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Problem posing: Students between driven creativity and mathematical failure

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We explore behaviors of high achievers 10 to 16 years old during a problem modification process by looking at the ways students vary the constraints of a given problem. We found that these children swing between low amplitude driven creativity and failure to pose mathematically consistent problems when it comes about understanding the deep structure of mathematical concepts and strategies.

Keywords: Problem posing, problem modification, constraints, mathematical creativity, endogenous and exogenous transformations.

INTRODUCTION

We exposed students from grades 4 to 10 to a problem posing context. We were interested in this case to see how mathematical creativity manifests in students and what is (if any) the relationship between their creative approaches and the quality of the mathematics problems they generate.

In general, in problem-posing contexts, students are stimulated to make observations, experiment through varying some parameters, and devise their own new problems (Singer, Ellerton, Cai, & Leung, 2013). In this paper, we accept that problem posing refers to the generation of (completely) new problems, and to the re-formulation/ modification of given problems (Silver, 1994). We specifically address here the context of problem modification.

Previous studies have shown that, in problem posing situations, mathematically able students tend to vary a single parameter in order to ensure the control over the relationships among all the elements of a posed problem, even when they make interesting

generalizations. This tendency of small-step variations was interpreted as a student's need to keep as coherent and consistent as possible his/her proposals by controlling (sometimes unintentionally) the consequences of the proposed changes (Singer, 2012a; Voica & Singer, 2013). In a problem-modification study, Singer, Pelczer and Voica have shown that the capacity of a student to pose coherent and consistent changes of a problem and to understand the (mathematical) consequences of these changes proves student's capability for deep transfer creative approaches (Singer, Pelczer, & Voica, 2011). Therefore, this capability could predict mathematical creativity, which has been characterized as being different from general creativity (e.g., Piirto, 1999).

In describing mathematical creativity, we use a framework based on cognitive flexibility, which is characterized by: cognitive novelty, cognitive variety, and changes in cognitive framing (e.g., Voica & Singer, 2013). In a problem-modification context, we consider that a student proves cognitive flexibility when she or he poses different new problems starting from a given input (i.e. cognitive variety), generates new proposals that are far from the starting item (i.e. cognitive novelty), and is able to change his/her mental frame in solving problems or identifying/discovering new ones (i.e., change in cognitive framing).

In addition, these studies have shown that the more the student advances in the abstract dimension of a given problem, the more mathematically relevant are his/her newly obtained versions. It seems that the students who are cautious and minimally change the problem are in fact mathematically advanced students who show proper insights on some mathematical concepts that exceed their age level. Actually, they pay careful attention to controlling the values of the vari-

ables, as mathematicians usually do. These studies conclude that a feature of mathematical creativity, at least for the students involved in the research (10–16 years old), is a type of cognitive flexibility characterized not so much by novelty, but by incremental representational changes in cognitive frames (Singer, 2012b; Voica & Singer, 2013).

In the present study, we start from the assumption, suggested by previous experimental research (e.g., Voica & Singer, 2014), that problem posing (and its particular case – problem modification) is a useful context for identifying mathematical creativity, and we try to go further in determining correlations between students’ mathematical creativity and the quality of their posed problems. Thus, we are trying to answer the question: What are the gains and the losses from the view of creativity, when students do mathematical problem posing?

METHODOLOGY

The sample of this study consists of students from grades 4 to 10 (10–16 years old), participants in a summer camp organized for the winners of a large-scale two-round competition (Kangaroo). The camp participants, representing the top 0.2% from a total of about 150 000 participants at the competition, are considered high achievers in mathematics. During this summer camp we launched a call for problems. The participants had to choose a problem from a list of three problems (given as reference points) and, after solving it, to pose three problems based on the starting reference problem: one simpler, one similar, and one more difficult. In addition, the students were supposed to write down the solutions of their posed problems and to explain how their proposals fit the requested criterion (simpler, similar, etc.). We used therefore a structured problem-posing situation (in the terminology of Stoyanova and Ellerton, 1996). Students had 3 days to formulate their answers. In all, 57 students submitted problems.

Each of the posed problems was evaluated by two independent evaluators who assessed it based on a set of criteria: correctness and completeness of the problem formulation and the proposed solution, originality of the text, and clarity of explanations. Based on this primary evaluation, we invited a few participants for interview. Among the selected students were those whose problems were highly ranked by the evalua-

tors. We also invited for interview two students who posed problems considered with potential for further development. In all, 20 students were interviewed. The interviews were based on a general protocol concerning issues such as: What are the similarities/differences between the posed problem and the starting one? What other problems are possible to pose? What make you think to such approach?

The interviews were carried out with two students at a time, thus creating a context for interactions between the children. This helped us to reveal students’ ideas through discussions with their peers (Vygotsky, 1978) and to get consistent feedback on their problem posing strategies and their level of mathematical understanding. The interviews were video recorded.

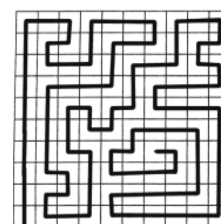
We focus the present paper on the problems generated by the students who have chosen the first problem in the given list (problem 1 below). Consequently, our sample consists of those 19 students (out of which 6 have participated in the interview sessions). The analysis that we do is a qualitative one, mainly based on students’ written submissions. Because of the reduced dimension of this paper, we just present a holistic view on students’ comments during the interviews to highlight our main conclusions.

CONTEXT

The starting problem

The following problem was initially given to students as a reference point: we will identify it as *the starting problem*.

Problem 1. The 10×10 grid is formed out of squares of 1dm. We glue a ribbon as shown in the image. What is the length of the ribbon?



A. 99 dm; B. 99.5 dm; C. 100 dm; D. 100.5 dm; E. 20 dm

We have chosen this problem from the Kangaroo contest to have statistical information on its degree of difficulty. In the competition, this problem was administered to a group of 5659 students from the 5th and the 6th grades. The Kangaroo contest takes place in two rounds and the participants have to answer 30 multiple-choice questions in 75 minutes; therefore,

	A	B	C	D	E	NA
Grade 5 (3278 participants)	10.2	18.5	37.2	8.9	7.7	17.5
Grade 6 (2381 participants)	10.4	25.1	33.9	7.7	7.0	15.8

Table 1: Frequencies of answers to the starting problem in the Kangaroo competition (Correct answer is B. NA stands for no-answer)

optimizing solution strategies is essential in obtaining a good score.

Statistical data about the performance of students participating in the Kangaroo contest show that they perceived the starting problem as difficult: this conclusion follows from the relatively low percentage of correct responses to this problem (see Table 1), the percentage of students that preferred not to answer this question, and the percentage of correct answers obtained by this cohort to other problems (around 75%). Even if we take together answers B and C (concluding that about half of the students had an insight concerning a strategy to optimize the solution to this problem), we can still conclude that the problem was perceived as difficult.

Components of a problem

To better understand the discussion that follows, we detail the components of the starting problem based on the methodology introduced by Singer and Voica (2013). In general, the text of a problem contains: a background theme, (numerical) data, operators (or operating schemes), constraints over the data and the operators, and constraints that involve at least one unknown value of a parameter (Singer & Voica, 2013). The background theme represents, briefly said, what is about in the problem. In our case, the background theme is represented by a grid and a trajectory marked on it. The background theme is characterized by one or more parameters: in our case, the parameter is the shape of the trajectory (of the ribbon). The data are (numerical or literal) values associated to these parameters: in the above problem, these are represented by the number of squares and their dimensions. The operating schemes are actions suggested by the text: here, these are the way of crossing, and the measurement of the trajectory. The constraints imposed on the data and the operators are restrictions that state the relations of the background theme with the data and the operating schemes. In the starting problem, some of the constraints are of *local* nature: for example, “the trajectory” unites the centers of the squares and crosses a square in one of the two modalities shown in Figure 1 (eventually rotated).

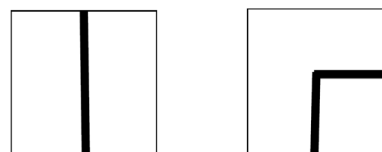


Figure 1: Local constraints for building the trajectory in problem 1

Other constraints are *global*, such as the filling condition – i.e. that the ribbon goes across each cell of the square. These constraints, in combination, allow a quick identification of the answer, since 99 out of 100 squares are crossed and the last one is only half marked (therefore, the correct result is 99.5). We believe that the difficulty of the problem comes from the fact that the solvers did not totally understand the local and the global constraints; usually, they try to perform sequential additions of the ribbon length, which can lead to miscalculations or to abandoning the problem because of the time limit crisis.

The starting problem contains two kinds of constraints: essential in-depth constraints that give the core-structure of the problem, and superficial constraints. The essential constraints manifest at both local and global levels: the global constraint refers to the grid coverage, which combined with the local algorithm (the trajectory crosses the squares in the midpoints of their sides following two different patterns, and there are no self-crossings) allows an optimal solution (as time is concerned). Superficial constraints refer to the dimension of the grid, the shape of the grid (including the base element of the grid), and to the particular path chosen within the grid.

Transformations

We classified the modifications made by the students when they developed a new problem in two categories: *exogenous* and *endogenous* transformations. The terms originate from the system theory, where endogenous transformations are transformations between models expressed in the same language, and exogenous ones are transformations between models expressed using different languages (Mens & Van Gorp, 2006). In problem modification (PM) situations, we consider that a student makes endogenous transformations when he/she uses the same types of operating

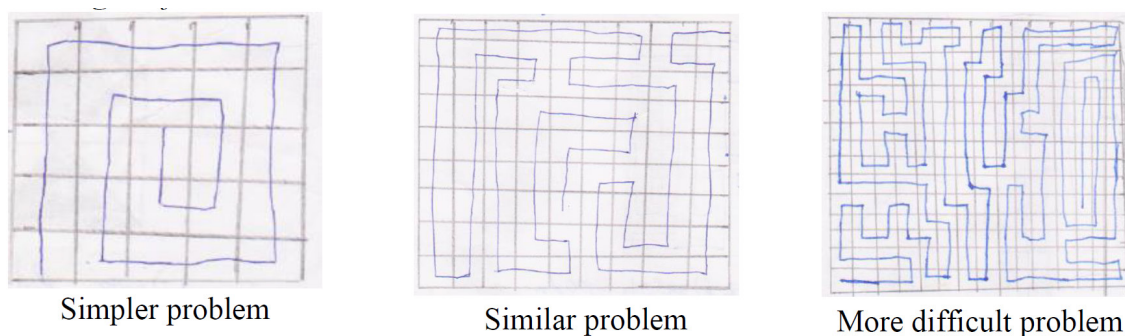


Figure 2: The drawings made by Victor (grade 4) in posing new problems

schemes and constraints as in the starting problem (i.e. makes a new mathematical problem using the same language - the same mathematical concepts as those of the starting problem); otherwise, we consider that the student makes an exogenous transformation. In other words: the endogenous transformation acts only on already present components in the starting problem, while an exogenous transformation will bring new components into the generated problem.

RESULTS

We present our analysis based on some illustrative examples.

Example 1. Victor (grade 4) proposed three problems based on the drawings contained in Figure 2. In all these problems, Victor formulated the same question: *What is the length of the ribbon?*

One can observe that Victor varies the dimensions and the shape of the trajectory. For the last problem (most difficult), Victor avoids two of the squares – therefore, he renounces to the complete coverage of the grid, but he keeps the same type of optimal strategy of solving. The transformations performed by Victor are endogenous: he poses a new problem using elements that exist in the starting problem.

Example 2. Doina (grade 8) based her posed problems on the patterns presented in Figure 3. She put the same requirements for all her problems: finding the length of the drawn broken line. Doina specifies in her problem texts the geometrical properties of the base cell (rhombus, in the first case; regular hexagon in the second; and equilateral triangle in the third). She is very careful in characterizing the local constraints; for example, in the last posed problem, she clarifies the fact that the ribbon crosses the centers of gravity of some of the triangles.

Overall, Doina kept both the local and global defining characteristics unchanged and modified only the layout and the basic element in the tessellation. The modifications took into account the unique association between each cell(s) and measure unit, the nature of the trajectory (no crossings) and the complete coverage of the grid, conditions that allow the optimal solution.

We can observe in this case cognitive flexibility in changing elements without changing the deep structure of the initial problem (solution). This case also illustrates endogenous transformations – since only the elements already present in the initial problem are manipulated.

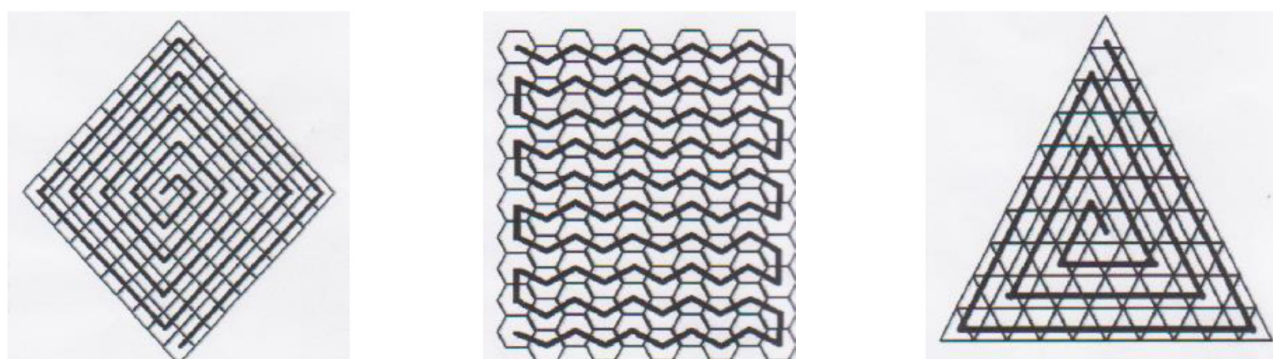


Figure 3: The drawings made by Doina (grade 8) in posing new problems

Example 3. The following problems were proposed by Silviu, a grade 9 student.

Simpler: Dan wants to cut out a shuttle from a textile covered by a square grid (Figure 4a). If the side of a square is 2, what is the remaining surface after the shuttle was cut out?

Similar: An ant crossed a piece of cake of an equilateral triangle shape with the side of 5cm. What is the length of the trajectory marked in the image (Figure 4b)?

More difficult (simplified phrasing): There are four animals competing on square-grid paths (Figure 4c); their speed is given in terms of relations between them. The question is on the order of arrival of the animals.

The “similar” problem has an exogenous modification on the global constraint. Although the student considered this proposal similar to the given problem, this is actually very different when its nature is considered: the only available strategy for solving is the counting of the line segments (two different kinds). Consequently, the student failed to use an optimized algorithm to solve this problem.

Silviu’s simpler and more difficult problems suppose also exogenous transformations. By adding new elements, the requirements changed, and the difficulty remains to be judged based on the amount of calculations to perform. By an exogenous transformation, the new problems are much farther from the given one, but they become procedural, as global solving is not possible.

Example 4. The following problems were proposed by three students (Mircea, Andrei, and Nguyen, grade 5), who decided to work as a group.

Simpler: A monkey wants to reach bananas. In the 7×9 square below the route is indicated (Figure 5a). Knowing that the side of a square is 5m and the monkey crosses 19m in 5 minutes, find how long does it takes the monkey to reach the bananas.

Similar: Ballerina lost her ribbon in the maze. The ribbon is represented in the square grid (Figure 5b). Knowing that the side of a small square is 3m, find the length of the ribbon.

More difficult: A maze is between the houses A and B. Jack wants to go from A to B, so he takes a string to indicate his way to go. On the 11×11 square grid below the Jack’s travel is drawn (Figure 5c). Knowing that a square has a side of 1.2m, find the properties of the number that shows the length of the road.

In their proposals, the three students renounced to the local constraints (the similar and more difficult problems have cross-overs), but also to the global constraints (the trajectory doesn’t fill in the grid). In addition, the problems contain a series of external elements, meant to increase the degree of difficulty. For example, they requested to calculate the time needed for crossing the given path (simpler problem) or the fact that problem 3 contains a reference to an 11×11 grid, although the given image contains a 11×12 grid (consequently, the last column should be ignored).

DISCUSSION

The data reported above highlight two situations. Some of the students vary superficial constraints by making endogenous transformations. For example, Victor (example 1) modifies the size of the grid and the shape of the trajectory, while Doina (example 2) modifies the generator element of the grid. In these cases, the students strictly follow the starting prob-

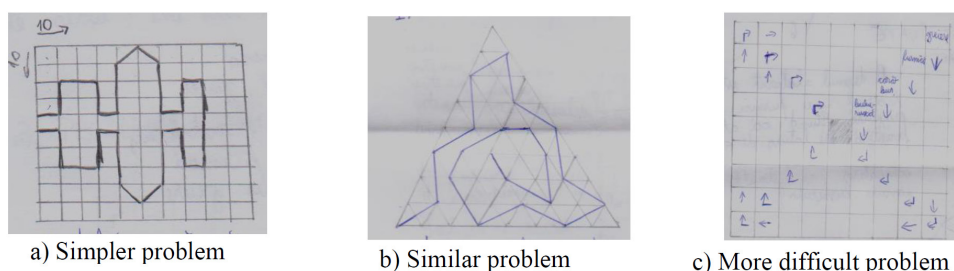


Figure 4: The drawings made by Silviu for his posed problems

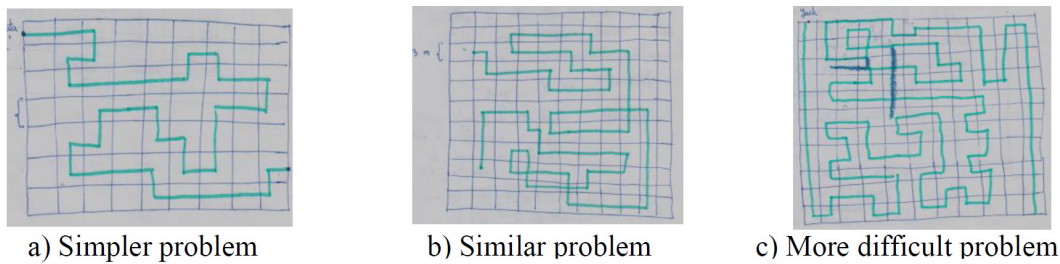


Figure 5: The drawings made by Mircea, Andrei, and Nguyen for the posed problems

lem’s structure and pose coherent and consistent problems that respect the nature of the given problem.

Some other students use their knowledge of advanced mathematical procedures and techniques to pose problems based on exogenous transformations. In these cases, the proposals are farther from the starting model, but at the expense of a lesser level of generalization. This is visible in the situations where students vary global constraints, as in the case of Silviu (example 3), or in the case of Mircea, Andrei, and Nguyen (example 4). In these cases, students introduce independent variables – that is, they perform exogenous transformations on the starting problem. In spite of their efforts to make their problems more difficult, they just increase the amount of procedural computations, but not the mathematical quality of the problem, measured by its mathematical consistency. Their interviews also revealed that they actually have limited understanding of the starting problem: this fact had an effect on their posed problems, which they were not able to solve using an optimal solution, but just making step-by-step procedural calculations.

Before getting to a final conclusion, we may consider that other factors could be at play in our analysis. The problems were posed for a problem-posing competition, and the students knew that the “value” of their problems would be judged. It might be that some students preferred to keep the structure of the starting problem, because it was already considered as being “valuable” (as it was a problem included into a real contest). Consequently, in appreciating the problems posed by students we need to consider their personal beliefs about what makes a good problem, as well as their ability to identify the problem structure and the mechanisms available for a controlled change of the problem elements. Thus, it is interesting to consider the nature of modifications at one-more finer detail.

Victor (example 1) has posed problems of identical nature to the given one, just by varying the size of

the net. Even though in his solutions we saw that he was aware of the constraint allowing an optimal solution, attending only to the modification of the grid suggests that he has not yet developed efficient mechanisms for changing more elements. Such situation is understandable if we take into account that Victor is a 4 grader, therefore his experience with different shapes and grid structures may be very limited. In contrast, Doina (example 2) seems to master such mechanisms, while keeping close to the problem type might be a personal preference. Somehow, Doina simplifies the starting problem because the pattern she has chosen for the “broken line” in each of her three proposals allows immediate identification of the grid-filling property. This simplification allows variations in the cognitive frame generated by the initial problem while keeping intact the analogy with the deep features of the starting problem. These variations in cognitive framing give an indication of her mathematical creativity.

Silviu (example 3), and Mircea, Andrei, and Nguyen (example 4) change many more elements of the starting problem. Furthermore, we found during the interviews that these students participated in intensive training programs (with their school teachers and with their parents). Their experience from such training might suggest that good problems are those with many data, which require breaking the problem into several pieces, and, in overall, ask for more fluid procedural work.

These cases reveal an essential fact: the extent of which the new proposal reflects an understanding of the hard core of the given problem drives the quality of the newly posed issues. We further analyze the students’ posed problems from this perspective. Beyond the students’ beliefs about problems and competitions, a dilemma still stands. On the one hand, we have more creative approaches but at the expense of mathematical quality, on the other hand, the newly

posed problems that reflect in-depth understanding are apparently less creative.

Some of the students who suggested changes of an exogenous type placed in their texts new mathematical concepts and usually provided traps to solvers. Although they seem to be more creative – when judged on the novelty dimension – they depart from a deep understanding of the starting problem structure. The problems they posed have a low degree of generality, and their difficulty is given by the solving procedure (overcoming traps intentionally included in the text and performing step-by-step calculations). The problems arising in our sample in this way proved weaker in terms of the mathematics involved. We interpret this result as a (mathematical) failure in posing problems that preserve significant aspects of the deep structure of the original problem, yet confirming a distinction between being creative and being *mathematically* creative.

What are the underlying factors that ensure for a new posed problem to be mathematically consistent? We found that, among the students who used endogenous transformations, those who kept the global way of solving of the starting problem made mathematically qualitative new problems. But, as they identified the optimal way of solving, the resulted problem remained close to the initial one, therefore in this case the student proved a low level of cognitive novelty, and the changes in cognitive framing were minimal. Their creativity was driven by the solution space, and inevitably, it had low amplitude.

Concluding, the evidence we got from problem modification tasks in our sample show that high achieving children swing between low amplitude driven creativity and failure to pose consistent problems when it comes about understanding the deep structure of mathematical concepts and strategies. Are these the only alternatives? Further research is needed to find a more definite answer.

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