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Emma's mathematical thinking, problem solving and affect

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This paper aims to understand one pupil's mathematical thinking through problem solving and mathematics related affect. The results reveal a successful, though quite unsure, problem solver whose affective state (connected to problem solving) seems to tell the same story as her affective trait (view of mathematics). The differences between results on affective state and trait seem to be connected mostly to emotions.

Keywords: Mathematical thinking, problem solving, affect.

THEORETICAL FRAMEWORK AND RESEARCH QUESTION

Developing mathematical thinking is one of the three tasks of instruction listed in the Finnish National Core Curriculum for Basic Education (Finnish National Board of Education [FNBE], 2004, p. 158). Some aspects of mathematical thinking are evaluated through tests at school, at the national (e.g., Rautopuro, 2013; Hirvonen, 2012) and international (e.g., OECD, 2014; Mullis, Martin, Foy, & Arora, 2012) levels. However, we lack a deeper understanding of the mathematical thinking pupils' take into their lives and possible further studies after comprehensive school.

Mathematical thinking is not defined in the curriculum, but a list of thinking and working skills is provided as evaluation criteria for every age group. These lists include learning objectives such as pupils' use of logical elements in their speech, judging truth of simple propositions and noticing parallels and regularities between different events (FNBE, 2004, p. 166). For sixth to ninth graders, 'thinking skills and methods' is also introduced as its own entity in the learning objectives parallel to core contents such as algebra and geometry (Mathematics curriculum, *ibid*, pp. 158–167).

When thinking skills and methods are listed in the curriculum, problem solving is repeatedly referred to. The term 'problem solving' is not defined. However, in final-assessment criteria for ninth graders, four problem-solving phases are introduced. These phases are similar to Polya's (1957) problem-solving phases. The process view on problem solving also guides this study where (in line with the curriculum) pupils' activities, actions and explanations during problem solving are interpreted as visible signs or expressions of their mathematical thinking.

In this study, thinking is considered mathematical when it relies on operations that are mathematical (Burton, 1984). Furthermore, a mathematical task is called a problem if the solver has to combine previously known data in a new way to her to solve the task (e.g., Kantowski, 1980).

When mathematical thinking is described, in addition to cognitive aspects, we should also explore affective factors (e.g., DeBellis & Golding, 2006; Vinner, 2004) and seek to understand the interrelationship between affect and cognition (e.g., Hannula, 2011; Zan, Brown, Evans, & Hannula, 2006). Instead of categorizing affective factors for instance as beliefs, attitudes or values, affect is seen as a mixture of cognitive, motivational and emotional processes:

Cognition deals with information (self and the environment), while motivation directs behaviour (goals and choices). Success or failure in goal-directed behaviour is reflected in emotions (e.g., shame). These emotions, in turn, act as a feedback system to cognitive and motivational processes. (Hannula, 2012, p. 144)

Affect is seen as a psychological domain with its state and trait aspects (Hannula, 2011). In connection to problem solving, we focus on rapidly changing af-

fective states. The more stable affective traits follow the categorization of pupil's view of mathematics introduced by Pehkonen (1995; discussed also in Op't Eynde, de Corte &, Verschaffel, 2002). These categories are mathematics, oneself as a learner and user of mathematics, learning mathematics, and teaching mathematics.

In an aim to understand the interrelationship between cognition and affect in mathematical thinking, we look at one pupil's, Emma's, problem solving and explanations on affect related to mathematics. The information from this exemplary case can later be combined with other cases to form a more informative view on mathematical thinking at the end of Finnish comprehensive school. So, with the question '*What characterizes Emma's mathematical thinking?*' we try to understand the mathematical thinking Emma takes from comprehensive school into her life and further studies (cf. mission for basic education in FNBE, 2004, p. 12).

METHODS

The data was collected in three cycles. Emma's results from the first cycle are discussed in Viitala (2015). This paper adds both problem solving and affective results and reports on findings from all three cycles of data collection.

Participant

Emma is a high achieving girl (mathematics grade 9 on a whole number scale of 4 to 10) who was selected for this paper based on a previous report (Viitala, 2015). The data collection was organized in the first semester of 9th grade when Emma was 15 years old.

Data collection

The data was collected from mathematics lessons and interviews over the course of three months. In each

of the three cycles, one mathematical task was solved in an ordinary classroom situation as a 'main task'. In Emma's case this meant that the pupils solved the tasks individually but they were allowed to talk about the tasks with a friend or ask for help from the teacher. In each of the three cycles, Emma was video recorded while she solved the task(s) in class and her solution on paper was collected.

The interviews took place either on the same day, or on the next day after solving the task in the classroom. The interviews contained two parts. The first part concentrated on affective traits and treated the following themes: pupil's background, mathematical thinking, and pupil's view of mathematics (following the categorization of Pehkonen, 1995; see example questions in Table 1). This part of the interview was semi-structured and focused (Kvale & Brinkmann, 2009).

The second part of the interview was about problem solving. The classroom data was used as stimuli when Emma's problem solving was discussed. Emma explained her actions and thinking and responded to questions such as, 'What are you thinking now?' and 'Why are you doing so?' Also, some affective and metacognitive questions were asked, for instance, 'What did you feel when you read the task?' and 'Did you think about your own thinking when solving the task?'

Finally, Emma was asked to assess her confidence before, during and after solving the problem(s), as well as her confidence in school mathematics using a 10 cm line segment (scale from 'I couldn't do it at all' to 'I could do it perfectly'; cf. estimation of certainty, e.g., in Merenluoto, 2001). All interviews were video recorded.

The tasks used in this paper are released PISA items. PISA tasks are well tested and based on real-life situ-

Theme	Example questions
Background	Tell me about your family.
Mathematical thinking	What does mathematical thinking mean? / How do you recognise it?
Mathematics	What is mathematics as a science? / Does it exist outside of school? (How? Where?)
Oneself and mathematics	Is mathematics important to you? / Does it help you think logically? (How?)
Learning mathematics	How do you learn mathematics? / Is it most important to get a correct answer?
Teaching mathematics	Does teaching matter to your learning? (How?) / What is good teaching?

Table 1: Interview themes and example questions

	Task	Given information	Why chosen?
Holiday, Q1	Calculate the shortest distance by road between Nuben and Kado.	Map of the area, Table of distances, Answer given in kilometres.	Complex situation, Combining different data
School excursion	Which (bus) company should the class choose, if the excursion involves a total travel distance of somewhere between 400 and 600 km?	Written explanation of the situation and rates that the bus companies charge.	Uncertainty, Decision making
Indonesia, Q3	Design a graph (or graphs) that shows the uneven distribution of the Indonesian population.	Table of the population of Indonesia and its distribution over the islands.	Open task

Table 2: Descriptions of some of the tasks used in the project

ations (e.g., OECD, 2006, p. 108). The 'main' PISA tasks, Holiday, School Excursion and Indonesia (ibid, pp. 77–78, 87, 111, respectively), were solved in the classroom. These tasks are open with respect to solution strategy (see task examples in Table 2). Indonesia also included two additional questions in which Emma's skill to read a table was tested (referred to as Indonesia Q1 and Q2).

In class, Emma also started to solve Carpenter (OECD, 2009, p. 111) which was added in case of extra time. In the interviews, Emma answered questions from the tasks Distance (OECD, 2006, p. 102; modified to 3 and 5 km), Growing up (OECD, 2009, p. 106; Q's 3 and 1) and Braking (ibid., pp. 128–129; also Q49 from a Web page of the Finnish Institute for Educational Research).

Analysis

The analysis was divided into two sections: Problem solving, and Affect related to mathematics. In problem solving, the main focus is on the cognitive problem solving process written in the curriculum for grades 6–9 as core content or final assessment criteria of thinking skills and methods. These phases follow the problem solving principles described by Polya (1957) and will be reported accordingly.

In connection to problem solving, pupil's thinking about their own thinking as well as control and self-regulation (e.g. keeping track of what is being done during problem solving) will be discussed as a part of metacognition (Schoenfeld, 1987). The pupil's motivation to solve the tasks as well as confidence and feelings during problem solving is reported as part of the psychological affective state.

In affect related to mathematics, pupil's view on mathematics is reported (see Table 1). These results can be referred to as affective traits. The discussion on metacognition concentrates on the aspects listed as

learning objectives in the curriculum (e.g., trusting oneself; FNBE, 2004).

The results are descriptive. More information related to methods and methodology can be found in earlier publications (Viitala, 2013, 2015).

RESULTS

Problem solving

Understanding the problem. Emma uses much of her time for understanding problems and the given information (in class 2–5 minutes which is some 30–55 % of total solution time). She seems very thorough and she says that she wants to understand every aspect of a task before starting to plan and solve it. On one hand, this seems to be a key element in her success as a problem solver. On the other hand, this might hinder her to solve a problem (not understanding all the mathematical expressions in Braking Q49) or to give a correct answer (she was prone to give an answer which she can completely understand in Braking Q49).

Emma uses graphs (maps and diagrams from the tasks) to assist her thinking when putting the given information together (e.g., marking routes and distances to the map in Holiday), understanding a problem (e.g., the graph in Braking for Q49) making a plan to solve a problem (e.g., routes in Holiday) and reflecting on the task when solving it (e.g., connecting the question to the graph in Growing up). The use of tools and drawings that assist thinking is part of mathematics learning objectives.

Making a plan. After taking the time to understand a problem and given information, Emma does not need much time to make a plan. Making a plan seems to happen on the third reading of the question, after reading the question quickly through on the first

reading and putting together the given information on second reading (according to her explanations, e.g., for Holiday Q2 and School excursion).

If a task feels hard, Emma says she thinks of alternative ways to solve it (in Holiday she calculated a second route to confirm her result). Additionally, if the task feels too simple, she might try to calculate the task further after getting the answer (Indonesia Q2, this calculation was later erased because 'it felt stupid').

Carrying out a plan. Emma seems careful and thorough in solving mathematical tasks. After understanding the task, Emma is fluent in transforming a text to a mathematical form (mathematical expressions e.g. in Holiday and School Excursion). She says she can return to the task description as a confirmation also in the middle of solving a task (e.g. in School excursion). She proceeds step-by-step with the tasks.

If Emma has different options to solve a task, she says that she chooses the one that feels more 'probable' (e.g., routes in Holiday) or has less doubt (e.g., choosing a point where to calculate School excursion). For a task that had more than one answer (Distance), she spontaneously found two answers and a third one after being probed. In most cases Emma was able to justify her actions and conclusions.

Looking back. Emma says she checks tasks only in tests. In the research project, the first class situation felt like a test situation for Emma and it was the only time she checked her answers (Holiday). Other answers to PISA tasks she checked from a friend (Holiday and School excursion) or left it until the interview. In addition, Emma feels that she does not need to check her calculations when a calculator is used.

If Emma is not sure whether she has understood the task or given information correctly, she chooses the interpretation that feels most reasonable and proceeds with that (e.g. table in Holiday, and Q1 and Q2

in Indonesia). This aspect of 'looking back' is done during the process of solving the task.

Affect related to problem solving. Emma feels unsure when faced a word problem. She might 'panic' if the task has a lot of text (Holiday Q2) or numbers (Indonesia), or she cannot understand all the given information (e.g. table in Holiday and mathematical expressions in Braking Q49). When she gets stuck with the task description, she seems to lack efficient tools to overcome the situation (Holiday Q2, Braking Q49, also visible in Emma's explanations about doing homework and learning mathematics; on getting stuck, see, e.g., Mason, 2015). In these cases, asking questions helps her to overcome the difficulties and proceed with the task.

Getting help (Holiday Q2, before solving the task) or asking the correct answer from a friend after solving the task (Holiday and School excursion) seems to have a direct influence on Emma's confidence. Similarly, not checking her answer (Indonesia) seems to make her feel very uncertain and anxious even in the interview (until the results were given). See Emma's confidence related to problem solving in Table 3.

Emma might experience many different feelings when facing, planning and solving a problem (e.g., in School excursion: nervous, unsure, doubtful, 'normal' and relieved chronologically, cf. Table 3) but she agrees that her feelings do not necessarily affect her more stable feeling of confidence. The main motivation for Emma to solve the given tasks was the video camera. However, when a mathematical obstacle was encountered, Emma was motivated to learn from it (e.g., not understanding some mathematical part of the discussion in an interview, such as graphs for companies in School excursion, or percentages when discussing Indonesia or Braking).

	Confidence <i>after reading</i> the task	Confidence <i>while solving</i> the task	Confidence <i>after solving</i> the task
Holiday, Q1	5.5	5	7.25
Holiday, Q2	3.75	6.25	7
School excursion	5	6.25	7.25
Indonesia, Q1-Q3	3.25	4	4

Table 3: Emma's confidence (0–10, ± 0.25 mm) for the tasks solved in the classroom

Affect related to mathematics

Mathematics. Emma's view on mathematics is very much tied to a school subject. For her, mathematics is calculating, both as a school subject and as a science. Mathematical knowledge is gained by calculating and correctness of mathematical knowledge can be verified by asking the teacher. Emma thinks mathematics is useful and needed for instance in other school subjects (e.g. civics, physics and chemistry). When asked, it is hard for her to see connections between mathematics and real life. Emma uses mathematics outside of school when she is shopping.

Oneself and mathematics. Emma is motivated to learn mathematics. The feeling of success drives her forward and succeeding with a difficult task makes her feel proud of herself (intrinsic motivation). She values the opportunity to show her skills to her teacher and classmates by going to the black board to calculate tasks. It feels rewarding and it motivates her to learn 'the next thing' (extrinsic motivation).

Emma likes mathematics and thinks it is 'quite fun' and interesting. However, in her own words, she does not feel 'very confident' in mathematics (cf. Table 4). However, she thinks that this might be a good thing: If you are too confident, you might not use that much time for thinking or check the calculations of a task. Emma thinks that confidence and mathematics grades are two separate things. Her grade (9) is the best she thinks she can achieve.

Interview 1	Interview 2	Interview 3
6.25	5.5	5.5

Table 4: Emma's confidence (0–10, ± 0.25 mm) in mathematics

Learning mathematics. Emma, together with her friends and family, values learning mathematics and thinks that mathematics is useful. She also agrees that the atmosphere with regards to mathematics in her class is positive. Emma seems to trust that if she studies mathematics she can succeed and get better in it. The feeling of success, the belief that mathematics is worthwhile and useful, and future studies motivate her to learn mathematics.

For Emma, learning mathematics is 'understanding', in addition to 'memorizing' and 'reasoning'. Understanding means you are 'able to use a method'. Learning as well as understanding takes time for

Emma. Understanding comes from calculating, asking questions, and proceeding little-by-little from easier tasks to more difficult ones. Rote learning is important. Emma learns new things as independent issues and does not actively seek for connections to previous knowledge. She 'forgets things quite quickly' and an indication of this was seen also in the interviews (calculations with percentages).

Emma's feelings in learning mathematics are also closely connected to understanding. Learning mathematics is fun when she understands or succeeds in mathematics. Not being able to understand is irritating. Sometimes learning mathematics is also tiring. Mathematics is easy for her 'but only a little' (cf. Table 4). Emma seems to take the responsibility of her own mathematics learning. She says that if she succeeds with a test, it is because she has studied for it and learned in class. Failure, on the other hand, means that she has not done enough work.

Teaching mathematics. Mathematics teaching methods and the mathematics teacher play a great role in Emma's learning of mathematics. She believes that without teaching she could not learn mathematics. A good mathematics teacher offers opportunities to ask questions, gives time for (rote) learning, does not proceed too quickly to the next thing, and proceeds from easier tasks to more difficult ones. These all are features that Emma's current mathematics teacher seems to possess (according to Emma's explanations and the researcher's observations from the classroom).

In some respect, Emma seems to connect her feelings and success in mathematics to her teacher. In elementary school she did not get along with her teacher. She was an average pupil (grade 7–8) who was not interested in mathematics, did not succeed in it and mathematics felt like torment for her. In lower secondary school Emma got a new mathematics teacher who she liked, and whose teaching she liked. Since then, Emma says she has liked mathematics and been a high achiever.

SUMMARY AND DISCUSSION

Results from Emma's problem solving and affect related to mathematics seem to give a well-matching picture of Emma's mathematical thinking. Emma is a reflective learner and problem solver who needs time for understanding. Her thoroughness and tendency to ask questions (both from friends and the teacher)

seem to be the key to her success in both respects. Emma is not very confident in mathematics or problem solving (though slightly positive, cf. Hirvonen, 2012) but, as a consequence, she seems to be very careful with her thinking and working. Moreover, her uncertainty might be a reason for her success both in problem solving and mathematics.

The results showing differences between problem solving and affect related to mathematics seem to be connected to less stable affective traits. As an example, Emma's confidence has more variance in problem solving than in mathematics and her feelings experienced during problem solving have more tendencies to negative feelings (e.g. unsureness) than her feelings in learning mathematics. What is notable regarding Emma's affect in mathematics is that, contrary to previous research results (e.g., Tuohilampi, Hannula, Laine, & Metsämuuronen, 2014), Emma's feelings towards mathematics have become more positive since elementary school.

Throughout the study, Emma worked in a sustained and focused manner with the problems. Even though she is not very confident in mathematics, she seems to trust herself as a mathematics learner (e.g., aiming to learn more mathematics so she can succeed in advanced mathematics in upper secondary school). She also seems to take responsibility for her own learning (e.g. reasons for succeeding or failing in tests). All these aspects (listed in the curriculum; FNBE, 2004) together with her problem solving skills seem to offer her a solid foundation for future studies.

In addition to preparing pupils for further studies, basic education must also provide opportunities to obtain the knowledge and skills pupils need in life (FNBE, 2004, p. 12) and mathematics teaching should help pupils to see the connection between mathematics and real life (ibid, p. 158). Nonetheless, even though the PISA tasks that were used were situated in the real world, Emma saw them purely as mathematics tasks. Additionally, she struggled to see where she uses mathematics in her own life outside of school (homework, shopping). After analysing all the cases in the project, we can see if this might be a possible trend among Finnish pupils.

The upcoming curriculum (FNBE, 2014; will be implemented in 2016) draws more attention to mathematical thinking and real-life connections. For

instance noticing connections between learned concepts and applying mathematics in other school subjects and surrounding society are written as individual learning objectives and as final-assessment criteria (ibid, p. 433–434). It is hoped that this will direct pupils' attention more towards their thinking and connections between mathematics and real life, at the same time making mathematics more worthwhile and enjoyable.

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