



**HAL**  
open science

## The mathematics teacher: An emotional rational being

Marina de Simone

► **To cite this version:**

Marina de Simone. The mathematics teacher: An emotional rational being. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.1167-1173. hal-01287339

**HAL Id: hal-01287339**

**<https://hal.science/hal-01287339>**

Submitted on 12 Mar 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The mathematics teacher: An emotional rational being

Marina De Simone

Università di Torino, Dipartimento di Matematica "G. Peano", Torino, Italy, [marina.desimone@unito.it](mailto:marina.desimone@unito.it)

*Starting from the "theory of rational behaviour", introduced by Habermas in 1998, I attempt to demonstrate how affective factors are entwined with those of rationality in the decision-making processes of the mathematics teacher. This type of analysis has been carried out by developing an adaptation of the concept of "emotional orientation", offered by Brown and Reid in 2006. In particular, I will present the case of one teacher with her grade 9 class, involved in the explanation of linear equations.*

**Key-words:** Rationality, emotional orientation, expectation, mathematics teaching, beliefs.

## INTRODUCTION

In this paper, I focus the attention on the discursive activity of the teacher when she is introducing linear equations. At the end of the twentieth century, the philosopher Habermas developed the "theory of rational behaviour", where he discusses how people discursive activities show their rational behaviour. In the last years, this philosophical and sociological framework has been re-elaborated and adjusted to mathematics education by a working group constituted by many researchers from Italian, French and Spanish universities. This collaboration has produced two different research forums, presented during the PME Conferences in 2010 (Boero, Douek, Morselli, & Pedemonte, 2010) and in 2014 (Boero & Planas, 2014). This paper is an expansion of the research presented within the last RF by F. Ferrara and myself and that I developed for my PhD dissertation.

Many of the educational studies about rationality according to the Habermas framework, were centred on the students in the mathematics classroom (e.g., Morselli & Boero, 2011). However, if, from the one side, this theoretical framework seems to be suitable to ex-

amine decision-making processes of a 'rational being', from the other side, also the study of the discursive activity of the teacher seems to be crucial examining the interactions which happen in the classroom. It is important to consider the decision-making of the teacher, because one peculiarity of the teacher is making decisions within the classroom. Several authors have recognized an essential role to the decision-making of the mathematics teacher. For example, Bishop considers it as the activity "... at the heart of the teaching process" (Bishop, 1976, p.42).

## THEORETICAL FRAMEWORK

Habermas defines a rational being as a human being who "can give account for his orientation toward validity claims" (Habermas, 1998, p. 310). He speaks about the concept of discursive rationality of the subject, explaining that it is not only referred to the discourse – as it could seem from the term at a first glance – but it has three different roots: the knowledge, the action and the speech, or, said in a different way: knowing, acting and speaking. Then, starting from the Habermas' assumption that for a rational being the discourse and the reflection on it (not necessarily explicit) are entwined, "the three rationality components - knowing, acting, and speaking - combine, that is, form a syndrome" (Habermas, 1998, p. 311) at a holistic level where reflection and discourse live together. Knowledge, action and speech constitute what he calls the epistemic, the teleological and the communicative components of rationality. They are inseparable, since a rational being acts in a specific manner to achieve a goal, on the basis of a specific knowledge, communicating in a precise way with the aim of being understood by the community. Hence, within the discursive activity, they are always present at the same time. According to Habermas, the epistemic rationality is connected to the justification of the knowledge at play: "We know facts and have a knowl-

edge of them at our disposal only when we simultaneously know why the corresponding judgments are true" (Habermas, 1998, p. 312). Concerning the teleological rationality, Habermas states that "all action is intentional", that is, every action is originated from an intention of the subject with the aim of the realization of a result. He speaks of teleological rationality when "the actor has achieved this result on the basis of the deliberately selected and implemented means" (Habermas, 1998, p. 313). Finally, Habermas states that the communicative rationality "is expressed in the unifying force of speech oriented toward reaching understanding" (Habermas, 1998, p. 315).

In his speculation about rational behaviour, Habermas seems to lack any reference to emotion or passion. He seems to avoid any emotion by claiming that the force of a good argument should be free of emotional tags. Several philosophers and social theorists complained that, in the development of his theory, Habermas doesn't take in account the emotional side of human beings. For example, Rienstra and Hook, quoting the philosopher Heller, posed the question that "Habermas leaves no room for "sensuous experiences of hope and despair, of venture and humiliation", accusing him of completely avoiding the "creature-like" aspects of human beings" (Rienstra & Hook, 2006, p. 13). Therefore, basing on the assumption that rationality is deeply linked with the emotional sphere, I looked for researches that confirmed this hypothesis in particular in mathematics education and human neuropsychology.

In the last years, research in mathematics education has progressively perceived the existence of a mutual interaction between the affective sphere and cognition in mathematics learning (Zan, Brown, Evans, & Hannula, 2006). As highlighted by Hannula (2012), many research studies, focused on mathematics-related affect, have been dated from the eighties. An important grow in the theory on mathematics-related affect was due to McLeod (McLeod, 1992), whose main goal was to build "an overall framework of mathematics-related affect that would be consistent with research that is cognitively oriented" (Hannula, 2012, p. 138). In McLeod's framework (McLeod, 1992), which is considered a cornerstone for the literature on mathematics-related affect, emotions occupy a fundamental place, because of their unstable or less stable nature than that of beliefs and attitudes. Unfortunately, the aim of constructing a general theoretical framework

that embrace all the research on mathematics-related affect has not yet been achieved. The most relevant problem is related to the terminology used in this field, because it is not universal. For example, as Di Martino and Zan discussed deeply (Di Martino & Zan, 2010), "some define attitude as positive or negative degree of affect, others identify emotions and beliefs as two components of attitude, while yet others define attitude as consisting of cognitive (beliefs), affective (emotions), and conative (behaviour) dimensions" (Hannula, 2012, p. 140). The recent research in mathematics-related affect has considered different affective concepts from those of the McLeod's (1992) framework such as values, identity, motivation, and norms. Zan and colleagues (Zan, Brown, Evans, & Hannula, 2006) have recognised the limited use of emotion in mathematics education research, even if it should be one the essential concept. They pointed out "how repeated experience of emotion may be seen as the basis for more 'stable' attitudes and beliefs" (Zan, Brown, Evans, & Hannula, 2006, p. 116). For Schoenfeld, emotional aspects are included in the wider category of beliefs, while goals is a motivational concept (Schoenfeld, 2010).

Human neuropsychology is another important field of research that studies the relationship between the affective and the rational sphere from a neurological point of view (Damasio, 1994, 1999). Specifically, Immordino-Yang and Damasio have shown the connection among emotion, social functioning and decision-making as a turning point for understanding the role of emotion in decision-making, the relationship between learning and emotion, and how culture shapes learning (Immordino-Yang & Damasio, 2007). While educational research often considered decision-making, reasoning and processes related to reading, language and mathematics as detached from emotion and body, these authors have shown that "learning, in the complex sense in which it happens in schools or the real world, is not a rational or disembodied process; neither is it a lonely one" (Immordino-Yang & Damasio, 2007, p. 4). For them, emotion is "a basic form of decision making, a repertoire of know-how and actions that allows people to respond appropriately in different situations" (Immordino-Yang & Damasio, 2007, p. 7).

This neurological research is becoming applicable also in the field of mathematics education. For example, Brown and Reid have developed and adapted the hypothesis of somatic markers (Damasio in 1994) for

studying the decision-making processes (Brown & Reid, 2006).

## EMOTIONAL ORIENTATION

Brown and Reid (Brown & Reid, 2006) analysing the processes of teachers and students' decision-making, have considered the notion of "emotional orientation" (Maturana, 1988). In particular, they have focussed on the "decision-making that happens before conscious awareness of the decision to be made occurs." (Brown & Reid, 2006, p. 179). Maturana (1988a, 1988b) referred the notion of "emotional orientation" to the criteria for acceptance of an explanation by members of a community, and considered emotions as being the foundation of such criteria. Reid adapted the concept of emotional orientation to the mathematics field, defining the "mathematical emotional orientation". The criteria for accepting an explanation in the particular case of the mathematical emotional orientation involve "the use of deductive reasoning, a basis in agreed upon premises, and a formal style of presentation" (Reid, 1999, p. 1). Moreover, there are many shared experiences and assumptions in mathematics, like the language used to talk about it. In the end, there are also many "actions" when someone does mathematics, like "drawing diagrams, generalizing statements, making conjectures" (Reid, 1999, p. 1). Emotions are still at the basis of these criteria. The concept of "emotional orientation" allows me to speak of the interconnection between rationality and emotion. In fact, as the words themselves suggest, the "orientation" of a subject oriented towards validity claims is "emotional", that is, affected by the emotions in a certain way. But there is still a methodological problem of how, practically, this entanglement can be analysed. Hence, I sketchily present an adaptation of the theoretical framework of the emotional orientation in order to speak practically about these two sides of the same coin. I define the "emotional orientation" of a subject (e.g. a teacher) in terms of "the set of her expectations": the term "expectation" is connected to her "emotions of being right" when she uses specific criteria for accepting an explanation by a community (e.g. a class) rather than other ones (Ferrara & De Simone, 2014). The most difficulty encountered in studying emotions is their "visibility" and, then, their "certain" identification. In this context, when I speak of emotion of the teacher I will refer to her *emotionality*, namely the set of "behaviours that are observable and

theoretically linked to the (hypothetical) underlying emotion" (Reber & Reber, 2001).

## METHODOLOGY

The study presented in this paper is part of the research for my PhD thesis whose focus is on aspects related to rationality of the teacher in the teaching of linear equations at secondary school. The participants were 3 teachers and their grade 9 classes, in a scientifically oriented secondary school in Western Italy. The teachers were selected assuming that rationality and emotions are proper of human beings and with the purpose of having different emotional orientations. Each teacher was first interviewed and asked about her personal beliefs on the topic of linear equations, on algebra in general and on how she uses the didactical materials. Each interview lasted roughly twenty minutes and was videotaped with one camera facing the interviewer and the subject. Then, the whole class activities conducted by the teacher and the students' working group activities were also videotaped. All voice and bodily movement during the interviews and the classroom activities were recorded. The videos were transcribed for data analysis. For the identification of the emotional orientation of the teachers I paid attention also to some indicators that allow me to say something about the emotions of the teacher. In particular, I considered as indicators the tone of voice, the words, the repetitions, the emphasis and the body language (facial expressions, gazes, gestures...). So, I identified the expectations of the teacher – that constitute her emotional orientation – starting from what she explicitly declared in an *a-priori* interview. Then, I tried to find them again, reflected in the class activities, through the indicators I listed above. At last, I analysed the transcriptions from both the emotional and the rational point of view, at the same time, because they are naturally entwined. Due to space constraints, I present the case of one of the three teachers involved in the whole research, whom I call Lorenza.

## AN EXAMPLE: THE EMOTIONAL ORIENTATION OF LORENZA

I identified different expectations that constitute the emotional orientation of Lorenza, but for the limited space, I show just one of them. From the interview, I identified her *expectation about the validity of the previous knowledge of the students that can be used for*

*constructing the new one*. With “previous knowledge”, I mean what students have learnt both in the middle school and with her. In order to highlight this expectation, I collected the moments of her interview from which this expectation could be detected. Lorenza explicitly declared: “Usually, I begin to treat linear equations starting from their previous knowledge *in order to see whether it is valid, or whether the students have misinterpreted the various procedures that they have been taught in the previous years. Anyway, I begin a new topic starting from the knowledge that the students already have*”. During the interview, I asked Lorenza when she introduces for the first time the letters in algebra. She answered that she usually uses letters for the first time in physics, but she commented: “*they are already able a little bit to manage it*” and, then, she repeated the same concept: “*even if when we speak of sets, the letter represents already something for them or also in the logic language the logic variable, then there is already a formalization from this point of view, we say*”. In another passage, it was asked to Lorenza when she speaks for the first time of equations and if she links the concept of linear equation with that of function. She stated that she makes this link for the first time in physics: “*in physics we have already said something about the equations, but just basics because I wanted to put them to work on inverse formula, then I said: “What do you know?” they know already something and they know to deduce or, in theory, they should be able to deduce an inverse formula given a formula*”. An interesting thing is that, during the interview, she explicitly made just the same question that she asks to her students, perhaps, because she is used to make it to her students for testing what is their knowledge and if it’s valid. Lorenza added that students have already known something about equation, but “*in a very naïve way*”, so they have to go in depth with her become aware of the link between the equations and the straight line.

From these pieces of the interview, it becomes quite clear that Lorenza believes that it is important to recall the previous knowledge of the students during all the lessons, not just when a new mathematical topic is introduced.

After detecting Lorenza’s expectations, I analysed the transcriptions of her lessons in which they are actually reflected. I will show how her “orientation” towards validity claims is “emotional”, that is affected by emotions. Using a metaphor, rationality and emo-

tions of the teacher can be seen as the weave and the warp of the fabric. As the weave and warp entwined constitute the fabric, the rationality and the emotions entwined shape the teacher as she actually is.

The first example I propose is taken from the first lesson after Easter holidays, during which the teacher was recalling the concept of identity – explained in the last lesson before Easter – with the aim of introducing, formally, the concept of equation.

- 1 T: before holidays, I hope that someone remembers just something, we have spoken about [pronouncing] identities, then, is there anyone who wants to give, for now, [tone of voice of a statement not of a question] the definition of identity and to do only an example of identity? Don’t be shy! [smiling] (Figure 1) [lifting up her chin and biting her lips] (Figure 2). Please [she addresses to S1 who is raising up his hand]



Figure 1



Figure 2

- 2 S1: it is an equality that is verified for each value that it is replaced to the letter
- 3 T: fine, it is an equality between two expressions, that contain letters, that is verified for each value we go to ascribe to

- the unknown. One example, we have done an example within the classical ones [smiling]
- 4 S1:  $(a+b)^2=a^2+b^2+2ab$
- 5 T: for example, the development of a special product is an equality between two expressions that contain letters and then it can be considered an identity and each value we go to give to the unknown  $a$  or to the unknown  $b$ , the result on the left and on the right of the equality sign must be the same and, conversely, what can be considered as an equation, do you remember? [speeding up] you have already seen them in the middle school, partly, no? yes [she is answering herself], we have already reviewed in physics since at the beginning of the year they serve us for working with formula etcetera, so we have already given indications. In the light of this path that we have done, any of you would like to hazard a definition of equation [tone of voice proper of a statement not of a question and, then, she lifts up her chin]? Try to hazard, Andrea!

The action of Lorenza of asking something that students already know (the definition of identity and an example of it) is aimed at constructing the concept of equation (#5). This action comes along with a particular tone of voice not proper of a question, but rather of a statement (#1). The affirmative tone of voice of the question and that facial expression (#1: she lifts up her chin after speaking) could show her expectation that someone remembers the concept of identity and will answer to her, because the class has already seen it a short time before. Waiting an answer, she laughs (Figure 1) and she bites her lips (Figure 2), probably, because she wants a feedback from the class. The action of asking something that the students should know is full of emotional hues linked to her expectation about the validity of the previous knowledge. This passage of the transcript highlights an *emotional teleological rationality* of Lorenza. It is not just a matter of what she is doing, but rather of *how* and *why* she is acting in that way. From the beginning, her speech seems to be charged by emotions (#1: “I hope”, “just something”, “Don’t be shy!”). These emotions are related to her expectation (“I hope”) that students remember the concept of identity, even “just something” (she can be easily satisfied, as long as, they are able to say something). She seems quite confident about their knowledge, thinking that her students don’t answer

because they are shy, indeed, she incites them into doing, using the imperative phrase “Don’t be shy!”. This “emotion-soaked” speech highlights an *emotional communicative rationality* related to her expectation about classroom culture. There is not only what she is communicating, but also *how* and *why* she is doing it that way. Requiring again the example of an identity (#3), after the answer of S1, could be interpreted as a way of involving more students in the discussion and to evoke the classroom culture as much as possible (#3: “One example, we have done an example within the classical ones”). Another time the teacher’s speech comes along with an emotional element (she smiles), because she seems to feel that students need to be comfortable for answering, even if they already should know the example.

Lorenza recalls just the term of identity to shift easily to that of equation: the former is an equality true for every value of the unknown, while the latter is an equality that may not be satisfied or, in the case it is satisfied, it can be undetermined or determined. This epistemic shifting comes along with an insistence of Lorenza on the fact that they already learn first degree equations both in the lower secondary school (grade 8) and with her in grade 9 (#5: “you have already seen them in the middle school, partly, no?”, “yes [she is answering herself], we have already reviewed in physics since at the beginning of the year they serve us for working with formula etcetera, so we have already given indications”, “In the light of this path that we have done any of you would like to hazard a definition of equation”). In addition, she asked her students “to hazard” a definition of equation, but with the tone of voice proper of a statement and not of a question (#5), probably because she is expecting that students construct new knowledge starting from the previous one. The insistence in the speech, the facial expression, the tone of voice linked to the knowledge into play could inform us about an *emotional epistemic rationality*.

Then, the discussion goes on as follows:

- 6 S2: it is an equality between two literal expressions in which the value of  $x$  is replaced by a unique value to make it true
- 7 T: we say that it is satisfied just f(or)
- 8 S2: for a single value of  $x$
- 9 T: always?! (Figure 3), do we always find it?!for you this value or (Figure 4), let’s try to think a little bit



Figure 3



Figure 4

- 10 S2: sometimes it's impossible  
 11 T: It could be, right.

After the answer of S2 (#6), Lorenza clarified his definition of an equation. This action is aimed at a first introduction of the different types of equation (determined, undetermined, impossible), which she will develop in the next lesson. The rhetorical questions (#9: “always?!”; “do we always find it?!”), the facial expressions in Figure 3 and Figure 4 and the general involvement in the discussion (#9: “let’s try to think a little bit”) accompany the actions she made on the basis of a certain knowledge into play, communicating in a specific manner. Particularly, in Figure 4, she seems to “catch” with her hands what they are in mind about the concept of equation. This frame brings out an *emotional teleological, epistemic and communicative rationality* of Lorenza.

## DISCUSSION

I presented an emblematic example of the coexistence of the emotional rational aspects in mathematics teaching. In particular, I showed how this merger outlines the decision-making processes of the teacher. As I highlighted in the analysis of the example, all the teachers’ decisions – about knowing, acting and speaking – are “visible” in language, but, mostly, in her emotional aspects. This doesn’t mean that emo-

tions explain the decisions, but, rather, that decisions are very often “visible” through emotions. I tried to study this complexity through an adaptation of the concept of “emotional orientation”, used by Brown and Reid in 2006, drawing on the work of Maturana (1988).

Referring to the transcript, from one side, the emotions of Lorenza are linked to her expectation about the class culture she developed from her own beliefs. These emotions can become clear from her tone of voice, her way of speaking, her body language. From the other side, the choices of Lorenza (starting from the identities to introduce equations, recalling explicitly with insistence the previous knowledge of the students...) are connected to this expectation of constructing new knowledge, basing on the validity of the previous one. Then, the emotions are strictly related to the choices and this gives the meaning of how the “orientation” of Lorenza can be “emotional”.

Hence, the analysis of the discursive activity of the teacher has naturally led to propose an enlargement of the Habermas components of rationality. As shown in the analysis of the excerpt, I tried to highlight the *emotional epistemic rationality*, the *emotional teleological rationality* and the *emotional communicative rationality* of Lorenza. In this context, I consider the *emotional epistemic rationality* as related to *why* the teacher uses that specific justification of the knowledge at play; the *emotional teleological rationality* as related to *why* the teacher makes that actions to achieve a goal and the *emotional communicative rationality* as related to *why* the teacher uses that speech oriented towards validity claims. These three adapted components of rationality are always present in the discursive activity of the teacher. Obviously, as testified by the example, during specific moments of the classroom activity, one component could emerge more than the others.

The role of the *a-priori* interview results particularly significant for this kind of analysis since it enabled to scrutinize the teacher’s beliefs and orientations for the teaching of the concept at stake.

## REFERENCES

- Bishop, A. J. (1976). Decision-making, the intervening variable. *Educational Studies in Mathematics*, 7(1–2), 41–47.

- Boero, P., Douek N., Morselli F., & Pedemonte B. (2010). Argumentation and proof: a contribution to theoretical perspectives and their classroom implementation. In M. M. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 179–204). Belo Horizonte, Brazil: PME.
- Boero, P., & Planas N. (2014). Habermas' construct of rational behavior in mathematics education: new advances and research questions. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 1, pp. 205–235). Vancouver, Canada: PME.
- Brown, L., & Reid D. A. (2006). Embodied cognition: Somatic markers, purposes and emotional orientations. *Educational Studies in Mathematics*, 63(2), 179–192.
- Damasio, A. (1994). *Descartes' Error: Emotion, Reason and Human Brain*. New York, NY: Putnam.
- Damasio, A. (1999). *The Feeling of What Happens: Body and Emotion in the Making of Consciousness*. San Diego, CA: Harcourt.
- Di Martino, P., & Zan, R. (2010). 'Me and maths': Towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teachers Education*, 13(1), 27–48.
- Ferrara, F., & De Simone M. (2014). Using Habermas in the study of mathematics teaching: the need for a wider perspective. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 1, pp. 223–228).
- Habermas, J. (1998). *On the pragmatics of communication*. Cambridge, MA: MIT Press.
- Hannula, M. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education*, 14(2), 137–161.
- Immordino-Yang, M. H., & Damasio, A. (2007). We feel, therefore we learn: The relevance of affective and social neuroscience to education. *Mind, Brain, and Education*, 1(1), 3–10.
- Maturana, H. R. (1988). Reality: the search for objectivity or the quest for a compelling argument. *The Irish Journal of Psychology*, 9(1), 25–82.
- McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Learning and Teaching* (pp. 575–596). New York, NY: MacMillan.
- Morselli, F., & Boero, P. (2011). Using Habermas' theory of rationality to gain insight into students' understanding of algebraic language. In *Early algebraization* (pp. 453–481). Berlin, Germany: Springer.
- Reber, A. S. (1995). *The Penguin dictionary of psychology*. New York, NY: Penguin Press.
- Reid, D. (1999). Needing to explain: The mathematical emotional orientation. In Orit Zaslavsky (Ed.), *Proceedings of the Twentieth-third Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4 pp. 105–112). Haifa, Israel: PME.
- Rienstra, B., & Hook, D. (2006). Weakening Habermas: the undoing of communicative rationality. *Politikon: South African journal of political studies*, 33(3), 313–339.
- Schoenfeld, A. H. (2010). *How we think: A theory of goal-oriented decision making and its educational applications*. New York, NY: Routledge.
- Zan, R., Brown, L., Evans, J., & Hannula, M. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics*, 63(2), 113–121.