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# Mathematics learning in whole class discussion: A design experiment

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*In this report, we examine the interacting use of whole class discussion and concrete manipulatives in the learning of geometry in a secondary mathematics classroom. We draw on our prior works on the generation and exploitation of learning opportunities in order to elaborate an example of opportunity for further reflection. In the analysis, two basic aspects are considered: (i) actions of collective argumentation and (ii) types of orchestration involved in the production of the students' learning. We illustrate the analysis through a moment of classroom talk in which a student is the initiator of an opportunity that leads to a situation of mathematics learning. We argue that the teaching activity is decisive in the joint exploitation of the opportunity.*

**Keywords:** Whole class discussion, students' mathematics learning, problem solving, collective argumentation, concrete materials.

## CONTEXT, QUESTION AND GOALS

The reported work needs to be situated in the context of a project strategy that includes the study of various mathematics lessons with different mathematical contents in use. All the work in the project is expected to contribute to the knowledge of mathematics learning by broadening the understanding of how mathematics learning opportunities may be created and exploited in classroom talk. Within this context, we address results from a research guided by the following question:

*What are the actions involved in the creation of mathematics learning opportunities in whole class discussion with problem solving and manipulatives?*

In the classroom for the conduction of the experiment, concrete manipulatives that could be physically handled by students were offered to explore and investi-

gate mathematical concepts and processes for finding solutions to geometry problems. The potential role of whole class discussion and problem solving with manipulatives in the generation of mathematics knowledge was a key assumption in the design of the three-lesson intervention. In particular, a concern to this work was that by examining the role of manipulatives as mediating artefacts in classroom talk, certain learning opportunities in mathematics might be better examined and characterised.

For the time devoted to whole class discussion in each lesson, the same procedure for the analysis was considered. We searched for moments of classroom talk where an approach to the resolution of the problem was being discussed. For each moment and when possible, we identified mathematics learning opportunities and related them to particular mathematical contents. What we present in this report is the analysis applied to one of the identified learning opportunities. Elsewhere (Ferrer, Morera, & Fortuny, 2014) we have detailed our procedure for the detection of learning opportunities and we have examined the role of the teaching activity. In what follows we outline our theoretical orientation, describe the experiment, provide our methods, and discuss our data, findings, and future directions.

## THEORETICAL PERSPECTIVES AND NOTIONS

A major problem in mathematics education research has to do with understanding and framing learning. Nevertheless, researchers in the field have reached important agreement on the fact that evidence of learning cannot be gathered in isolation, neither at the level of individuals nor at the level of groups (Sfard, 2001). Without denying the importance of the individual, social theories take the system of actions and practices as the starting and explanatory main component of learning.

In the line of social theories of mathematics learning (Goos, 2004), our work is placed within the tradition of design experiments in mathematics education research. As said by Cobb, Confrey, diSessa, Lehrer and Schauble (2003), this type of experiment aims at identifying and explaining successive patterns emerging from the study of connections between the students' learning and the classroom circumstances in which it is developed. For the conduction of an experiment, the phases are planning, implementation, evaluation and iteration. In this report, we refer to data coming from the first round of implementation of the planned teaching sessions.

As part of our project strategy, design experiments are supported to develop knowledge on mathematics learning, and in particular around mathematics learning opportunities. The notion of mathematics learning opportunities is central to our research as a way to link the social aspects of classroom activity to the students' development of mathematical ideas (Planas, 2014). In our analysis of practices that potentially foster learning opportunities during classroom talk, we give priority to actions of collective argumentation and types of orchestration, which actually are specific types of sequenced actions. To this respect, we plan design experiments in which conceptual and procedural forms of mathematics learning are expected to be facilitated by means of an interacting system involving inquiry-oriented tasks and pedagogical resources such as physical artefacts.


As claimed by Miranda and Adler (2010), there is little literature on the role and use of concrete manipulatives in the teaching and learning of mathematics. From the perspective of the teaching activity, it is argued by these authors that the presence of manip-

ulatives in the development of a task needs explicit and reasoned justification so that students are told the importance of using those resources. This is why we collaborated with the teacher to foster responsibility for a form of teaching activity that privileged the use of manipulatives not only during the time for group work but also during the presentation of the task and its discussion in whole-class talk.

### THE CLASSROOM EXPERIMENT

The experiment consisted of three lessons in a classroom of 12 years-old in a school of Barcelona, Catalonia-Spain. What was first selected was the teacher, on the basis of her expertise in teaching mathematics for several years and her active involvement in our research team. She was given in advance the sequence of three geometry problems and she made relevant contributions in order to adapt the wording and content to the particular group of students. The lesson dynamics was also negotiated with the teacher. The students were first asked to read the problem, to work in small groups, and finally to participate in a whole class discussion. During group work, the students were provided with problem-based concrete materials and had to produce written individual responses; after thirty minutes of group work, the teacher took the materials away in order to encourage finishing the responses. The teacher had the materials for manipulation in her guiding of the interaction with the students during whole class discussion. It was possible, and in fact it was promoted, to complete, revise or modify responses up to the end of the lesson. The students in this classroom were used to similar dynamics but the work with manipulatives was new to them.

**Packing glasses**



We have 12 glasses, each of them measuring 92mm (height) and 74mm (diameter of the major circle). We want to find the cheapest box for all glasses, that is, the box requiring the least material. In addition we want:

- The base of the box to be rectangular.
- All glasses to be facing up in the box.
- No glasses inside each other.

Under these conditions:

What are the minimum dimensions for the box to contain all the glasses?  
Which data will you give the shop owner to order the box?

Figure 1: The problem of the second lesson

'Packing glasses' (see Figure 1) was the problem for the second lesson. It was adapted from a problem created by the Millennium Mathematics Project at the University of Cambridge (for the original wording see <http://nrich.maths.org/880>). The problem was thought of as useful for dealing with early geometrical modelling and optimization of area and perimeter. Several approaches and resolutions are possible, as well as follow-up questions depending on the evolution of the students' talk. The teacher was asked to first introduce the problem and the material, and then to handle the final class discussion with attention given to the mathematical talk of the students. There were two main objects: glasses and boxes. All small groups were given 12 plastic glasses. In the facilitation of whole class discussion, the teacher had three scaled boxes that represented the three possible solutions (with glasses being aligned in one row of twelve, two rows of six, and three rows of four) and the box that represented the case for one glass (one row of one). It was assumed that the manipulation of glasses and boxes would help better understand the problem and the required optimization processes to solve it.

## METHODS OF A TWO-SIDED ANALYSIS

Lessons were video-taped and whole class discussions were transcribed. Each transcript was organized into shorter transcripts around moments of class discussion with students exploring an approach to the resolution of the problem. The difficulty of determining the exact turn on which the discussion of an approach started and finished, was addressed by including the turns that were dubious for some reason. On the other hand, participants commented on the same approach at different stages of the discussion; this is why the transcript of a moment did not necessarily consist of consecutive turns. In fact, the moment for illustration in this report is an example of non consecutive turns having been grouped together on the basis of the resolution strategy being under discussion. Its transcript stands for the explicit talk around the required quantity of material for any of the solution boxes.

The construction of transcripts was followed by the search for learning opportunities arising from the interaction among participants in classroom talk. Drawing on the notion of learning opportunity, special attention was paid to reactions of students that serve for clarification, exemplification, generation of new questions..., and which might be explained as

provoked by prior interventions of other participants in that lesson. In case of differing interpretations within the team, we went back to the videos until we agreed on a decision. Actions and reactions were initially associated with opportunities to participate and interact in classroom talk, and only when mathematical content was at focus, they were regarded as mathematics learning opportunities. It was during the observations and analyses conducted in other school settings (Ferrer, Morera, & Fortuny, 2014), that the relationship between opportunities to interact in classroom talk and opportunities of mathematics learning was decided as an effective way to approach the detection of mathematics learning opportunities.

## Actions of collective argumentation and types of orchestration

After having linked the transcript of a moment to a mathematics learning opportunity by reflecting on the mathematical contents in use, we split the analysis into two parts according to the characterisation of actions of collective argumentation, on the one hand, and types of orchestration, on the other. The selection of this two-sided analysis responds to the idea that the kinds of talk in the mathematics classroom can be interpreted in terms of the quality of the collective argumentation (Krummheuer, 2007) and the variety in the orchestration (Ferrer et al., 2014). Elsewhere (Planas & Morera, 2011) we have commented on collective argumentation among students in the construction of mathematics knowledge in two secondary classrooms.

The distinction of actions of argumentation was carried out at the micro-level 'within-the-moment'. All turns of talk were studied in order to decide whether mathematical reasons for particular statements were being provided. Further, we observed how the different contributions were taken up, either individually or collectively in public classroom talk. There may be a wide range of mathematical reasons depending on the complexity of the inquiry activity (Goos, 2004): from the mere description of an answer, a property or a fact, to the thoughtful proof of a conjecture. Mathematically wrong reasons in the context of the considered statement were not excluded as they helped to understand the coherence in the progression of mathematical talk. We drew on the idea that mathematics learning occurs in the coordinates of the combined potential movements between mathematically wrong and correct actions of collective argumentation in classroom interaction.

The distinction of types of orchestration was also carried out at the micro-level ‘within-the-moment’. We used the following six types: exploring the artefact, explaining through the artefact, connecting artefacts, discussing the artefact, discovering through the artefact and experimenting the instrument; and we added the type ‘replacing the artefact’ to refer to situations in which participants pointed to different artefacts (blackboard, boxes, applets, glasses...), but did not establish connections among them. The importance given to the use of artefacts in the broader project (for findings about mathematics learning in whole-class discussion with dynamic geometry software, see Ferrer et al., 2014) comes from our interpretation of who the relevant others in the interaction are. Classroom talk is developed in interaction with subjects and objects, and consequently the others are not always individuals, students or teachers, but may be represented by sorts of artefacts.

The two-sided analysis applied to moments in which learning opportunities had been identified became a powerful tool for the detection of mathematics learning. It allowed us to conclude that certain opportunities had been exploited into mathematics learning. To this respect, convincing evidence was found, either in classroom talk or in students’ written responses, of changes in the understanding of mathematical contents that had been involved in actions of argumentation in interaction with the orchestration of specific artefacts. Exhaustiveness in the identification of learning opportunities and learning was not attempted through the application of our two-sided analysis, and in fact it was in some cases deliberately avoided, since the extent to which mathematical talk had evolved in some turns was difficult to interpret.

### EXAMPLE OF LEARNING OPPORTUNITY

We have selected an example of mathematics learning opportunity where it can be acknowledged the role of a student, Aloma, in the introduction in classroom talk of ideas that lead to cohesive reasoning and progress in the resolution of the problem (see Figure 1). It is an example in which a diversity of actions of argumentation and types of orchestration are involved with respect to three main artefacts: blackboard, glasses and cardboard. We draw on the analysis of this example as evidence in support of the potential of the teaching activity in interaction with classroom talk, collective argumentation, and problem solving with

manipulatives. We begin by reproducing most of the transcript of the moment related to the opportunity (Roman numerals are included for organization of cross-referenced contents in Table 1):

- 1 Teacher: Come on, Aloma.
- 2 Aloma: A way for knowing it would be, for instance, to take into account that each glass, it touches up and down...
- 3 Teacher: That is, if we put each glass in an individual box [working on the blackboard], it would touch all four [pointing to the drawn lateral] walls; but in this one [indicating the drawn one row of twelve], the second glass would touch here and here [pointing to two drawn lateral walls] and those in the corner would touch three [pointing to three drawn lateral] walls.
- 4 Aloma: <sup>I</sup>This way we use plenty of cardboard, <sup>II</sup> but with the box of three per four we need less cardboard for the glasses in the corners.
- 5 Teacher: <sup>I</sup>This glass, for instance, it only touches one wall, while this one does not touch any [pointing to the box with three rows of four]. <sup>II</sup> Here we have built a model for the individual box with the real measures as said by Aloma. (...) Aloma’s reasoning is great! <sup>III</sup> She is trying to reduce all these walls as much as possible. Thus, if I want to build a box for the twelve glasses and place the individual small boxes within the three per four, then when two walls get in contact, we can eliminate them because we only want those that are external [in the video Figure 2]. <sup>IV</sup> That is, the glasses in the four corners contribute with two pieces, the others with only one piece, and those in the middle with none. (...)
- 6 Teacher: What Aloma explained, is it clear? What she said about saving walls, eh? Here [showing the box with three rows of four] we need a total of fourteen walls. But with this [showing the box with two rows of six] we need sixteen walls, and finally, this [showing the box with one row of twelve] needs a total of twenty-six walls. Yes, everyone? Clàudia?
- 7 Clàudia: The less glasses touching the walls, the best.



8 Teacher: The less glasses touching the walls, the less quantity of cardboard.



Figure 2: Teacher explaining through cardboard

The eight turns that constitute the moment in this transcript were analysed according to the detection of actions of argumentation and types of orchestration. Table 1 situates the turns in relation to actions and types, and summarises some of the curricular mathematical contents involved in the explicit talk. The joint analysis of the transcript and Table 1, along with the

video of the lesson indicates the creation of at least a learning opportunity around the optimization of the geometric variable corresponding to the surface of a rectangular volume. Aloma is the student who acts on the glasses and the cardboard to justify the need for establishing the individual box for only one glass as the surface and volume unit of measurement (i.e., the small box is presented as a box of capacity one glass), and simultaneously as the unit of counting on the basis of how many individual boxes can be placed in the considered solutions and how they can be placed. Through talk, manipulation and interaction with the teacher, Aloma grounds her reasoning on the quantity of cardboard for the different considered boxes by comparison with the unit box.

The talk initiated by Clàudia provides evidence of exploitation of the identified opportunity into learning. The teacher also provides further evidence of exploitation of the reasoning introduced by Aloma and expanded by Clàudia. This happens, however, later in whole class discussion, when the teacher modifies

Turns	Action of argumentation	Type of orchestration	Curricular content
1 Teacher	Communication of empirical evidence	Experimenting the instrument (glass)	Bidimensional representation of a rectangular prism
2 Aloma	Communication of empirical evidence	Experimenting the instrument (glass)	
3 Teacher	Particularization and study of a possible solution	Explaining through the artefact (blackboard)	
4 Aloma	I Affirmation with empirical support	Replacing the artefact (blackboard → cardboard)	Optimization of the perimeter of the prism base given its surface
	II Comparison among possible solutions	Discovering through the artefact (cardboard)	
5 Teacher	I Particularisation and study of an alternative solution	Explaining through the artefact (cardboard)	Units of measurement and counting
	II Establishment of unit of measurement and counting	Exploring the artefact (cardboard)	
	III Validation and inference of reasons grounded on the unit	Linking artefacts (blackboard & cardboard)	Relative positions of a unit
	IV Classification of space positions relative to the unit	Explaining through the artefact (cardboard)	
6 Teacher	Emphasis and conclusion on a solution from all options	Explaining through the artefact (cardboard)	Optimisation of the lateral surface of a rectangular prism
7 Clàudia	Affirmation with empirical support	Discovering through the artefact (cardboard)	
8 Teacher	Formalisation of an informal reasoning	Explaining through the artefact (cardboard)	

Table 1: Actions, types and contents in a moment of the second lesson

the condition of a rectangular base for the box. It is a modification that leads to different options for the solution boxes, but still keeps valid the conjecture, “The less glasses touching the walls, the best.” From there the mathematical discussion moves toward the problem of isoperimetric space figures and the visualization of cylinders.

By examining how mathematics learning opportunities can be promoted through whole class discussion with problem solving and manipulatives, we have detected opportunities for our conceptual understanding of mathematics learning and the classroom environments in which it may occur. Our research shows that certain actions of argumentation in interaction with certain types of orchestration are positively related to the creation and exploitation of learning opportunities around mathematical contents that are not strictly procedural (see Table 1). We are striving to understand, however, features of the teacher’s teaching activity that are decisive in the process that goes from classroom talk to learning opportunities, and from there to mathematics learning. Although important, we have not started looking at whether differences in teaching activity explain differences in learning moves.

## FINAL CONSIDERATIONS

We have examined the relationships between actions of argumentation and types of orchestration in the generation of mathematics learning opportunities. We still know little about why and how mathematics learning occurs. Nevertheless, our study provides some light on lessons with problem solving and manipulatives in which learning relates to the development of collective argumentation in interaction with the orchestration of particular materials. The progressive sophistication of argumentation is probably fostered by the role and use of materials, but in the empirical context of our study it is too early to conclude on this. What we already know is that these issues, actions of argumentation and types of orchestration are clearly important for the understanding of the dynamics of learning. First, the notion of collective argumentation has been used to illustrate the importance of interaction among participants –subjects. Second, the notion of orchestration has been used to illustrate the importance of interaction with materials –objects. Other two-sided analyses would be possible under the same idea of keeping balanced

the interaction with subjects and that with objects in the mathematics classroom.

Researchers interested in the role and use of artefacts may focus their questions on processes and tools through which students elaborate their thinking strategies in interaction with dynamic geometry software, physical manipulatives, virtual manipulatives, etc. Other researchers concerned with the role and use of talk may focus their questions on outcomes of pair work, small group, whole class discussion, etc. Similarly to Fetzer and Tiedemann (2015), our conceptualization of learning as learning-by-talking-and-doing seeks for integrated approaches. We are aware of the risks of misrepresenting any approach, and more generally, of the few empirical studies in the field taking a balanced subject-and-object perspective. By means of the example in this report, we have attempted to explain that the integration of subjects and objects, instead of its distinction, is at the root of our social view of mathematics learning. Manipulation of objects (either Platonic mathematical objects or concrete manipulatives) and talk with subjects (either one self or others in the class) are expressions of the same basic realization called mathematics learning.

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