

The link between the cognitive structure and modelling to improve mathematics education

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This study focuses on the aspects in the cognitive structure that should be trained to develop Dutch students modelling without actual modelling lessons. The research used 16 fifth-grade beta coursed students to study the development of the cognitive structure and its relation to modelling. The methodology is based on the Conceptual Content Cognitive Map Method of Kearney and Kaplan (1997), while Tall's (2013) theory is the theoretical framework. Data are collected by a cognitive test and a modelling test. The results show that students with a rich cognitive structure develop compression. Thus, teachers should focus on the connections between mathematical concepts as a solution for this problem.

Keywords: Modelling, cognitive structure, student learning, cognitive map.

INTRODUCTION

Modelling is an important discipline in beta sciences. However, it does not belong to the curriculum of Dutch secondary schools, which results in a gap between secondary school and university (Renkl, 1997; Savelsbergh, 2008). This problem has been studied from different points of view. Barquero, Bosch, and Gascón (2008) investigated a new didactic device to teach mathematical modelling at university. Verhoef, Zwarteveen-Roosenbrand, Joolingen, and Pieters (2013) studied the themes for mathematical modelling that interest Dutch students in secondary school. Lots of studies searched the cognitive structure, but only a small amount used quantitative analyses for measurement. Therefore this research focuses on the relation between the cognitive structure and modelling in a quantitative way, by use of cognitive maps. To quantify this, the study was well-delineated to trigonometry. This leads to the following main question:

Which aspects of the cognitive structure of secondary school students are determining factors in their development of modelling?

To answer this question the theoretical framework contains theory of the cognitive units, the cognitive structure and modelling. The method is formulated by means of the Conceptual Content Cognitive Map method of Kearney and Kaplan (1997), because of the quantitative analyses.

THEORETICAL FRAMEWORK

Cognitive units and structure

The human brain is a complex biological structure that forms a multi-processing system. It is able to make decisions and focuses on important information as a result of the electrical communication among nerve cells, also called neurons. Signals are sent from a neuron's axon and will be received by another neuron's dendrite. The signal frequency of the neurons produces neurological activity. Increasing activity results in stronger connections, as decreasing activity results in weaker connections. The more often a connection is used, the thicker the connection will be. This is a result of long-term potentiation, which is a long-term increase of the spiking frequency of the neurons. This is very important in the formation of brain structures. It is directly related to the memory (Purves, 2008) and makes subconscious actions possible (Barnard & Tall, 1997; Starzyk, Li, & Vogel, 2005). The plasticity of neuron connections makes change of thoughts possible by opening and closing the connections. The working memory, that can be used to solve problems, is a result of this plasticity in closely connected neurons (Barnard & Tall, 1997; Crowley & Tall, 1999; Starzyk et al., 2005; Vogel, 2005; Widrow & Aragon, 2013). Such densely packed neurons form interneurons. Activation of a neuron in

such interneuron group, activates all connected neurons if the connections are strong, called compression (Barnard & Tall, 1997; Purves, 2008).

In this study a schematic representation of the brain is used to simplify this theory. Terms as cognitive unit and cognitive structure are used. A cognitive unit is a small piece of information that the brain can focus on. As Tall (2001) says: "A cognitive unit consists of a cognitive item that can be held in the focus of attention of an individual at one time, together with other ideas that can immediately be linked to it." (Tall & Barnard, 2001, p. 2). We define a rich cognitive unit, when the cognitive unit contains a great amount of connections between small pieces of information (Vogel, 2005). The cognitive structure can be described in different ways. According to Hiebert and Carpenter (1992) it can be described by the two metaphors; vertical hierarchies or webs. Tall and Barnard (2001) combine both and define the cognitive units as the nodes of the cognitive structure. The related cognitive units are connected by the threads of the web. In a rich cognitive structure there are many connections.

Compression is also an important schematic term in mathematical thinking. It describes the way in which small, rich cognitive units are formed within a cognitive structure. The junctions in the spider's web are so close that they touch each other (Tall & Barnard, 2001). This process is important because all cognitive units will be activated if one part of the information have been evoked, which is important in the thinking process. According to Tuminaro and Redish (2007), secondary schools focus too much on the students' results, while the connections and the learning process are more important.

Modelling

Modelling activates learning processes and confronts the scientist with the effects of his theories. Blum studied the difficulties with modelling and tried to explain these difficulties by the students' cognitive demands of these tasks. He emphasised that mathematical modelling has to be learnt specifically by students, and that modelling can indeed be learned if teaching obeys certain quality criteria, in particular maintaining a permanent balance between teacher's guidance and students' independence (Blum & Ferri, 2009).

In our study we focus on the cognitive aspects that are required for the development of modelling. By

understanding the differences in the cognitive structure between students that are bad in modelling and good in modelling, we try to find the key aspect that is necessary in teaching modelling. For this study a mathematical model will be used which should comply with the conditions of modelling. According to Blum (2002), a modelling process contains five steps: a) Simplifying the real problem into a real model; b) Mathematizing the real model into a mathematical model; c) Searching for a solution for the mathematical model; d) Interpreting the solution of the mathematical model and; e) Validating the solution within the context of the real-life problem the real model into a mathematical model.

The Conceptual Content Cognitive Map Method

Lots of studies investigated the cognitive structure, but only some used quantitative analyses for measurement. Because this study also requests a quantitative measurement of the cognitive structure, the methodology is mainly based on the Conceptual Content Cognitive Map (3CM) Method¹ of Kearney and Kaplan (1997). Not only does their research show to be reliable and valid, their method is also a good basis for this study (Kearney & Kaplan, 1997; Somers, Passerini, Parhankangas, & Casal, 2014).

RESEARCH METHOD

Participants

Sixteen 16–17 aged students were subjected to the tests. They all had a beta programmed curriculum (physics and chemistry) with mathematics education.

Research instruments

This study used four instruments: a) the cognitive test; b) its evaluation, c) the modelling test and d) the grade list. Cognitive structures and modelling are respectively measured by the cognitive test and the modelling test. The study started with a cognitive test as a benchmark. One week later the modelling test followed, testing the students' modelling and their increasing insight in trigonometry and the ability to make connections between concepts. A week later, the second cognitive test studied the development in modelling concerning trigonometry. This second test was the same as the first cognitive test, so the differences in the results could be used to study the

1 For a detailed description of the 3CM Method, read <http://eab.sagepub.com/content/29/5/579.full.pdf+html>

cognitive development. After each cognitive test the students were asked to fill in the evaluation form and to note their grades for mathematics, physics, chemistry and if such is the case, informatics. Figure 1 shows an overview of the research method.

a) For the construction of the cognitive tests (used for the first and second cognitive test) mathematics concepts were collected. Therefore new participants were selected by their expertise, experiences, field of study and/or age. This results in a list with the following participants: a professional mathematician (f), ten master students (m/f) and one secondary school student (f). The participants made mind maps as a spider's web, which resulted in 68 concepts.

To test the cognitive test two of above-mentioned participants were used. They were chosen by their result of the mind map, because they had respectively the widest and deepest order. On basis of this pilot study the instructions were improved. This was tested by two secondary school students, which showed no further additions were necessary.

For the implementation of the cognitive test all participants (fifth-grade students) got an envelope filled with 68 concepts, one instruction form and one A₂ paper with in the middle the term trigonometry. The instruction form explained the participants to make a mind map around the word trigonometry in 15 minutes time. They could only use the concepts on the cards and a pen to connect them. Not all concepts had to be used. After 15 minutes the result was photographed and later analysed. The cognitive test measures the cognitive structure by constructing a mind map, called a cognitive map.

b) After each cognitive test the participants filled in an evaluation form. The answers to these questions

were an underpinning of the results of the cognitive tests. The form contained the questions:

- *What was the assignment according to you?*
- *Did you know all used concepts?*
- *Would your cognitive map be different if you had more time? If so, what would be the differences?*

c) The modelling test was based on the theoretical framework of model conditions. The participants for this test's pilot were five master students (m/f) and two secondary school students (f). The pilot study showed that an illustration of a schematic representation had to be added to the test instruction. The modelling test was a trigonometry based problem, so the students should find connections between the right triangle, unit circle and sine as signal as function in time. The problem was a riddle that had to be solved to crack a safe. This riddle had to be decoded in goniometric steps. The students were asked to answer this problem by constructing a mathematical model. The problem mainly focused on the first two steps of a mathematical model (Blum, 2002), but could be solved with all modelling steps. The results were analysed by the following scale: insufficient, sufficient, good and very good. If the results didn't satisfy the first steps of a mathematical model it was graded insufficient. Any further step in the model resulted in a better grade.

d) The student's grades were collected during the cognitive tests. The students wrote their grades of mathematics, physics, chemistry and informatics (only if such is the case) on the envelopes, which were collected at the end of the tests and the grades were listed.

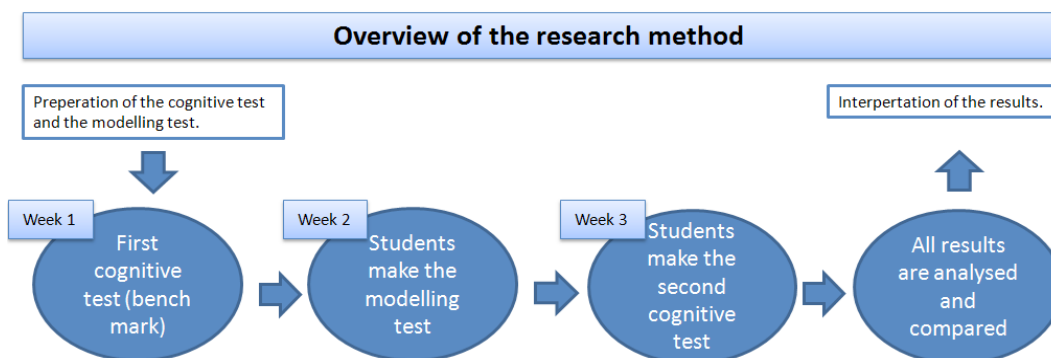


Figure 1: Overview of the research method

Data analysis

The four research instruments were each individually analysed and gathered in the overview table, which relates the data.

a) Cognitive test

The results of each participant were categorised by the depth of the cognitive map. As Figure 2 shows, the depth was determined by the amount of concepts counted from the main concept 'trigonometry'. For instance, if the maximum depth of a cognitive map was two concepts, the participant was placed in class 1. The class division was counted up to 5, as the pilot showed that a maximum of class 5 was enough. The pilot also showed that students who have a depth further than five can be placed in class 5.

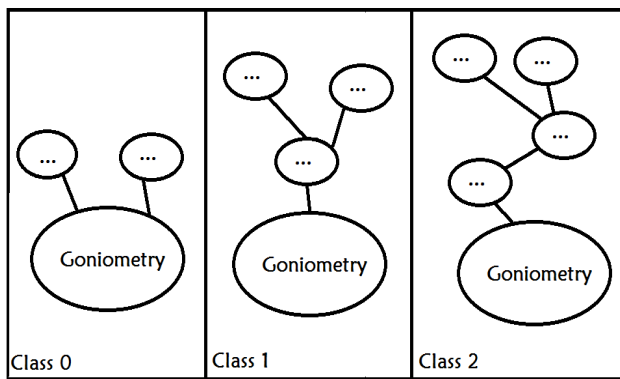


Figure 2: Examples cognitive map classification

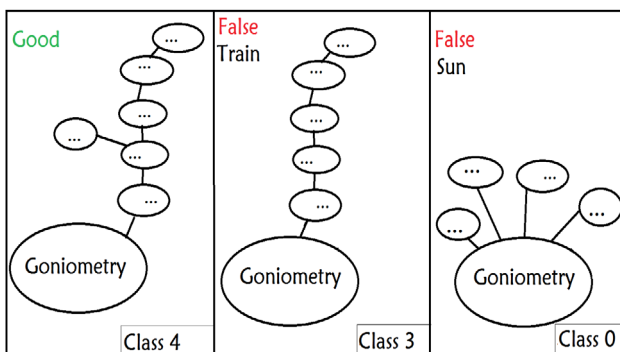


Figure 3: Cognitive maps containing 'trains' or 'suns'

There are some conditions in the classification of the cognitive maps. If students create 'trains' or 'suns' (Figure 3) in their maps it shows lack of depth. As 'sun' has no depth and a 'train' can be divided in more branches. That's why 'trains' longer than three concepts will be counted as three. Every deeper concept won't be counted, because it is always possible to further connect a concept this deep. This could mean that a cognitive map with depth 4 will be classified in class 3. The results of each participant were translated to a matrix, irrespective of classes.

The results were processed in three different kinds of matrices. Each a 68x68 matrix, constructed of *concept x concept*. Matrix type I was formed for each participant. The matrix cells were filled with '1' if the concerning concepts were connected. If the concepts were not connected the cell was filled with '0'. In matrix type II each [ij] element states the percentage of participants that connected the concept *i* with concept *j*. So, if 80% of the participants connected concept *i* with concept *j*, element *ij* was filled with 0.8. Matrix type III is a deviate of matrix type II. Each percentage smaller than 0.5 was equalled to 0. Percentages of 0.5 or larger than 0.5 were equalled to 1.

The correlation between matrix type I and matrix type III was calculated. This was done per participant per class. That means that the results of each participant was compared with the results of the other participants in the same class. The results were listed in the overview table (Table 3).

b) Modelling test

The results of this test were graded by the scale shown in Table 2.

c) Grade list

The school grades of mathematics, physics, chemistry and informatics (if such is the case) of each participant were listed. They are scaled from 1 to the upper val-

Result	Grade	Abbreviation
No form of a model and clearly no attempt to	Insufficient	I
Attempt to model, contains some elements of a model	Sufficient	S
Model, but result misses some model elements	Good	G
Model, clear and strong structure	Very good	VG

Table 2: Explanation for the evaluation of the modelling test

ue 10. The results were collected by forms that were handed out during the cognitive test.

d) Overview table

The results of each participant were collected in Table 3. The results of the cognitive test were expressed in the correlation between the students' test and the class average. This correlation represents the degree of connected concepts in the student's cognitive test compared to the class average. Students with a correlation smaller than 0.25 were selected. Based on the 3CM method, the results of these students deviate from the average results. This means that a student makes more or less connections than the average student.

RESULTS

Table 3 shows an overview of the collected results, sorted per student. It is noticeable that most students rose in classes and that more than half passed the modelling test. Also most students show sufficient results for the school grades.

Students 3, 4, 6, 7, 9, 12, 14 and 16 show correlations smaller than 0.25, but Table 3 shows no mutual connections. That means every cognitive map had to be analysed individually. Starting with the students that have a correlation smaller than 0.25, three students are insufficient graded for their modelling test. As

students 3, 6 and 14 have different cognitive maps, all show little connections between the used concepts. The students with a correlation smaller than 0.25 and with a "very good" for the modelling test are students 12 and 16. Both seem to use many connections between their concepts. The results of the three other students that are graded with a "very good", 5, 10 and 13, show the same. In general, most striking cognitive maps are those of student 5 and 12. Both students use many concepts that are linked in many ways. Figure 4 shows the cognitive map of student 12. The student makes many connections, that even form loops which is not by all students. In fact, the opposite can be found in the results of student 2, 8 and 11.

CONCLUSION AND DISCUSSION

The results show some noticeable students that should be discussed, like students 3, 4 and 16. As the grades of student 3 were very high, the modelling test was graded insufficient. So it seemed that this student is highly talented. Student 4 showed a huge increase in classes. According to his cognitive maps, this seems to result from the amount of used concepts and the development in the used connections. This student seemed to give a clear result regarding the influence of the modelling test and the repetition of the cognitive test. The same applies for student 16. This student showed no progress in classes and used a small amount of concepts. Though he increased the used

	Class test 1	Correlation test 1	Class test 2	Correlation test 2	Modelling test	Grade 'wiskunde B'	Grade 'wiskunde D'	Grade physics	Grade chemistry	Knew all used concepts test 1	Knew all used concepts test 2	More time, better test 1	More time, better test 2
Student 1	4	0,55	5	0,40	S	6	-	7	7	x	x		
Student 2	3	0,37	5	0,63	I	6	-	7	7	x	x	x	
Student 3	3	0,00	4	0,59	I	7	9	9	9			x	x
Student 4	2	1,00	5	0,13	G	5	-	6	7				x
Student 5	5	0,29	5	0,43	VG	8	6	7,5	6,5	x	x		
Student 6	4	0,12	5	0,46	I	6,3	-	6,4	7	x			x
Student 7	5	0,23	-	-	S	6	-	6	6			x	
Student 8	4	0,40	5	0,34	I	6	-	6	8		x	x	x
Student 9	3	0,25	-	-	S	5	-	6	6	x		x	
Student 10	3	0,37	-	-	VG	7	-	8	9			x	
Student 11	3	0,37	4	0,80	I	5	-	7	7		x	x	
Student 12	5	0,15	5	0,16	VG	9,5	9,5	8	8	x	x		x
Student 13	5	0,35	5	0,38	VG	6	-	6	6	x	x	x	x
Student 14	5	0,30	5	0,09	I	8	-	7	7	x		x	x
Student 15	3	0,28	5	0,52	G	5	-	6	7			x	
Student 16	3	0,00	3	1,00	VG	8	-	7	7	x	x	x	x

Table 3: All collected data listed for each student*

* None of the participants had informatics, so this is not present. The evaluation results of the cognitive test are represented as an 'x' for positive answers.

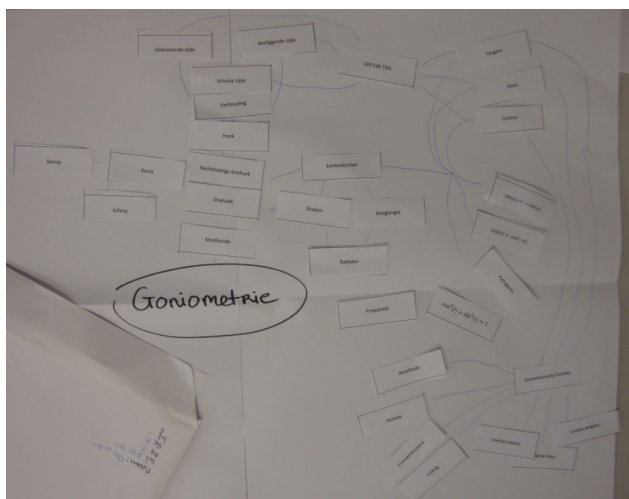


Figure 4: Cognitive map of student 12

connections in the last cognitive test. Only, it is not clear what influenced this development, was it the modelling test or the repetition of the cognitive test? This effect should be studied in further research. The results of this study prove to be a good indication for a large scale study. Although, it would be better for the analysis of the modelling test to use a more detailed checklist for evaluation. This would make it more reliable and valid.

It follows from the results that Dutch students are better in modelling if they have many connections between the concepts in their cognitive maps. This can be interpreted as the connections in the students' cognitive structure. It also follows from the results that the amount of the used concepts defines the category of the cognitive map. This represents the cognitive unit. Evident examples for this conclusion are the result of student 3, 4, 8, 12, 13, 14 and 16. So it can be stated that:

Students that are good in modelling have a rich cognitive structure. Students that are placed in a high class have a rich cognitive unit. When a student is good in modelling and is placed in a high class, there is compression. These students have a rich cognitive unit as well as a rich cognitive structure.

The cognitive structure can be developed to a rich cognitive structure by repetition. This seems to emerge from the connection between the results of the first and the second cognitive test. Repetition is definitely of indirect significance in the development of modelling

RECOMMENDATIONS

As this study shows good and useful results, it can be recommend to repeat this study with a larger group of participants. Further research should be focussed on the influences of the repetition, regarding the modelling test on the development of the cognitive structure. This study focussed on the mathematical subject trigonometry, but it can be useful to focus on different mathematical items.

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