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How communicative teaching strategies create opportunities for mathematics learning

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The overall interest in this study is actions of the diverse participants in the mathematics primary classroom. More specifically, attention is put on typical communication strategies that teachers and students, in our research context, use during mathematics lessons. We present some examples with classroom data to illustrate how the strategies are used and what opportunities to learn mathematics are offered as an outcome from the implemented actions by teachers and students.

Keywords: Mathematics education, communication strategies, primary classroom.

INTRODUCTION

The language and how it is used is of significant importance for what is possible to learn in all education (Säljö, 2000). The way the teacher manages to direct the communication in the classroom and how students and the teacher talk to each other is crucial for students' learning in terms of mathematical content as well as how they view themselves as mathematical performers (Franke, Kazemi, & Battey, 2007). The main interest in this study is communicative teaching strategies that are used in the mathematics classroom and what these strategies can offer in terms of students' opportunity to learn mathematics.

The findings in this paper extend the results of a larger study by Engvall (2013), to which Samuelsson and Forslund Frykedal contributed importantly. That study discusses teachers' and students' actions in the mathematics classroom in primary school when written calculation methods are in focus. The theoretical framework is built on activity theory (AT), which here refers to Engeström's (1987) model of the activity system. According to Rezat and Sträßer (2012) this theory is one among other socio-cultural or semiotic theories that has been shown to be successful in math-

ematics education research. In AT, the theory about tool mediation is a mainstay (Cole & Engeström, 1993). It is pointed out that any meeting with mathematics is mediated through either material tools as for example textbooks and rulers or non-physical tools such as language, visual representations and gestures (Rezat & Sträßer, 2012). A major feature in Engeström's model is that the context has been extended with some additional mediating factors such as rules, community and division of labour in order to allow better analysis (Engeström; 1987; Goodchild, 2001; Rezat & Sträßer, 2012). Yet, in this paper, we focus on verbal language as it is used by teachers and students in some classes during mathematics lessons in primary school, or in other words communication strategies. There are two main questions to be elaborated here:

- (1) What kind of communication strategies do teachers and students use in the mathematics classroom?
- (2) What do different communicative teaching strategies have to offer the students in terms of opportunity to learn mathematics?

Communicative teaching strategies in the mathematics classroom

In this paragraph some characteristic communicative teaching strategies in mathematics classrooms will be presented. In particular, we pay attention to the language used for teaching, i.e. the kind of language used to demonstrate, explain and exemplify mathematical connections, drawing from Löwing's (2000) instrument for analysis of communication in the mathematics classroom. Löwing makes a distinction between formal and informal language for teaching, where the latter is used e.g. together with manipulatives or when an everyday situation is taken as a starting point for explaining a calculation. In this presentation we focus on the formal language since, according to Setati and

Adler (2000), one of the challenges in mathematics education is to get the students to gradually change from informal to the formal language, which is characteristic for mathematical activities.

According to Löwing (2000), two main categories of spoken language strategies can be distinguished within the formal teaching language, namely *descriptive language* and *conceptual language*. By using descriptive language, teacher and students put attention on procedures, as e.g. when teacher and students together are performing calculations like $53+25$ on the board and somebody describes adding the tens by saying “five plus two”, which means that the tens are treated verbally as if they were ones. In this case the language is representing the calculation procedure. Conceptual language, on the other hand, is visible when mathematical words with a specific meaning, e.g. “five tens plus two tens” are used in order to explain and not only describe a step of a calculation like this.

The following strategies, presented in the community of international researchers in mathematics education (Mercer, 1995; O'Connor & Michaels, 1993), can be associated with the category of conceptual language. Our first strategy to be presented here is *key questions*. Researchers have demonstrated that teachers use key questions or certain phrases to create structures for supporting students' learning (e.g., Mercer, 1995). Some students pick up these questions or other common phrases from the teacher's spoken language and use them as a kind of support when performing his or her tasks.

Another distinctive strategy connected to the second category is *revoicing*, since revoicing directs the attention towards thinking and conceptual understanding (O'Connor & Michaels, 1993). Revoicing can actually be defined as at least three communicative teaching strategies: (a) *repeating*, (b) *rephrasing* and (c) *recasting*.

With reference to Mercer (1995), repeating can be used for directing the students' attention towards something specific in a student's answer/expression/utterance so this can support the students' learning. When a teacher instead rephrases a student's utterance the class will get another chance to grip what just has been said but in a version that is more consistent with what the teacher wants to point out. Finally, if a student has expressed something almost non understandable in class, this can be further developed by

the teacher's recasting so the meaning thereby can be explained for the students (Mercer, 1995; O'Connor & Michaels, 1993).

The content in this section is interwoven with our first research question regarding the use of communication strategies in the mathematics classroom. Our second question is aimed at paying attention to the idea of possible learning, which therefore will be followed up in the next section. This theoretical overview will be concluded by a short presentation concerning different aspects of mathematical knowledge.

Opportunities to learn and mathematical knowledge

One of the most firmly established links between teaching and learning is the idea of “opportunity to learn” (Hiebert & Grouws, 2007). This means that “the students learn best what they have the most opportunity to learn” (p. 378). Although it is impossible to predict learning outcomes in mathematics based on the use of a specific teaching strategy it is still likely to reason about students' opportunity to learn with regard to teachers' and students' strategies during mathematics lessons (Hiebert & Grouws, 2007). Further, this means that it is possible to discuss what teachers' and students' verbal action can offer when it comes to opportunity to learn during mathematics lessons. This is a core message in this paper.

In various frameworks (Kilpatrick et al., 2001; Niss, 2003) mathematical knowledge is presented as something multifaceted which comprises different types of “tightly interwoven” competences. Procedural knowledge, conceptual understanding, communication, reasoning and strategic competence are some examples. In Sweden, as in other countries, this approach has influenced the curriculum in mathematics education. This means that students during mathematics lessons will get more opportunities to be involved in activities of communicating together with thinking and understanding instead of putting big efforts into skill practice and remembering procedures (Anghileri, 2001; Clarke, 2006). For more than 20 years, this approach has also given rise to a common trend for the teaching of arithmetic, which is an essential part of the mathematics content in primary school. Before the students get any instructions on traditional algorithms they will focus on strategies for mental calculation. From a Swedish point of view this means that traditional algorithms gradually have been replaced in the textbooks

by other written calculation methods. For example the addition $56 + 28$ can be calculated as follows, $56 + 28 = 70 + 14 = 84$. The “intermediate” $70 + 14$ is written down in order to facilitate the mental work.

In this paper, we focus on teachers’ and students’ communication strategies with respect to procedural knowledge and conceptual understanding when the content is written calculation methods for addition and subtraction.

METHOD

The collected data in this observation study consisted of video-recorded mathematics lessons in five different classrooms. Besides, an audio recorder has been used together with the video camera in order to get a more complete sound reproduction. In addition to the recorded material there were also field notes. The participating classes belonged to four schools with students from areas with comparable socioeconomic status. The number of students in each class was 24–25. Every teacher had the primary responsibility for the mathematics education in her/his class. Most of the teachers did also teach all lessons in almost every subject in the class. Collection of data started during the spring when the students were in second grade and continued during the autumn, when the students were in third grade. The video-recorded material comprises a total of 24 lessons. In all the lessons one and the same content has been in focus, i.e. written calculating methods for addition and subtraction with numbers exceeding 20.

The collected research material has been analysed in two steps. The first step, with inspiration from Braun and Clarke (2006), can be described as empirically oriented and thematic. This part of the analysis has been carried out in order to discern phenomena that form patterns in the material and thus point to the characteristic actions in mathematics classrooms. The result derived from the first step of the analysis has then been used as the basis for the second step, where the analysing tool, inspired by Engeström (1987), has guided the analysing process. Engeström’s model for activity system gives the researcher opportunity to analyse not only actions mediated by tools, but also by other mediating factors such as rules, community and division of labour. Yet, the focus in this paper is on the outcome of the analysis regarding a non-physical tool,

spoken language, or more generally, communicative teaching strategies in the mathematics classroom.

ANALYSIS AND FINDINGS

In the following presentation we will report some typical communicative teaching strategies that have appeared in the empirical material. These will be presented in relation to descriptive and conceptual language. Thus, here we put attention to just two aspects of mathematical knowledge, procedural and conceptual knowledge.

The content taught in the classrooms in the study is, as already mentioned, written calculation strategies for subtraction. In the first example below, initially, the teacher invites a student to tell how to calculate the subtraction $55 - 21$. Almost at the same moment the teacher clarifies that the goal is to tell how to write the intermediate but the outcome of the subtraction is not going to be emphasized.

Transcript 1

- Teacher [WRITES $55 - 21 =$ ON THE BOARD] I don’t want to hear the outcome. I want to hear the intermediate...Peter!
- Peter (stud.) Fifty minus twenty makes thirty.
- Teacher [WRITES 30 AFTER $=$]
- Peter And the plus.
- Teacher [WRITES $+$]
- Peter And then it is five minus ten...four ... four...forty... [ADDRESSING THE TEACHER WHO WAITS A MOMENT BEFORE WRITING THE NEXT DIGIT], a four.
- Teacher [WRITES $4 =$] And that makes??
- Peter Err, thirty-four.
- Teacher [WRITES 34] Thirty-four. Exactly! Yes, it’s important to keep track of whether you have to use the tens or the ones.

From this sequence we notice that the dialogue is dominated by the number words together with some words which are representing other symbols. Thus, the student’s communication strategy can be defined as descriptive language, since it is mainly used for representing the different steps in the calculation which is performed by the student while the teacher is writing on the board. The student’s third reply indicates some uncertainty regarding the value of the digit one in 21. The teacher’s comments are very limited.

However, in the last lines the teacher mentions some fundamental concepts concerning number value but this does not seem to be aimed at clarifying the meaning of these concepts, rather to give the student an advice. Accordingly, in this example attention is directed towards the procedure.

The next transcript illustrates the first steps of a teacher's instruction to the whole class on how to calculate the subtraction 58–34. Together with the written subtraction on the board, the teacher uses manipulatives, especially adapted for putting on the board, representing the number of tens and ones.

Transcript 2

Teacher /.../ If we are going to calculate fifty-eight minus thirty-four, which numbers are we going to begin with? ... Jenny!

Jenny (stud.) The tens.

Teacher We begin with the tens. In fifty-eight there are five tens [MAKES A RED MARK RIGHT BELOW THE DIGIT 5], In thirty-four there are three tens [MAKES A RED MARK BELOW THE DIGIT 3]. Five tens minus three tens [POINTS AT THE DIGIT 5 AND THE DIGIT 3]... yes, here we have five [POINTS AT THE FIVE MANIPULATIVE TENS ON THE BOARD] and then we take away one, two, three [TAKES AWAY THREE TENS]. How many tens do we have left? ...Mika!

Mika (stud.) Twenty.

Teacher We have twenty left. Two tens or twenty. It's the same thing just different ways to say it. [WRITES 20 AFTER =]....Mm. Which numbers are we going to continue with now?

We can observe that the teacher clarifies the number of tens in each term, e.g. "in fifty-eight there are five tens". By that, the ten's names and the number names synonymous with each other appear at the same time, e.g. two tens and twenty. Here, the strategy of conceptual language is prominent.

In both sequences above there are examples illustrating how teachers use *revoicing* although they do it differently. This is particularly evident in the last teacher sentence in each transcript.

Excerpt from Transcript 1

Teacher [WRITES 4 =] And that makes??

Peter Err, thirty-four.

Teacher [WRITES 34] Thirty-four. Exactly! Yes, it's important to keep track of whether you have to use the tens or the ones.

Excerpt from Transcript 2

Teacher /.../ How many tens do we have left? ... Mika!

Mika (stud.) Twenty.

Teacher We have twenty left. Two tens or twenty. It's the same thing just different ways to say it. [WRITES 20 AFTER =]....Mm. Which numbers are we going to continue with now?

What these last teacher sentences have in common is that the teacher is *repeating the student's reply*. On the other hand, a clear distinction is visible when we compare the teachers' actions in the two examples. In the first example, besides repeating the student's reply, the teacher makes a comment without any further explanation while the teacher in the second example makes a typical rephrasing. This rephrasing, "Two tens or twenty. It's the same thing just different ways to say it", offers the possibility for the class to take part in what has been said, but in a version that is more in harmony with what the teacher wants to point out. In this situation attention is put on the meaning of tens and ones and what the digits are representing in terms of place value. Thus, in the second example conceptual knowledge is focused while that is not really the case in the first one.

In transcript 2 it is also illustrated that the teacher puts attention to the tens and ones, each in turn, by asking questions like "Which numbers are we going to begin with?" and "Which numbers are we going to continue with now?" We recognize this from research as *key questions*. These are typified by a seemingly procedural course of action. However, depending on how these questions are formulated, the repeating character offers opportunity to develop not only procedural but also conceptual knowledge. We can notice that the student in this example uses a place value word to reply on a key question. Therefore, this kind of questions has an important function in that they can help the students to make structures. It is not un-

usual that the students pick up the teacher's phrases and use them later on when performing similar tasks.

Finally, the following transcript represents the final part of a session when a class is performing the subtraction $45 - 22$ together with their teacher. They have already jointly agreed that the result of the performed subtraction of the tens is twenty. $45 - 22 = 20$ is now written on the board and when students are invited to perform the subtraction by the ones, the teacher gets two different suggestions from students about what to write next, either plus five or minus three. The teacher then repeats the question about how many ones there are left, which is the opening sentence in the transcript below. The way this question is formulated together with the teacher's illustrating the calculation by fingers, makes it almost impossible for the students to give anything else than a correct reply. The teacher's utterance in line four "Now you said minus three" is a comment to the student who just recently suggested that the teacher should write minus three to complete the intermediate.

Transcript 3

- Teacher How many ones have you left then? ... If you have five [SHOWS FIVE FINGERS], and you are going to take away two?
- Student Three.
- Teacher You have three ones left. Now you said minus three, but if you have something left, what sign do you think you should use, when you have something left?
- Student Plus.
- Teacher Plus [WRITES + TO THE RIGHT OF 20]. It sounds like plus is a very suitable word when you have something left, doesn't it? ...If something was missing you shouldn't use it, should you?

This transcript makes visible how the teacher uses a specific expression "have left" in order to put the students on the right track about which sign they are supposed to use when writing the intermediate in performing written subtraction calculations. In the bottom line we can also notice that the teacher makes a contrast by using the word "missing" in order to demonstrate an association with the minus sign. This strategy we call *key words*. This strategy resembles other strategies where focus is on remembering rules, which makes it closely related to *descriptive*

language and consequently, attention is put on procedural knowledge.

CONCLUSION

Even if mathematical knowledge is multifaceted, our concern in this paper has been concerned with opportunities to learn in terms of conceptual and procedural knowledge. We have presented a number of communicative teaching strategies that teachers and students use in the mathematics classroom. The two categories, descriptive and conceptual language, are represented to various extents in the five classrooms in the study. Depending on which verbal strategies teachers and students are using, different opportunities to learn are offered to students.

With this paper we want to illustrate teachers' and students' actions when they use these communication strategies. The result indicates that strategies that direct attention to procedures are also focusing on conceptual knowledge and vice versa. As an example, when the teacher repeats a student's reply more or less attention can be directed to concepts depending on what type of formal language the students used. In a classroom where descriptive language is frequent, it is more likely that the strategy repeating a student's reply will put emphasis on procedures. However, this is in conflict with O'Connor & Michaels (1993) who claim that revoicing directs the attention towards concepts and thinking. Furthermore, using another strategy such as key words can be characterized as procedural. On the other hand, the key words carry some meaning and thereby this strategy might offer opportunities to develop conceptual understanding.

What has been presented above can also be illustrated in the following figure. The horizontal line expresses the communicative teaching strategies and the vertical line symbolizes opportunities for developing either procedural or conceptual language. The two bigger crosses illustrate that descriptive language offers possibilities to develop procedural knowledge while a similar relationship can be described regarding conceptual language and conceptual knowledge. Also, conceptual language strategies offer possibilities to develop procedural knowledge but to a lesser degree than from descriptive language strategies. A similar reasoning can be applied to the relationship between descriptive language and conceptual knowledge.

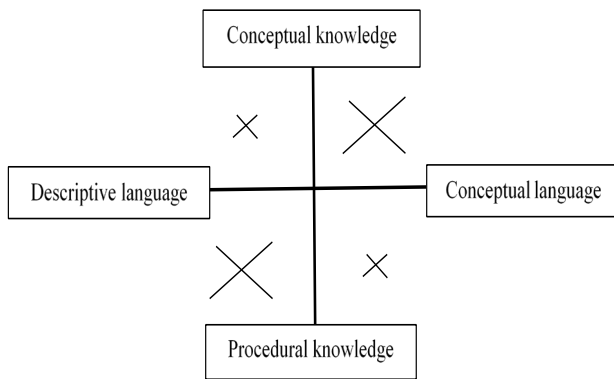


Figure 1: Communication strategies and opportunities for learning mathematics

This paper illuminates how teachers' and students' use of communication strategies influence opportunities to learn with respect to procedural and conceptual language in the mathematics classroom. It can be used for further discussions on pedagogical implications in teacher training programmes as well as in in-service teacher training.

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