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# Teachers' individual beliefs about the roles of visualization in classroom

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*This paper investigates mathematics teachers' beliefs regarding the roles of visualization for teaching fractions and algebra. The paper discusses the construct of beliefs and theoretical roles of visualization. We further give rationale for the qualitative approach of our study. In the results we compare two teachers' beliefs from a larger sample. We work out that, although both teachers use visual representations in a similar way, they assign different roles to their respective use. Their beliefs appear to be stable across subdomains. We discuss our impression that the teachers emphasize other roles of visualization than educational research implies.*

**Keywords:** Teachers' beliefs, visualization, domain-specific, fractions, algebra.

## INTRODUCTION

The use of visualization is regarded as important for the teaching and learning of mathematics. Possible roles of visualization are for example to promote students' understanding (Presmeg, 2006; Duval, 2006), to enable problem solving (Heinze, Star, & Verschaffel, 2009; Duval, 2014; Arcavi, 2003), to discover, to describe, and to explain (Rivera, Steinbring, & Arcavi, 2014).

The demands on teachers to appropriately teach using visualization are high, e.g. to use visual representations in a conscious and active way (cf. David & Tomaz, 2012; Presmeg, 2014). Accordingly, nowadays it is emphasized to incorporate strategies on how to use visual representation into teachers' education (e.g., Presmeg, 2014). Thus, on the one side, we gained a lot of knowledge about a theoretical framework of visualization and a framework for teaching with visualization. On the other side, teachers seem to have complex attitudes regarding visualization (e.g., Gómez-Chacón, 2015). Yet we gained scarce results regarding the question of how

mathematics teachers define the role of visualization for their classroom practice, or why they define the role of visualization in an individual way. However the teachers' thinking or rather their beliefs are understood to crucially impact on both these teachers' classroom practice (Calderhead, 1996) and their willingness to receive new information about teaching and learning with visualization (Chapman, 1999).

For this reason, our study aims to investigate teachers' beliefs about visualization in more detail. We seek to identify individual perspectives and reasons as to how and why teachers do or do not use visualization in classroom and which roles the teachers assign to visualization. In this report we try to give first answers to the following research questions, by means of a comparison of two teachers' beliefs:

(1) What are secondary teachers' beliefs about the roles of visualization in fractions and algebra? (2) To what extent do these roles differ in specific domains?

For this, we first outline our theoretical framework, including definitions of the constructs of beliefs and visualization and including an overview of the roles of visualization. Afterwards we discuss our method. The main part of this paper refers to results relating to the beliefs of two mathematics teachers from a larger sample. These teachers point out their understandings of visualization and their beliefs regarding the role of visualization. The results represent an empirical counterpart to our theoretical knowledge about teaching with visualization.

## THEORETICAL FRAMEWORK

### Definition of beliefs

In order to answer our research questions the multifaceted term "beliefs" has to be clarified. We chose

a description given by Philipp (2007) who combines many similarities of existing definitions in the context of teaching and learning:

Beliefs - psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes. (p. 259)

For this study, several aspects are important:

- As individual judgments about the validity of statements beliefs differ from knowledge, but they contain a cognitive component (Baumert & Kunter, 2006).
- Beliefs can be understood as lenses in the way that they form a specific world view. They are assigned to influence perception and action (Philipp, 2007).
- Being gradual incorporates that a belief regarding a specific role of visualization can individually be more or less important for a person (cf. the term of central beliefs; Thompson, 1992).
- Beliefs are classified as rather stable (cf. also Hannula, 2012) and refer to specific content (Eichler & Erens, 2015). Regarding visualization, we look for the specificity of beliefs referring to a mathematical domain, but also for similarities if different domains are regarded.

Calderhead (1996, p. 719) identifies “five main areas in which teachers have been found hold significant beliefs”. Of these, “beliefs about learners and learning” as “assumptions teachers make about their students and how their students learn”, “beliefs about teaching” as “beliefs about the nature and purposes of teaching”, and “beliefs about the subject” as “epistemological issues - what the subject is about” are of interest when investigating beliefs about visualization (ibid.).

### Definition of visualization

This study uses a broad definition given by Arcavi (2003, p. 217):

Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings.

The definition integrates many facets of visualization. The following are especially relevant for this study:

- It integrates process and product. In everyday language, which is likely to be used by teachers, both terms are often used synonymously.
- It integrates different ways of how to handle visualization, especially the meaning of making something visual for oneself (often: to visualize) as well as to make something visual for somebody else (often: to use a visual representation). Both aspects are important in classroom.
- It mentions diverse kinds of pictures. Thus, everything not being completely symbolic could be considered as visualization.
- It defines visualization as goal-oriented and enumerates purposes like advancing understanding. Yet the purposes are not considered to be exhaustive.

### Roles of visualization in theory

The purposes of visualization lead to the different roles of visualization: Quoting different researchers in the introduction, we mentioned a list of possible roles of visualization, i.e. promoting understanding, enabling problem solving, facilitating discovery and explaining. Partially overlapping, Arcavi (2003) enumerates in his definition the purposes “depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings”. From an epistemic point of view the roles which Giaquinto (2008) mentions are also specific for mathematics: proving, discovering, heuristic aid, augmenting understanding, and enabling calculation.

A comparison of the different definitions of visualization yields that it might be common to these roles that they could be grouped in (1) understanding, explaining (2) discovery, problem solving, heuristic aid, and developing ideas, (3) describing, depicting, and communicating information, (4) enabling calculation, (5) proving.

### **Empirical studies on teachers' beliefs regarding the role of visualization**

However, there are few studies about teachers' beliefs regarding the *role of visualization*. For this report we chose three studies. Stylianou & Silver (2004, p. 353) report in a study about problem solving that "experts and novices perceive visual representation use as a viable strategy. However, the two groups judge visual representations likely to be useful with different sets of problems."

Biza, Nardi, & Theodossios (2009) investigate how teachers use visual arguments in proving in the domain of functions. Teachers who at first glance appreciate visual arguments reveal differences in the acceptance in follow-up interviews.

Research results from text-picture-integration suggest that teachers for different domains have different views on the use of visualization (McElvany et al., 2012).

These studies suggest that the role of visualization from a teachers' point of view might be related to a specific content. They also support the idea of a qualitative approach, as all of them show that it is important to investigate the teachers' thinking in detail.

### **METHOD**

The research questions are part of a qualitative study about how teachers want to use visualization in classroom. The study is carried out with twelve secondary teachers. Further, the study covers the domains fractions, algebra, functions, and calculus, since these domains cover secondary mathematics of all ages, and visualization can be used in different ways, e.g., as a typical means of illustration (like the pizza in fractions), as a mathematical object (e.g. the graph of a function), or with a rather structural focus (e.g., drawings in algebra). The teachers are chosen by a "theoretical sampling" (Glaser & Strauss, 1967)

which aims at contrasting cases (e.g., regarding type of school, age).

In semi-structured interviews lasting three-hours the teachers talk about how and why they use visualization in classroom. The interviews address the same questions for each domain, e.g. regarding the teachers' use of visual representations, their aims, their perspective on students' learning and their use of technology. It is completed by general questions about mathematics and mathematics teaching.

The interview transcripts are analyzed according to Grounded Theory (Glaser & Strauss, 1967). In open and axial coding we developed categories concerning the function and the relevance of visualization for the teachers. The objective is to generate theoretical concepts. In part, the concept aims at a consistent perspective on teachers' beliefs regarding the role of visualization in classroom. The roles might be different from existing theoretical frameworks.

The study does not investigate how teachers act in classroom. To explain teachers' practices, many other factors, e.g. pedagogical orientation and context (Philipp, 2007, p. 275), are to be considered. However, the study is based on the assumption that teachers' beliefs impact their classroom practice.

### **RESULTS**

In this paper, we compare the roles of visualization in classroom for two mathematics teachers. Alan is a teacher at a comprehensive school with about ten years of teaching experience. Claire has been teaching at an upper secondary school for about twenty years. We develop the roles of visualization regarding fractions and algebra (quest. 1), and we analyze in how far their beliefs regarding these roles are domain-specific (quest. 2). First, we analyze Alan. The part regarding fractions is rather detailed in order to provide also an insight into the development of the codes. According to grounded theory the resulting hypotheses influence the analysis of Alan's statements about algebra. Then we analyze Claire in the same way, but more briefly.

#### **Alan: Fractions**

For Alan it is very important that his students first develop a conception of a fraction as a "part of the whole". From his point of view it is paramount that

they can relate fractions with images, see for instance the following quotation:

Alan: There [in fractional arithmetic] it is very important that, at the beginning, students have an idea of what a fraction is in the first place. [...] And this is something which they always have to keep in mind. [...] They really have to have this image in their heads.

He stresses that students develop his intended conception of fractions if they can use many different geometric forms. Also as regarding calculation in fractions, Alan considers visualization helpful. Expressions like “see”, “have pictures of something”, and “understand” are used synonymously, as he did already for the conception of a fraction. Alan believes that *visualization supports students' understanding*:

Alan: For many, this [addition of fractions with a common denominator] is not clear at first. If you just draw it, they see it immediately.

At the same time, visualization is for him a methodical aid for *explanation*. Alan appreciates good reasons. Visual means can support him with this:

Alan: Visualization also helps with improper fractions and mixed numbers [...] This you can somehow also do well with pieces of pizza [...].

Alan: Reducing and expanding with rectangles, [...] this can be done perfectly with horizontal and vertical lines, like here. And then, once they have figured it out, I move on quickly to the purely algebraic.

In addition, the preceding quotation, and also the following, highlights his objective of withdrawing from visualization and his orientation towards algebraic objectives, for example the mastering of calculation techniques:

Alan: You see, when calculating with fractions, we do not always think of anything graphic, either. And that is where I want to guide the students, too, of course.

At first glance, a contradiction seems to arise: Alan regards it as very important that his students have a visual image of a fraction. He also uses visualization for explaining calculation. However, at the end, for Alan calculation is a symbolic manipulation. Taking into account the above quotes, for Alan it might be sufficient to show the deduction of constructs or methods with the aid of visualization. This hypothesis *visualization supports justification* is supported by the following quote:

Alan: In this case, it is really a lot easier to have remembered that you divide by a fraction by multiplying by the reciprocal, and so on. Nevertheless, it is important for them, of course, to have first understood the principle why this is so. And this just works well with easy visualizations. But then they quickly have to get past this.

Alan expresses the value of using visualization when introducing a subject as a basis for the following:

Alan: That is why this introduction phase with the drawings always takes quite a long time. As far as this is concerned, I'm of the opinion that this is the basis for everything else which comes later.

But he only gives justification for an operation if he thinks that this is feasible with drawings that are easy to use (cf. reducing and expanding fractions). On the other hand, when he does not know an easy visualization he does not give a justification:

Alan: This [division by a fraction] is difficult, because this is about how often one fraction fits into another fraction. You can visualize that wonderfully if the result is a natural number. But if a fraction fits four seventh times into another fraction that becomes difficult to visualize.

Concluding this paragraph we can say that Alan values visualization for *understanding*, for *explaining* and as *justification*. He only uses drawings which he finds easy to understand. Mathematics seems to have a deductive character for him: New things have to be logically derived from the well-known. As soon as



they are justified they can be used without knowing the reason or the visualization.

### Alan: Algebra

Alan's beliefs regarding visualization regarding algebra are very close to his beliefs expressed regarding fractions. For example, for solving linear equations Alan uses a drawn beam balance to support students' *understanding*:

Alan: None of them know a beam balance any more, but intuitively they grasp it immediately.

The aim of using visualization is that the students get a plausible reason for the later procedure (*justification*):

Alan: This will enable them to see what the effect of what we are illustrating is in algebra. And this is intended as a first explanation as to why or how equivalence transformations work.

He further consistently expresses the belief that for him the deduction is very important. If a plausible deduction is easier with other means than visualization, he uses other means. Visualization is not automatically related to deduction, but when he uses it, then often for deduction.

### Interim summary 1

Alan regards the role of visualization as an aid to *understanding, explaining, and justifying* (quest. 1). These beliefs can be identified in two domains. This could be an indication for a belief that is similar in different (mathematical) domains (quest. 2).

### Claire: Fractions

We compare the results regarding Alan with Claire's beliefs. She also uses visualization intensively when introducing a subject. Afterwards her aim is to establish schematic calculation. We would like to know if she assigns the same roles to visualization. Claire, too, develops the conception of fractions using many drawings:

Claire: That is very important, so that they just get this idea of a fraction. Divide something in five equal parts, take three of these parts. That you also just visualize it.

Similarly she uses visualization for the illustration of most calculation operations. But she is not as convinced as Alan that visualization is helpful for understanding:

Claire: Four fifth divided by three, however [...] How does it suddenly get into the denominator? Thus, I again made a drawing. [...] But this is one which is difficult to draw. I doubt that this really helps the students. I rather did it because of the requirement that I once learned that you always have to visualize, and how important this is.

She rather uses visualization because she learned it in that way. To visualize seems to be a *norm*: "Teachers use visualization when they introduce a new theme." On the other hand, Claire sees a high benefit for her students in the fact that they can remember the contents later because of the visualization:

Claire: I always try to select something that still is easy to remember.

Although Claire shows quantities and development of the use of visualization in fractions similar to Alan, she associates different roles with it. She rather uses it as a *mnemonic aid*. Even if she is not convinced, she uses visualization for explanation. She rather seems to conform to an educational *norm* she learned at university.

### Claire: Algebra

In algebra, too, Claire - like Alan - uses a balance for solving equations. She appreciates the possibility to *reactivate* knowledge once the subject is manifested:

Claire: That is also something you can well remember later; once this has been understood, you can always come back to the balance [...], standing there with your hands on the left and right side.

She does not claim to present every subject in a visual way, as algebra seems to be a rather abstract domain, not related to visual elements:

Claire: Algebra is something which is incredibly abstract, actually. [...] To get somewhere

in algebra, it is not always visualizations that does the trick.

### Interim summary 2

One important reason why Claire uses visualization is a *mnemonic aid*. In fractions, she additionally associates visualization with a *norm*, in algebra she does not (quest. 1). The role of a mnemonic aid seems to be similar in different domains. Also her orientation on perceived norms, which might influence her beliefs about the role of visualization, seems to be similar in different domains (quest. 2).

## DISCUSSION

This paper identified and compared beliefs regarding the role of visualization when teaching fractions and algebra, analyzing two secondary teachers, Alan and Claire. Both intensively use visualization as part of the introduction of a new subject, and both teachers tend to emphasize schematic calculation. However, in a deeper analysis, they show different beliefs about the roles of visualization. This result is in accord with Biza and colleagues (2009) who found teachers' beliefs about visualization to be similar at first glance, but to be considerably different upon closer inspection.

The identified beliefs about the roles of visualization are about learning (*understanding aid, mnemonic aid*), about teaching (*explaining aid, norm*), and about the subject mathematics (*justify*). The investigated beliefs appear to be rather stable across domains.

In comparison to existing frameworks, we find *understanding* as a role for both teachers, with high intensity for Alan. Both teachers use understanding "in the sense of one's grasp of a definition [...]" (Giaquinto, 2008, p. 36). Alan also uses it "not only [for] grasping the correctness [...], but also [for] appreciating why it is correct" (ibid.), which is related to the role of *justification*. Both teachers do not express much consciousness about the "conversion of representations" (Duval, 2006, p. 121) as a source of understanding. *Explaining* and *justifying* could be beliefs that are in a cluster (Thompson, 1992) with *understanding*.

From a teaching point of view the aspect of classroom practice as learned in teacher education, the *norm*, is important for Claire. This could be related to sociological and cultural aspects of using visualization in mathematics teaching (cf. Arcavi, 2003).

Finally, the *mnemonic aid* is a role which does not seem to be very important in the literature about teaching and learning of mathematics, but possibly in practice.

For these two teachers we found only a few indications related to discovery and problem solving (Heinze et al., 2009; Duval, 2014; Arcavi, 2003), especially when making a sketch related to an application-oriented task. There are also indications that visualization can serve to "enable calculation" (Giaquinto, 2008, p. 39); both teachers recognize the "visio-spacial nature" (ibid) of some algorithms they teach.

In our study, we did not prove that the teachers enact their professed beliefs. However, "under specific conditions the teachers' espoused beliefs could explain the teachers' enacted beliefs" (Eichler & Erens, 2015, p. 197).

Prospectively, we will compare our results to more domains, e.g. also functions and calculus, to see if the teachers' beliefs are stable across other domains. We further expect to find other beliefs regarding the role of visualization like using visualization as a method for overcoming students' fears, seem to be relevant. We hope to develop a theory of beliefs regarding the role of visualization for teachers.

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