The interplay of language and objects in the mathematics classroom

Marei Fetzer, Kerstin Tiedemann

The interplay of language and objects in the mathematics classroom

Marei Fetzer[1] and Kerstin Tiedemann[2]

1 Goethe-University of Frankfurt, Frankfurt, Germany, fetzer@math.uni-frankfurt.de

2 University of Cologne, Cologne, Germany

Language appears to be a crucial aspect in mathematical learning processes. At the same time, objects like artefacts or other didactical tools play an important role in those processes. For that reason, we are interested in the interplay of both: We focus on the association of mathematical language and objects in classroom interaction. How does mathematical language vary in relation to objects and their participating in mathematical learning processes? Based on empirical examples, three different forms of interplay are presented.

Keywords: Objects, association, orality, literality.

INTRODUCTION

There is no doubt that mathematics teaching and learning are closely connected to aspects of language. Language is a central learning medium in every class and a content of learning, too. In some cases, it turns out to be a barrier to learning. Empirical research in mathematics education is familiar with research projects about language and mathematics learning (see TWG "Mathematics and language" at CERME over the past years). However, our picture of interactions in mathematics classrooms will be incomplete if we solely focus on oral or written language and, thereby, solely on human beings. There are not only students and teachers who contribute to classroom interaction. Material objects like manipulatives, rulers or diagrams influence the course of action as well. They make a difference in mathematical learning processes. Meanwhile, a sociological perspective that accepts objects as actors in the course of classroom interaction is only on the rise (see Fetzer, 2013, 2015; Kalthoff & Röhl, 2011). Latour introduces a sociology of objects that accepts objects as well as human beings as actors and participants in the emergence of social reality. His actor network theory offers us a new perspective on the interplay of language and objects in the mathematics classrooms (Latour, 2005).

LANGUAGE BETWEEN ORALITY AND LITERALITY

When talking about language in classrooms, researchers often need to specify what kind of language they actually mean. In this regard, Koch and Oesterreicher (1985) offer a helpful distinction that focuses on two fundamental aspects of language: on the form of language and on its function. Firstly, they differentiate the medium of language. Thus, an utterance can be phonic or graphic. For the context of this paper, we only focus on phonic language on face-to-face interactions in mathematics classrooms. Secondly, they identify two different conceptions that an utterance can have. It can be conceptually oral or conceptually written. This affects the question of communication strategies used. More precisely, conceptually oral language is often used when interlocutors are directly related and can refer to a given situation. This is what, consequently, gives a specific form to the language. For example, at any time, the interlocutors can ask questions of understanding, show emotions and influence the course of the interaction. For that reason, sentences may be short and even incomplete. Referring to the given situation, the speakers use deictic expressions and gestures. Thus, orality is characterized by interlocutors who spontaneously negotiate their roles and the course of their interaction. Koch and Oesterreicher call this a language of nearness. Examples are a conversation in the family (medially oral) or a chat among friends (medially written). In contrast, conceptually written language is used when the interlocutors are not necessarily in direct relation and the processes of speech production and speech reception might be separated from each other. Thus, aspects of the situational and cultural context have to

be made explicit. As a consequence, sentences are longer and more complex. For example, the writer forms main clauses, but also subordinate clauses to express the relations he wants to inform about. Moreover, he uses more specific terms, e.g. mathematical terms, to be precise and explicit. Koch and Oesterreicher call this a language of distance. Examples are a text of law (medially written) or a scientific lecture (medially oral).

Against this background, one can classify the registers of everyday language and academic or technical language (see Cummins, 2008; Duarte, 2011; Gogolin 2009). Halliday (1985, p. 29) describes a register as "a variety of language, corresponding to a variety of situation". According to Halliday, the use of the term 'register' points to the assumption that individuals usually adapt their use of language to a given situation. Thus, the register of everyday language is rather conceptually oral, irrespective of its medium. It has to fulfil the function of a fast and unproblematic communication in our everyday life. As such, the oral language may be, at any time, supported by gestures or by reference to a context. Words do not have to be clearly defined, sentences may be short or even incomplete. In contrast, the register of academic language is conceptually written, again irrespective of its medium. In academic contexts, language should be as explicit and precise as possible and intelligible without any reference to a specific situation. For that reason, words have to be well defined; sentences might be complex in order to reflect the relations that the author wants to talk about.

REASSEMBLING THE SOCIAL

Latour goes beyond the traditional understanding of the social. His Actor Network Theory (ANT) is a radical change of perspectives proposing a sociology of objects (Latour, 2005). Latour extends the list of actors assembled as participants fundamentally. He does not only accept humans as participants in the course of action. Instead, he integrates and gathers all sorts of actors. "Any thing that does modify a state of affairs by making a difference is an actor" (2005, p. 71). Following Latour, objects participate in the emergence of social reality. Consequentially, Latour recommends a broader understanding of agency. "Objects too have agency" (2005, p. 63), and appear associable with one another, but only momentarily. They "assemble" (2005, p. 12) as actor entities one moment and combine in new associations the next minute. Accepting objects as participants in the course of action, Latour gives up the idea of stable and pre-defined associations and actor-entities. He reassembles the social.

Looking through Latour's sociological lenses, not only the traditional understanding of agency has to be re-defined, but also the notion of action. Objects participate in the course of action and take effect. However, it can be noticed that their mode of action is different from the way human participants contribute to the social interaction.

Mathematics education has to deal with all sorts of (material) objects, didactical tools, artefacts and manipulatives, diagrams and signs. All those objects leave their traces in the emergence of mathematical learning processes and take part in the course of action. Mathematical learning appears to be closely connected to objects and non-human things. Even if Latour himself does not suggest any methods of empirical analysis, Latour's approach proves to be a fruitful background theory for empirical research in mathematics education (see Fetzer, 2013, 2014). Accepting objects as participants in the course of action and following the idea of objects having agency helps us to get a better understanding of mathematical learning processes.

METHODOLOGICAL BASES

The goal of our research is to analyse the relationship between language and objects in everyday mathematics lessons. Probably in every classroom, you can gain insights referring to that topic. To start, we have focused on primary classrooms. However, it was not important to us to observe specific lessons, but rather a wide range of occurring mathematics lessons. Therefore, we observed several mathematics lessons in three different German primary schools. On the bases of videos of the whole lessons, we filtered out those scenes in which humans as well as objects participated. Those scenes were transcribed and became objects of analysis. In order to include a wide spectrum of scenes, we distinguished episodes with and without a teacher.

To get access to interaction processes in mathematics classrooms, our analyses are of a reconstructive manner. They are analyses of interaction (see Cobb & Bauersfeld, 1995). This method refers to the inter-

actional theory of learning and is based on the ethnomethodological conversation analysis (see Sacks, 1996). This method was devised by a working group directed by Bauersfeld. In contrast to conversation analysis, it focuses on the thematic development of a given face-to-face interaction rather than on its structural development. For that reason, it is especially suitable for our research because it allows us to analyse the relationship between language and objects while teachers and students negotiate mathematical meaning. However, reconstructing empirically the way objects participate in learning processes remains unaccustomed. Objects' contributions to classroom interaction become accountable in the process of interweaving. As soon as object-actors become associated with other actors they enter the course of action. Their traces render perceivable and can be captured by a turn by turn analysis (see Fetzer, 2013; Sacks, 1996).

To illustrate our results, we selected four strongly contrasting scenes. 'Diagonal' is a teacher-orientated scene. In contrast, both '1000 dots' scenes are group work situations without a teacher and show an interesting change in the relationship between language and objects. In addition, we chose the very short example 'party hat' in which the object is not present at all, but still a point of reference.

EMPIRICAL EXAMPLE I: 'Diagonal'

The following scene is taken from a lesson about the hundred field. The teacher and the students start by repeating what they already know about the hundred field. Thus, a student says that a column „goes from top to bottom or from bottom to top". This utterance is confirmed by the teacher. Then, another student (Danis) puts up his hand and is asked, but the teacher does not pay attention to him. The scene starts when the teacher turns back to Danis. On the mentioned hundred field, no numbers can be seen. They are all covered by coloured squares, the upper half is red, the lower one is blue. In the transcript, the squares are named by the corresponding numbers in order to facilitate the reading.

26 Teacher: [...] How does the diagonal run?

27 Danis and 100 field: *[going to the blackboard:]* Well, like this.
[on the 100-field from the 60 to the 96] Here for example. Then, I move like this.

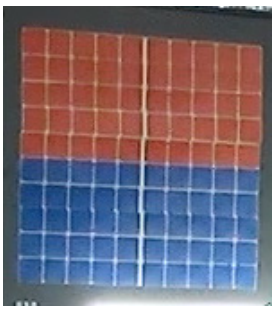

Figure 1: Material of the hundred field

28 Teacher: Can you also explain that with words? Now, you have already shown that to us.

29 Danis: *[sits down again]* Well, that works for example like this. .. that I am atthe ten and the 91. I can because the corner and the diagogal runs simply [*moving his finger in front of himself from top right to bottom left]* like this. .. Dialogal. .. But it can't go up and it can't go down or right or left.
[Comment: In German, Danis says "diagogal" and "dialogal" which are, like in English, not the correct words.]

In this scene, Danis and the teacher pick out the diagonal as a central theme. As the teacher is talking about "the" diagonal, one could think that there is only *one*. In contrast, Danis shows two different diagonals, one from 60 to 96 and another one from 10 to 91. One can only guess what his definition of a diagonal might be, but one can see that he identifies more than one diagonal on the hundred field. His two references to a diagonal show a difference concerning our research question. In the first one, Danis points at the hundred field with his finger and, thereby, integrates it into the discourse as a participant. In connection with the object, Danis' utterance gets intelligible. Thus, one can say that the object changes its status and becomes an active participant in the discourse. Danis and the hundred field get interwoven. With the object completing Danis' utterance, the boy's language gets reduced and deictic ("like this", "here", 27). Taking over the turn, the hundred field allows Danis to use an easier, more oral language. He only forms main clauses that are not clear without reference to the concrete situation including the hundred field.

In the second part of the scene, we can reconstruct the opposite. The teacher explicitly asks Danis to give his description in words. Thus, he is confronted with the task to describe the same mathematical phenomenon, but without any direct reference to the object (and

maybe even without gestures which do not belong to the category of words either). This interpretation is confirmed when Danis goes back to his chair, sits down again and, thereby, distances himself from the hundred field. Now, the object is no longer a participant of the discourse. Instead, Danis develops a second description of a diagonal solely by the use of words and a gesture. One can see that excluding the object from the course of action means to exclude its actions. Now, Danis has to take over again: He has to compensate the missing object by being more specific in his language. For example, he now names the two "corners" of the figure whose diagonal he is talking about. Furthermore, he starts forming more complex sentences ("because the corner...", "... and...", 29). In summary, one can state that the language is more orientated towards written language.

EMPIRICAL EXAMPLE II: '1000 dots'

In contrast to the first scene, there is no teacher participating in the scene '1000 dots'. Two girls, Martina and Sonja, are sitting at a table. On the table, there is a sheet with the task 'Share 1000 dots fair-mindedly between three children', a pair of scissors and a box containing 100-dots-cards (small cards with each showing 100 dots arranged in lines of ten). First, the two girls try to figure out the task on their own. Once they get a leftover of ten dots they get stuck. This is when the selected scene starts.

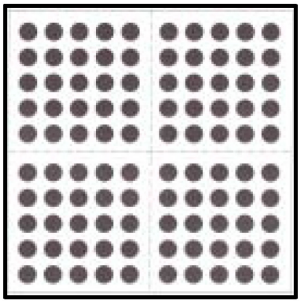

Figure 2: Material of dots-cards

167 Martina and cards: *[reaches with her left hand for the box with the 100-dots-cards]* Shall we make use of them?

Figure 3: Box with the dots-cards

168 Sonja: *[looking at the cards in Martina's hand]* Yes.

169 Martina, box, cards: *[takes the pile of cards out of the box and pushes the box away]*

Figure 4: Student in interaction with the material

170 Sonja: Cause it's difficult, isn't it?

171 Martina and cards: OK. *[puts cards from her hand down on the table like sharing out a deck of cards]* One, two, three.

As soon as the girls get stuck in their solving process, they invite the 100-dots-cards to participate ('cause it's difficult' <170>). These object participants breathe new life into the solving process. Assembling with the girls and the table, the cards embody (at least) one fundamental intermediate step in the solving process: instead of just representing the situation, they *are* the 1000 dots that have to be shared between three children. Their lasting quality contributes to a better orientation within the social solving process. Being arranged in a pile of ten cards they take over the part of a reliable basis. Thus, one can see that objects are not only useful for expressing one's own ideas, but also for developing new ideas that can push the solution process forward. The girls' language changes once the object-actors enter the stage as participants. The association of cards and girls can be reconstructed in the reduced character of the chosen language: short utterances like "of them" <167> or "one, two three" <171> intertwine with the objects and their manifest and structuring message.

Some minutes later, the girls approach a mathematical result: 999 dots lie there separated on three piles. While Martina is cutting the last dot in pieces, Sonja is worrying about the correct result.

917 Sonja Yes, but how much *do* the little ones *count?* How *much?*

918 Martina, pieces, scissors *[laying a little piece of her last cutting action on the middle pile]* A fourth, half of a fourth. *[laying another little piece on the right pile]*

While the solving process is coming to an end on the level of objects, Sonja is worrying about an appropriate language for what the girls can see in front of them (917). The non-human participant was irreplaceable in the solving process. But, subsequently, in order to fix the result and present it to others, mathematical language gets relevant. The girls obviously do not regard the task as completed until they can express their result by means of language, too. Thus, mathematics as a social practice needs a precise language to fix, to communicate and to discuss its results. While the language was relieved during the intensive search for a solution by means of the available object-participants, it now gets more demanding again. Martina forms a description of what the "little ones" count which is comprehensible regardless of the concrete objects. Her language develops from conceptually oral to conceptually written forms. She expresses a mathematically *new idea*, the idea of fraction. By this step in their work, the girls facilitate a precise mathematical communication with others, e.g. their classmates.

Considering both succeeding scenes, they illustrate the development from an orally orientated language in the first scene and the beginning of scene two towards a rather written conception of language including mathematical terms ("a fourth, half of a fourth"). In both examples the object-actors can be reconstructed to be participants. However, their mode of action varies within the emergence of the solving process. At the beginning, they contribute their structuring and lasting quality to the course of interaction. They invite the girls to share the dots out right to the end. Thus, the cards and dots prompt Sonja and Martina to cut the last dot into bits and pieces. Later lying on the table, the mini-objects release the strong requirement for a precise term to label them. They call for language, they demand for mathematical description on a rather written level.

EMPIRICAL EXAMPLE III: 'Party hat'

In the last scene, there is actually no object participating at all. It is only present in the mind of the human participants. In the lesson, the students have worked on their own around the following task:

> Which of the two flatplans fits the three-dimensional figure?
> Why does one of the flatplans match and the other one does not?

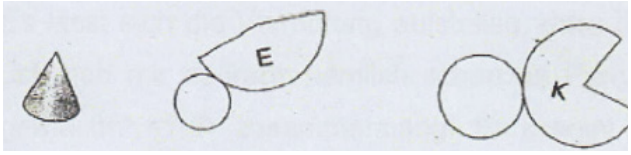

Figure 5: Two flatplans and a three-dimensional figure

Later the students compare their results in the whole class. In the selected scene, it is Dominik's turn to present his findings.

1	Dominik:	K works because the gap is smaller. Cause I have already made myself a party hat of that, you know.
2	Some pupils	*[whisper and nod]*

Dominik puts forward an empirical argument that relies on (his own) experience. He creates the context for his argument exclusively on a phonic level with spoken words. By talking about the party hat, Dominik includes the hat as an actor within the course of action. As such the party hat seems to have the power to convince the other children of the verbally presented decision 'K works'. The party hat is not present at all, but it is replaced by language. In other words, the party hat solely takes part in the interaction in form of its linguistic representative.

EMPIRICAL RESULTS AND CONCLUSION

Objects do play an important role in mathematical learning processes. They may become participants in the course of action and contribute to the emergence of social classroom reality. In our research, we basically differ between two learning situations: Those situations, when objects are to be seen as participants in the course of classroom interaction, and those, when human participants act without reference to objects. In the context of our study the first case might be called 'Language and objects' and the second case can be characterised as 'Language without objects'.

Concerning situations we call 'Language and objects', there are two cases to be observed. Sometimes, object participants and human actors *assemble in their actions*. In these cases, *objects take over part of the turn*. Action is no longer attributed to one single actor but to actor entities, actions are experienced as combined actions. If objects and students assemble in their actions and humans have to take over only part of the turn, language proves to be rather conceptually oral (compare the scene 'Diagonal'). However, the assem-

bled action appears to be 'complete'. Other participants can reconstruct the meaning that emerges. This is the crucial point. If objects participate in the course of mathematical action and take over part of the turn, they thus relieve students on the level of language. This might be an initial opportunity for children with poor language performance to act mathematically.

In other situations, *human actors and objects interact in turns*. In these cases, objects take over whole turns, and students react to those turns. If human actors invite the objects to be a part of the discourse, objects can challenge the students to find a new language (compare the second part of the scene '1000 dots'). We could observe that in these situations language tends to be rather conceptually written. This aspect is of special interest from a didactical point of view. Objects cannot only relieve students on the language level, but also challenge them. They can be 'supporters' in the development of mathematical thinking and of a precise mathematical language.

When looking at situations that we call 'Language without objects', we observe that associations between objects and humans dissolve. Students are obliged to take over each turn on their own. Accordingly, they become responsible for the idea and for the formulation of the intended utterance (see the second part of the scene 'Diagonal'). For that reason, students have to be as explicit as possible in their language. They might use well-defined mathematical terms and rather complex sentences. They might give many explanations about the context of culture and of content in order to be clear about the given topic. Language turns out to be conceptually written.

Summing up, we see conceptually oral language when students and objects assemble in their actions. In contrast, we find conceptually written language, when associations between human and non-human actors dissolve. Finally, we observe a development of language from conceptually oral to rather conceptually written forms, when human actors and objects interact in turns. These situations of language development can be seen as the situations of mathematical learning. Maybe, further research can show how the interplay of language and objects depends on the kind of material that is part of the discourse.

REFERENCES

Cobb, P., & Bauersfeld, H. (Eds.) (1995). *The emergence of mathematical meaning: interaction in classroom cultures.* Hillsdale, NJ: Lawrence Erlbaum.

Cummins, J. (2008). BICS and CALP – Empirical and theoretical status of the distinction. In B. Street & N. H. Hornberger (Eds.), *Encyclopedia of language and education* (pp. 71–83). New York: Springer Science+Business Media LLC.

Duarte, J. (2011). *Bilingual language proficiency: a comparative study*. Münster, Germany: Waxmann.

Fetzer, M. (2015). Mit Objekten rechnen: Mit Latour auf den Spuren von Materialien im Mathematikunterricht. In T. Alkemeyer, H. Kalthoff, & M. Rieger-Ladich (Eds.), *Bildungspraktiken. Körper- Räume- Artefakte*. Weilerswist.

Fetzer, M. (2013). Counting on objects in mathematical learning processes. Network theory and networking theories. In B. Ubuz, C. Haser, & M. A. Mariotti (Eds.), *Proceedings of the 8th Congress of the European Society for Research in Mathematics Education* (pp. 2800–2809). Ankara: METU.

Gogolin, I. (2009). Zweisprachigkeit und die Entwicklung bildungssprachlicher Fähigkeiten. In I. Gogolin & U. Neumann (Eds.), *Streitfall Zweisprachigkeit– The bilingualism controversy* (pp. 263–280). Wiesbaden, Germany: VS.

Halliday, M. A. K. (1985). Part A. In M. A. K. Halliday & R. Hasan (Eds.), *Language, context, and text: aspects of language in a social-semiotic perspective* (pp. 3–51). Oxford: OUP.

Kalthoff, H., & Röhl, T. (2011). Interactivity and interobjectivity. Knowledge objects and discourse in class. *Human Studies, 34*(4), 451–469.

Koch, P., & Oesterreicher, W. (1985). Sprache der Nähe– Sprache der Distanz. Mündlichkeit und Schriftlichkeit im Spannungsfeld von Sprachtheorie und Sprachgeschichte. *Romanistisches Jahrbuch, 36*, 15–43.

Latour, B. (2005). *Reassembling the social. An introduction to actor-network-theory.* Oxford: Oxford University Press.

Sacks, H. (1996). *Lectures on conversation.* Cornwall, UK: Blackwell Publishers.