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# Identity and rationality in group discussion: An exploratory study

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*In this contribution we study group work in mathematics adopting a socio-cultural perspective and combining two theoretical lenses: the construct of identity and that of rational behavior. More specifically, we show how individual identity and dimensions of rationality in interaction may influence conceptual change. Data analysis is performed on excerpts from a group work (age of the students: 12) on negative numbers.*

**Keywords:** Group work, identity, rationality.

## INTRODUCTION

Group work as a methodology is often advocated in mathematics education, and its value is often taken as obvious by researchers and also teachers. Nevertheless, working in group does not immediately turn into a search for a common solution. What happens to the students when they see the others doing something different from them? Does every student care about the agreement with classmates? Research may help figuring out the possible causes of failure of group activities. Conversely, research may also show to those teachers, who ask their students to provide a common solution for group activities, how difficult is for students to reach on their own an agreement and how far is a solution from being “common” and accepted by all the members of the group. In this contribution we study group discussion adopting a socio-cultural perspective and combining two theoretical lenses: the construct of identity and that of rational behavior. By means of a networked analysis, we aim at better understanding what happens during group interaction, in particular what makes a group interaction efficient or not for students involved in a task-based activity designed with the aim of stimulating a conceptual change.

## LITERATURE REVIEW AND THEORETICAL FRAMEWORK

### Group work and sociocultural perspective

The pioneering analysis carried out by Vygotskij (1978) concerning the crucial role of social activities mediated by signs and language in the development of mathematical thinking traced a path in which a whole thread of researchers placed their roots. The effects of students’ interactions in classroom activities have been studied, described and interpreted in mathematics education since its origins, see Radford (2011) for an overview. We rely on a sociocultural perspective, according to which the learning of mathematics takes place in a social context through interactions and is deeply affected by culture (Radford, 2006; 2011). Radford (2006, p. 58) affirms:

Certainly, the students were actively engaged in what has been termed a “negotiation of meaning”. But this term can be terribly misleading in that it may lead us to believe that the attainment of the concept is a mere consensual question of classroom interaction. [...] meaning also has a cultural-historical dimension [...]. It is in fact this cultural object that shapes and explains the teacher’s intervention [...] classroom interaction and the students’ subjective meaning are pushed towards specific directions of conceptual development. Cultural conceptual objects are like lighthouses that orient navigators’ sailing boats. They impress classroom interaction with a specific teleology.

Students are involved in a double-faced problem: they meet at the same time the culture and the others and have to find a place in both the cultural and the classroom discourses, that are related but not necessarily equal. In particular we will focus on the classroom

discourse side. Even if the attainment of a concept is not a mere consensual question, the agreement between students is very important in mathematics group activities, as we will show. We refer to Radford's interpretation of cultural-historical activity theory (Roth, Radford, & Lacroix, 2012). This theory is rooted in Leont'ev and Vygotskij's dialectical psychological theories. The keyword *activity* is defined as a common place in which cognition and consciousness arise; through activity individuals relate not only to the world of objects but also to other individuals. Learning is the result of a shared common practice that involves students' subjectivities and in which subjectivities moves towards others and culture to find and transform themselves. In Radford (2008) it is pointed out that students' interaction in a small group is a complex process in which students are involved at many levels, not only at the cognitive one. The processes of objectification (students align their thoughts with culture) and subjectification (a *thinking and becoming* process of being-with-others mediated by alterity) that take place in the teamwork are mediated by culture.

### Conceptual change

In our perspective, learning is a *thinking and becoming* process in which students are involved not only at the cognitive level, and a continuous negotiation of meanings between one individual and another mediated by culture (Radford, 2008). We conjecture that conceptual change, as part of mathematics learning, cannot be seen just as "change of concept" at cognitive level: it is a social act deeply related to the subjectivities of students involved in group activity. Drawing from the definition of concept as an emergent object that condenses or generalizes the previous practices (Sfard, 1991), we speak of *group concept* as an object emerging from the individual or shared activities, whose validity is recognized and accepted by all the students.

### Identifying and subjectifying

In the thread of Leont'ev and Vygotskij's dialectical psychological theories presented before, we analyze group activity by means of one analytical tool that was introduced by Heyd-Metzuyanim (2009). The author presents a useful tool to distinguish the different ways of interacting of each student in terms of individuality, in particular in a mathematics group work. The tool "allows to point out how identity and emotional processes influence the effectiveness of learning.

Subjectifying may help in mathematizing or obstruct it" (Heyd-Metzuyanim, 2009, p. 2). The subjectification process is linked both theoretically and operationally to the identity construction process and to the mathematizing activity in group work. The starting point is the definition of identity (Sfard & Prusak, 2005, p. 1): "Identity is a set of reifying, significant, endorsable stories about a person." This definition is deeply related to the *commognitive perspective* (Sfard, 2008), whose cores are the notion of thinking and communicating. Since thinking is a form of human doing, it can only develop as a collective patterned activity: "Thinking is an individualized version of (interpersonal) communicating." (Sfard, 2008, p. 81). Heyd-Metzuyanim frames mathematizing and subjectifying in the commognitive perspective: mathematizing is communicating about mathematical objects, subjectifying is communicating about participants of the discourse. In this perspective the construction of identity takes place as an internalization of collective discourses that make stories about the self arising. These stories can talk about the way in which a person relates to the mathematics and so can influence the participation in the teamwork, the engagement, and definitively, success or failure in mathematics activities. In her work, Heyd-Metzuyanim (2009) looks at verbal and non-verbal acts of subjectification, distinguishing participation and membership. She operationalizes the notion of *resistance to participation*, seen as a type of subjectifying action always interpreted according to context, especially to the reactions of other participants, especially the teacher. Then she analyzes these acts, deciding if they are identifying processes or not. Identifying utterances (verbal or non-verbal) are "those that signal that the identifier considers a given feature of the identified person as permanent and significant." (Heyd-Metzuyanim, 2009, p. 2). The prototypical cases of different aspects of the relation between subjectifying, mathematizing and identifying are exemplified in Table 1.

### Rationality

The construct of rationality was presented by Habermas (1998) in reference to discursive practice and later adapted to mathematical activity (see Morselli & Boero, 2009 for the special case of mathematical proving). According to Habermas, rational behaviour may be seen as three interrelated dimensions: epistemic dimension (related to the control of the propositions and their chaining), teleological dimension (related to the conscious choice of tools to

achieve the goal of the activity) and communicative one (related to the conscious choice of suitable means of communication within a given community).

### Identity, rationality and conceptual change

We hypothesize that the conceptual change is a social act and that acceptance of this change may depend on a group-coherence, i.e., a sort of agreement reached using cultural tools provided by the teachers or personal tools. Subjectification and participation can be considered at the same time stimulus for engaging in a group conceptual change process and an obstacle in the individual conceptual change process. In order to describe individual contributions within the group work, we add the construct of rational behavior. Since we will deal with peer interaction, the communicative dimension will have a relevant role. Moreover the epistemic rationality, encompassing the possibility of changing opinion, seems to be linked to group work and conceptual change:

Someone is irrational if she puts forward her beliefs dogmatically, clinging to them although she sees that she cannot justify them. In order to qualify a belief as rational, it is sufficient that it can be held to be true on the basis of good reasons in the relevant context of justification - that is, that it can be accepted rationally. The rationality of a judgment does not imply its truth but merely its justified acceptability in a given context. (Habermas, 1998, p. 310)

## RESEARCH QUESTIONS

We wonder whether identity and rationality may have a role in influencing (positively or negatively) the conceptual change that may occur during group problem solving activity. The initial research focus on group work can be turned into the following research questions: 1) Is it possible to describe group interactions in terms of identity and subjectification? 2) Are there cases of resistance to participation? 3) Is group conceptual change an act of social agreement, and this necessity of agreement influence the individual conceptual change? 4) What is added by the analytical tool of rationality?

## METHODOLOGY

### The task

We present and analyze data from a teaching experiment carried out in grade 6 (age of the students: 12).

The teaching experiment concerned the concept of negative numbers. At first, students were asked to answer individually these questions: 1) *What is a number?* 2) *What is it possible to do with numbers?* Afterwards, they worked in group on a task to be solved on the Cartesian axes. The negative part of the axes was used by the students and such a solution was after institutionalized by the teacher. Finally, the students were asked to answer in group to the following questions: 3) *You said that numbers are [reference to their former individual answers]... and with numbers you can do [reference to their former individual answers]... Do you confirm your opinions now?* 4) *Negative numbers are numbers in the sense you intended before?*

The aim is to analyze the conceptual change that occurs when moving from natural numbers to negative numbers to whole numbers, that is to say a wider set that contains both positive and negative numbers. Here we confine to one episode referring to the group work on questions 3 and 4.

### Using the analytical tools

We perform a networked analysis, combining different theoretical tools with the aim of reaching a fuller understanding of the episode at issue. The lens of conceptual change allows to characterize the acts of mathematizing analyzed in our research. The first analysis aims at detecting verbal and non-verbal acts that are signal of participation or resistance to participation, membership or non-membership, mathematizing, identifying. The lens of rationality can provide information about the rationality dimension of these sentences. The joint analysis addresses the topic of relating mathematizing and subjectifying acts to rationality.

At first we analyze the episode in terms of identity, subjectification and conceptual change, as derived from Heyd-Metzuyanım's paper (2009). Some sentences from the transcript are interpreted following the criteria proposed by the author in Table 1 and labeled with the codes: identity (I), subjectification (S) and conceptual change (C). Also the specific codes for subjectification used by Heyd-Metzuyanım will be used. Afterwards, we add the analytical tool of rational behavior. We refer to the epistemic dimension when one sentence is linked to a mathematical fact, and we speak of lack at epistemic level if some assumption is taken per se, without the need for a justification. For instance, in the very first part of the working group

one student says that negative numbers are to be considered numbers (“*Because she (the teacher) spoke us of negative and positive*”): she is just relying on the authority of the teacher, without paying any effort towards a real understanding. We may say that her statement lacks in terms of epistemic rationality. We refer to the teleological dimension when the action is clearly linked to a goal (and we report a lack in teleological rationality when the reference to the final goal is missing). We refer to communicative rationality when a special care is paid to the organization of the discourse, so as to make the listener to understand. For instance, one student’s wide use of drawings and diagrams may be linked to her effort in making her positions understandable to others, thus to a communicative dimension.

## DATA ANALYSIS

At the beginning of the group work, students discuss to decide whether it is possible to deal with negative number as numbers. The selected episode refers to a connected crucial issue: whether it is possible to perform operations with negative numbers.

### Analysis in terms of identity (I), subjectification (S) and conceptual change (C)

In the subsequent part, we present an example of analysis of one episode in terms of identity (I), subjectification (S) and conceptual change (C), with more specific

codes used by Heyd-Metzuyanim (2009). Some labels are assigned even if in the transcript the utterances are not recurrent, because they are repeated many times in the whole transcript.

Other isolated episodes allow to characterize students’ behaviors in terms of identifying and subjectifying acts and to link students’ way to attend the group work and conceptual development. *Nor* always works on her own, never speaks, even if the teacher asks to do it. Every non-verbal act performed by *Nor* may be read in terms of *resistance to participation*. 6 on 8 students look for agreement but with different aims: some of them to convince the classmates, some to make the work of others coherent to their personal discourse, other to be accepted from the classmates rather than thinking at mathematical contents (as student *Ari*). From the analysis we grasp different ways to participate in the group, that affect personal concept development: *Mar* and *Nor*, working on their own, develop personal concepts; *Ari* and *Luc* abandon their conceptual change to find an agreement; *Bea* and *Er* follow the group conceptual development and only ask for clarifications; *Giu* imposes her point of view on all the group affirming her personal opposition to conceptual change. She identifies clearly herself as good in math and influences the whole process, even when she’s not right. The agreement is not reached and the group conceptual change doesn’t occur. This failure drives all the students but *Giu* to abandon

	Type			Example	Identifying?
Mathematizing	Mth				No
Subjectifying	Pe: participation evaluating	Sp: related to a specific performance		Saying “I don’t understand”	No
		Ge: generalizing		Saying “I hate doing this”	If consistent with other data
	Me: membership evaluating	Vb: verbal	Di: direct	Saying “I am a math person”	Yes
			Id: indirect	Changing the subject of discourse, which can be interpreted as “I don’t want to talk about this”	Depends on the nature and frequency of the utterance
		Nv: non verbal	Di: direct	Raising one’s hand, which may be interpreted as “I wish to speak about this”	Only if recurrent
			Id: indirect	Groaning at a given task, which may be interpreted as “I don’t like this”	Only if recurrent

**Table 1:** Prototypical cases and labels (Heyd-Metzuyanim, 2009)



11 <i>Giu</i> : it must be for all the operations, and then -3 times -2 is equal to?	<i>Giu</i> is mathematizing but also identifying as good in math. (I) [Me Nv Di] <i>Giu</i> provokes a change in the discourse generality: "If it's true, it has to be always true, for every kind of operation" (C)
12 <i>Ari</i> : just a minute, I wrote: This means that it is negative number, but anyway you can do $-2+3$ [in column] you get -5.	
13 <i>Teacher</i> : and how do you get it?	
14 <i>Ari</i> : the sign means that it is a negative number, then I can do... that sign does not mean anything! It just means that it is a negative number, so...	<i>Ari</i> is mathematizing and suggesting a path for a conceptual change from numbers as natural to numbers as positive and negative (C).
15 <i>Giu</i> : no, A, what you are saying is meaningless.	<i>Giu</i> introduces the question of the sense of the operations with negative numbers. This change the status of the following statements (C). She also talks about <i>Ari</i> (S) [Pe Sp] and confirms <i>Ari</i> 's identity as person not good in math, as she did for the whole discussion saying that as usually she doesn't understand (Is) [Me V Di]
16 <i>Ari</i> : yes.	
17 <i>Giu</i> : -16:-4? [laughing]	<i>Giu</i> laughs when she doesn't agree with <i>Ari</i> 's proposals, showing self-confidence in math (I) [Me Nv Di]
18 <i>Ari</i> : -16? You must do $16-4$ [after, she adds the minus before the numbers, in column].	
19 <i>Giu</i> : try and do the division. The division.	<i>Giu</i> is mathematizing but also identifying as good in math and saying to <i>Ari</i> what she has to do (I). [Me Nv Di]
20 <i>Ari</i> : ok.	<i>Ari</i> interrupts her activity accepting <i>Giu</i> 's request (S). [Pe Nv Sp]
21 <i>Luc</i> : wait a minute! [she takes the pen].	<i>Luc</i> wants to find a place in the discussion (S) [Me Nv Di]
22 <i>Giu</i> : let me speak!	<i>Giu</i> wants to participate (S) [Me Nv Di]
23 <i>Luc</i> [she does -4 divided by -2, and she writes -2]. Here you are! -4 divided by -2 is -2.	<i>Luc</i> tries to answer to <i>Giu</i> 's provoking question but doesn't satisfy <i>Giu</i> 's request (C).
24 [ <i>Ari</i> does again $16:4$ ].	

Table 2: Transcript and analysis

their personal path to conceptual change because of group disagreement. So neither the group conceptual change nor the individuals occur and the first event seems to be cause of the second.

### ANALYSIS IN TERMS OF RATIONALITY

The theoretical lens of rationality is used to gain understanding of interactions. Here we confine our analysis of *Giu* and *Luc*'s excerpts, with a special focus on the part of discussion on the possibility of performing operations with negative numbers. A first result is the crucial role of teleological rationality. *Giu*'s initial position is that negative numbers are not numbers as positive ones and she does not want to modify her position. According to this goal (*teleological rationality*), she challenges the propositions of her groupmates. Her interventions are mainly on the *epistemic* level:

she suggests that, in order to have negative numbers a numbers, it must be possible to perform operations, and she asks for justification and meaning for such operations (interventions 15, 19). *Giu*'s requests to the groupmates are at epistemic level, she pretends justification and meaning. *Luc*'s initial position is that negative numbers are not numbers like positive ones (she sees the number and the minus as separate objects). Nevertheless, she is keen to change her opinion: her priority is to gain a group solution. According to this aim (*teleological rationality*) she puts great effort in sharing her ideas with the mates, for instance using diagrams (*communicative rationality*) (intervention 26). When challenged by *Giu*, she is ready to find solutions (e.g. how to perform operations; see intervention 23) and she is rapidly satisfied. She does not put much effort in justifying or giving meaning to the methods. In her exchanges with *Giu* the commu-

nicative dimension is prevailing; she does not move to the epistemic level, then she does not come to an agreement with *Giu*.

## DISCUSSION AND PRELIMINARY CONCLUSIONS

The first part of the analysis shows that the students participate in the group work in different ways. There are even two cases of *resistance to participation*. We argue that different participation sometimes affects personal concept development and, of course, hinders conceptual change as a group. At the end of the session, there is not a general agreement. We wonder why it was not possible, in spite of individual good ideas, to reach an agreement and why some students abandon their attempt if groupmates don't agree. We turn then to the theoretical lens of rationality. The analysis in terms of rationality shows that teleological rationality may refer to different goals and that some interventions are clearly on communicative or epistemic level. Combining the two analysis, we can state that individual *participation or resistance to participation* and also *membership or non-membership* may be described in terms of *dimensions of rationality*: if individual interventions are on different levels (epistemic vs communicative), it seems very difficult to reach an agreement. If a dimension prevails, some students can avoid to participate. Moreover, individuals may have different aims and act accordingly (teleological rationality), may consider the epistemic dimension or not, and this may affect individual/collective conceptual change. We hypothesize that group work, in order to be efficient, should take into account the three dimensions (in particular, the epistemic dimension can not be neglected); moreover, group interactions are not fruitful if the groupmates focus on different dimensions.

The preliminary results of this study suggest further research. From one side, we plan to analyze other data (including long-term observations of the students), in order to test our working hypothesis concerning the link between identity, conceptual change and rationality. Moreover, we see other issues that need further exploration:

- 1) The link between identity and teleological rationality brings to the fore the relationship between identity and goals. This could also be linked to the work of Gómez-Chacón (2011), who draws from Camilleri and colleagues (1990) the idea of identi-

ty strategies as “processes or procedures set into action (consciously or unconsciously) by an agent (individual or group) to reach one or more goals (explicitly stated or situated at an unconscious level); procedures elaborated in function of the interaction situation, that is in function of the different determinations (socio-historical, cultural, psychological) of this situation” (p. 24).

- 2) The role of the teacher is crucial in helping students to interact at the same level; furthermore, we hypothesize that other kind of tasks, for instance aimed at comparing individual solutions rather than providing immediately a group solution, could be more efficient.
- 3) Finally, we wonder whether there is a link between identity and rationality: more specifically, we wonder whether the resistance to participation depends on the dimension of rationality that most characterizes the identity.

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