, giving her fellow classmates an opportunity to voice their opinions. Reuven commented, with a bit of a humorous tone: "so what you proved here is that every number has a prime divisor". Jasmine retorted, quite excited "No! That's not what I proved here. That's totally not what I proved here! I was relying on it" [144].

At this point, it seems that Jasmine was quite frustrated. She was still convinced her proof was correct, but was not able to follow the instructor's counter arguments. The instructor tried explicating a mathematical meta-rule "so look... like, every claim you make here requires clarification". Yet Jasmine did not understand this simply as a meta-rule of mathematical proving. Rather, she interpreted it as a comment undermining her identity as a successful student, as

could be seen in her short, high laugh and her retort "you said I think weirdly, so this is my weird thinking" [154].

From this point on, most of the communication in the class revolved around identity issues. The instructor, trying to alleviate Jasmine apparent distress, said "No, I don't have any problem (with it), it could be true and it's possible you'll find a new claim and write a paper about it" [156]. Jasmine replied cynically "yeah, sure... physics maybe, math no" [159]. Other classmates started giggling and immediately Jasmine's attention turned to them (though they claimed not to have laughed at her). The only mathematical claim that Jasmine still tried to make was that 1,814,401 (by using N=10 in (N!+2)/2) is a prime number. She said so quite excitedly, sticking her calculator in the face of her classmates. Here, not only was she not acting according to mathematical meta-rules of proof, Jasmine was also making a wrong assertion about the mathematical object (1,814,401 is not a prime).

There was no resolution to this argument. The instructor tried to encourage Jasmine to re-enter the mathematical discussion and re-state her proof but Jasmine gave up stating that "she's tired" and that she got "totally confused".

Aftermath

After the session was over, during a casual conversation with the instructor and other fellow students, Jasmine remarked: "I think you embarrassed me in front of the whole class, I stood helpless". To explain her discontent with the instructor's conduct she said to another student "I'm standing at the board, proving a problem, and every second she's interfering with it.... But it was true too. I proved it. I proved it completely and you (instructor) didn't believe (it)!" [9]. It is clear from this description that Jasmine's framing of the situation was very different than that of her instructor or of any experienced participant in the mathematical community. While Jasmine saw counter-arguments as "interferences" that unnecessarily "confused" and embarrassed her, such counter-arguments and rigorous search for the veracity of every claim are the hallmark of mathematical argumentation. Expecting the instructor "to believe" her simply because she said she proved it was another signal that Jasmine's framing was misaligned with that of her instructor.

DISCUSSION AND CONCLUSIONS

In this report we took a close look at an episode of high level mathematical argumentation coupled with intense emotional expressions. Our goal was to examine what may be learned from participants' emotional reactions about their framing of the situation, or the meta-rules of mathematical discussion they were following. The analysis revealed that the interaction included three types of communication failures. One was mathematical, at an object level. Jasmine and her instructor talked differently about "prime numbers" and about the objects signified by a "P". Communicational failures resulting from interlocutors referring to different mathematical objects despite using similar visual mediators or words, are a common cause for ineffective teaching-learning interactions (Heyd-Metzuyanim & Sfard, 2012), yet they do not necessarily involve frustration or embarrassment.

Another failure of communication was at the meta-level of mathematical discourse and related to what "proving" was all about. Jasmine saw "proof" as a process where one outlines her reasoning, not necessarily justifies every step. Her instructor followed the mathematical community's meta-rules according to which every claim has to be examined by searching for counter-examples and by following its' implications. Jasmine's difficulties with proof are not unique. Much literature shows that students often do not understand the role of examples and counterexamples in the process of proving. Such misalignments of meta-mathematical rules have a much stronger potential of producing frustration and heightened emotional expressions, if only for the reason that participants usually are not aware of them and therefore do not have access to the reasons that their communication fails.

Finally, we saw here conflict at the level of meta-rules of identification, or what certain actions and re-actions say about the *identity* of interlocutors. While Jasmine saw any disclaiming of her proof as a cause for embarrassment, and disruptions as mistrust in her ability, the instructor saw the process of making arguments and counter-arguments as a purely mathematical activity without any identifying content attached to it. Such discrepancy with regard to the identity stories that should be constructed as a result of a certain activity is, we conjecture, the most

potent fuel for intense emotional agitation that may lead to failure in communication.

Yet, despite the distress this particular episode caused for Jasmine, it was probably an important and useful step towards the ultimate goal of initiating her and her fellow classmates into the mathematical community. In this summer camp, such interactions (though usually not so emotionally intense) were quite common. As could be seen here, the instruction in the summer camp was quite different than in school settings and resembled much more the process of apprenticeship (Barab & Hay, 2001). One of the main indications for this could be seen in the role that the instructor took, that of an equal (though somewhat more experienced) participant in the problem solving process. The importance of such a non-authoritative stance has been noted by many. However, in this setting it seems this role was truly authentic, probably because the problems dealt with were far from being trivial even for the instructor.

Finally, we wish to point to the need, which receives only a partial and very preliminary response in our study, to study gifted or extremely high achieving students' learning from a socio-cultural perspective. Such studies might be able to point to the processes that accelerate these students' learning, thus weakening prevalent notions about mathematical giftedness being a purely biological, permanent and individual trait. Furthermore, they can deepen our understanding about how to enhance instruction in schools with normally achieving students.

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