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*The present study addresses the differences in the construction of ninth graders' situation models, when they were confronted with an application problem that is found in middle-school mathematics textbooks in Mexico. This study focuses on the reading-comprehension phase, better known as the situation model, which is considered a prerequisite for the mathematical model construction of word problems. Four versions of the problem "The broken tree by the wind" were given to a total of 192 ninth- grade students. Students' drawings were analyzed in order to identify any possible causes by which few of the students were able to build an adequate situation model. The results suggest a potential influence of problem wording on students' visualizations and ways of interpreting mathematical problems during the situation model construction.*

**Keywords**: Situation model, word problems, mathematics textbook.

## INTRODUCTION

Text comprehension in mathematical word problems has been investigated in order to improve problem-solving processes (Cummins, Kintsch, Reusser, & Weimer, 1988; Mayer & Heagarty, 1996). Researchers have been trying for some time to get an insight into the causes that affect such performance in order to improve it (Kintsch & Greeno, 1985). Students' difficulties increase when problems imply a mathematical modelling process (Galbraith & Stillman, 2006).

It is already well established, both theoretically and experimentally, that the modelling process comprises several phases (Borromeo-Ferri, 2006). The first one consists of the situation model (SM) construction that the problem refers to, and it is necessary for the understanding of the mathematical problemand its further resolution, which is considered as "...a mental representation of the situation described by the text..." (Kintsch, 1986, p. 88).

Some results in a recent exploratory investigation (Juárez & Slisko, 2013) show that middle-school students have great difficulties to build the situation model of a problem named "The fallen tree". The mental images or models of the situation were studied through the requested drawings. The statement of the problem is the following:

> The wind has broken a tree trunk in such a way that two of its parts form a right triangle with the ground. The upper part forms a 35° angle with the ground, and the distance, measured on the floor, from the trunk to the fallen top of the tree is 5m. Find the height of the tree (Mancera, 2008, p. 333).

Juárez and Slisko (2013) found that after thirty middle-school students had been given "The broken tree" problem, only ten of them were able to depict the actual situation described in the text. However, these ten students interpreted it in different ways.

This problem and its various formulations appear in several middle-school textbooks published in Mexico authorized by the Ministry of Education. In some of them, it is just an application problem (Briseño, Carrasco, Martínez, Palmas, Struck, & Verdugo, 2007, p. 223; Waldegg, Villaseñor, García, & Montes, 2008,

p. 205; Farfán, Cantoral, Montiel, Lezama, Cabañas, Castañeda, Martínez, & Ferrari, 2008, p. 202). In certain books, its historical origin is mentioned (Pérez & Pérez, 2008, pp. 227–228; Arteaga & Sánchez, 2008, p. 75; Sánchez, 2008, p. 216).

The results mentioned above motivated us to conduct a more comprehensive study, where the factors and processes involved in the situation model construction as a prior step leading to a resolution of the problem could be investigated. In particular, we were interested in analyzing the possible effects of changing certain elements from the text of the problem, i.e. the title and the object of the problem, on students' text-comprehension process and their corresponding situation models.

Results of previous studies based on students' drawings have shown that students find it difficult to imagine the situation. What makes it hard for most of them, is the mental image of a "tree" (the trunk's thickness and the treetop formed by branches and leaves), because it is hard for them to "get rid" of the treetop. This study central assumption is that if students get a version of the problem, where the tree is changed for a simpler "tree" plant, like a "bamboo" (it has a straight shape and it does not have any branches that hamper the floor support), they will perform better in the development of the situation model than the students who get "the broken tree" version.

So, our research question is the following: Do changes in the problem's title, as well as in the object used in the text have an effect on middle-grade students' situation model construction?

## THE CONSTRUCTION OF THE SITUATION MODEL AND MATHEMATICAL WORD PROBLEM SOLVING

Polya's model for problem solving has played a significant role in Mathematics Education since it has clarified many of the cognitive processes involved in it (Polya, 1976). In the first stage, the understanding of the problem, strategies can be found that help with the representation of the situation and the comprehension of the problem's conditions. The importance of using some heuristics is also emphasized here, like 'draw a picture or a diagram'.

Nathan, Kintsch and Young (1992) propose a theoretical model, where students have to read and understand the statement first, and then the information obtained is "organized into a (qualitative) situation model and mapped into a (quantitative) problem model that captures the algebraic structure" (p. 332).

According to Diezmann (2000), to make a drawing of the problem situation may be crucial for a person trying to solve it. In this way, many word problems describing a real situation are presented to the student as if its mental representation was an immediate process. However, it is a translation process that involves linguistic information decoding and visual information encoding. In this sense, Heagarty and Kozhevnikov (1999) investigated the relationship between mathematical visualization and mathematical problem-solving, being the first one understood as an individual's ability to use images or diagrams. They classified visual-spatial representations as schematic or pictorial and, in their study, schematic representations, which encode spatial relations described in the problem, were positively correlated with success in mathematical problem solving. On the other hand, pictorial representations, which are related to the visual appearance of the objects described in the problem, were negatively correlated.

In the same line, Edens and Potter (2007) examined the relationship between student performance on particular drawing tasks and their achievement in mathematical problem solving. They provided evidence that teaching strategies based on drawings can help teachers in obtaining useful information about their students' level of spatial understanding.

From a cognitive perspective, several studies have focused on the complex process of reading comprehension during word problem-solving, both in algebra and realistic problems, as well as in arithmetic problems (Vicente & Orrantia, 2007). These researchers acknowledge the need to create a model of the problem situation, by applying the real-world knowledge possessed by the student. Meanwhile, Reusser (1988) suggests that it is necessary to produce what he calls an "episodic model of the situation" between the text base and the problem's mathematical model.

One of the consequences that the lack of an appropriate situation model construction would have, for example, is the student's behavior strongly linked

to practices, as mentioned by Reusser and Stebler (1997):

> As a result of schooling, students' behavior is pragmatically functional if they take into account any information they can draw from both problem texts and contexts. That is, their mathematical sense-making is functional if they actively and continuously construct a mental representation not only of the specific task (problem model...), but also of the socio-contextual situation which they are in (construction of a social context model)... (pp. 325–326).

Despite the amount of research indicating the importance of the situation model for understanding the text of the problem, some researchers such as Voyer (2010) argue that the question of the influence of the model constructionon student performance in solving word problems remains open. This conclusion might be correct for some arithmetical problems used in Voyer's research. Nevertheless, it seems that in trigonometric problems, the situation model construction influences significantly posterior students' problem-solving performance. It is easy to agree with van Dijk and Kintsch (1983), who emphasize that "... we know very little about the conditions that promote or inhibit the construction of situation models from texts ..." (p. 346).

## METHODOLOGY

The participants were 192 students in 9th grade at a public middle school in central Mexico, all divided into four different groups. In terms of the curriculum, the ninth graders had already been exposed to the Pythagoras Theorem and trigonometric ratios.

Each student was given a worksheet containing one of the four versions of "the broken tree" problem and they were asked, after reading the text of the problem, to draw the described situation. There was no time limit, and they were not asked to solve the problem.

Each version of the problem was applied to a different group. Two central objects were used, i.e. the tree and the bamboo, resulting in two distinct texts. In addition, two titles for each one of them were drafted, yielding four versions of the problem that are shown in Table 1.

The data collected for this study included only students' drawings, since the focus of the study was on the situation models built by the students. In order to describe them, the collected drawings were categorized via our definition of situation model mentioned above. Students' responses were grouped by identifying those drawings that had common characteristics in their situation models, in the sense that they were leading to the mathematical model construction.

## RESULTS AND ANALYSIS

From the analysis of the drawings, four categories were obtained: Situation Model with a Right Triangle (SM-RT), Situation Model with an Arbitrary Triangle Related to the Situation (SM-AT-RS), Situation Model with an Arbitrary Triangle Not Related to the Situation (SM-AT-NRS) and Situation Model Without Triangle (SM-WT).

| | Version 1 | Version 2 | Version 3 | Version 4 |
|---|---|---|---|---|
| Problem's title | The broken tree that forms a triangle | The broken tree | The broken bamboo that forms a triangle | The broken bamboo |
| Problem's statement | The wind has broken a tree in such a way that two of its parts form a right triangle with the ground. The upper part forms a 35° angle with the ground, and the distance, measured on the floor, from the trunk to the fallen top of the tree is 5m. Find the height of the tree. | | The wind has broken a bamboo in such a way that two of its parts form a right triangle with the ground. The upper part forms a 35° angle with the ground, and the distance, measured on the floor, from the bamboo's base to the fallen top is 5m. Find the height of the bamboo. | |

**Table 1:** Four different versions of the "the fallen tree" problem

| Different SM for the version "The broken tree" | | | |
|---|---|---|---|
| SM-RT | SM-AT-RS | SM-AT-NRS | SM-WT |
| 29.5%<br>13/44 | 29.5%<br>13/44 | 9%<br>4/44 | 32%<br>14/44 |

**Table 2:** SM classification for the first version of the problem

### Version 1: The broken tree

The number of students that constructed a particular type of SM for the first version of the problem "The broken tree" is shown in Table 2.

The above table shows that almost 30 percent of the students produced the SM considering a right triangle, but only one of them drew up the situation model correctly. This finding is consistent with what Diezmann (2000) reported with primary school children, that although 'drawing a diagram' is advocated as a useful strategy for solving problems, to generate an appropriate diagram is problematic for many students.

On the other hand, the same number of students built their SM with an arbitrary triangle related to the situation described in the problem. Figure 1 shows the SM of one of these students, Victor, where its construction could have been affected by the interference of his real-life knowledge evoked while imagining the situation.

### Version 2: The broken tree that forms a triangle

The number of students that constructed a particular type of SM for the second version of the problem "The broken tree that forms a triangle" is shown in Table 3.

In most of the situation models, a high number of realistic elements related to the scenario were included, such as wind, tree branches, and leaves, as well as clouds and rain. When compared to the first version of the problem, the same percentage of students did not include any triangle in their situation models, even though the expression "...that forms a triangle" was present in the title of the second activity. It seems not to have influenced the understanding of the situation or the corresponding situation model. These results suggest that the change in the problem's title had no effect on the construction of the SM.

However, the percentage of students that included an arbitrary triangle not related to the situation increased when compared to the first version. Figure 2 shows a situation model, for example, where Daniel drew an arbitrary triangle that has nothing to do with the situation described in the text. It seems that the explicit reference to a triangle in the activity's title forced the students to draw a triangle, without taking into account the rest of the information included in the text.

This behavior can be explained according to what Borromeo-Ferri (2006) claims:

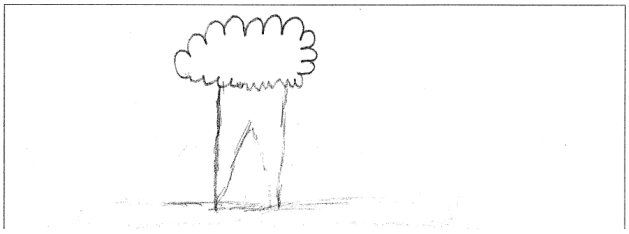

**Figure 1:** SM drawn by Victor (arbitrary triangle related to the situation)

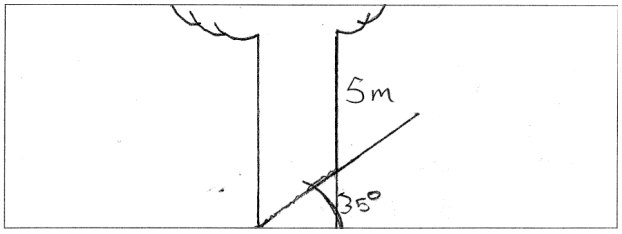


**Figure 2:** SM drawn by Daniel (arbitrary triangle not related to the situation)

| Different SM for the version: "The broken tree that forms a triangle" | | | |
|---|---|---|---|
| SM-RT | SM-AT-RS | SM-AT-NRS | SM-WT |
| 20%<br>(10/50) | 32%<br>(16/50) | 18%<br>(9/50) | 30%<br>(15/50) |

**Table 3:** SM classification for the second version of the problem

> The individual has a mental representation of the situation, which is given in the problem. This MRS can be very different, for example depending on the mathematical thinking style of the individual: visual imaginations in connection with strong associations to own experiences; or the focus lies more in the numbers and facts given in the problem, which the individual wants to combine or relate. (p. 92)

### Version 3: The broken bamboo

The "broken bamboo" version was given to 48 students. Table 4 shows the percentages for each SM. Only 12 of them (25%) included a right triangle in their model. In the second column it can be observed that 40% of the students constructed a SM with an arbitrary triangle related to the situation. This could mean that, changing the tree for the bamboo could have been the cause for more students to include a triangle in their situation model. In Figure 3 one example of this type of situation model is shown.

### Version 4: The broken bamboo that forms a triangle

For the problem's version "The broken bamboo that forms a triangle", 26 students out of the 50 (52%) that were given this version of the problem, were able to draw the right triangle required in the statement. This percentage is significantly higher when compared with versions 1, 2 and 3, thus confirming our hypothesis about the positive effect of including the "bamboo" instead of the "tree" in the statement of theproblemand "...that forms a triangle" in the title. Figure 4 shows an example of such drawings. Also, 26% of the students included an arbitrary triangle related to the situation in their SM.

The number and percentage of students' types of situation models for the problem's version "The broken bamboo that forms a triangle" is shown in Table 5.

## FINAL COMMENTS

After analyzing the drawings made by students when confronted with a mathematical word problem, we note that the construct 'situation model' as conceptualized in the literature, was effective to account for the various productions and elements that hamper reading comprehension, and therefore the construction of mental representations corresponding to the situation described in the problem.

One of the first interesting findings in this study is the fact that a very low percentage of students created the situation model so as to enable them to solve

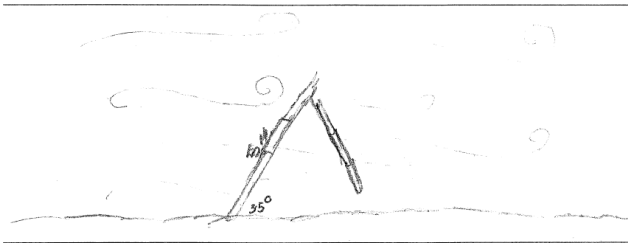


**Figure 3:** SM drawn by Monica (arbitrary triangle related to the situation)

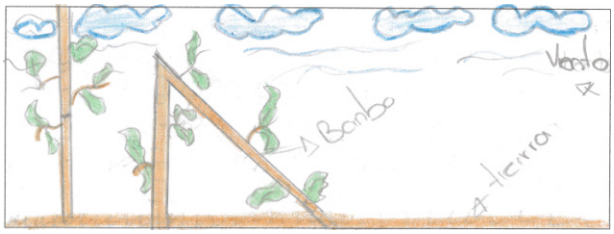


**Figure 4:** SM drawn by Giselle (right triangle)

| Different SM for the version "The broken bamboo" | | | |
|---|---|---|---|
| SM-RT | SM-AT-RS | SM-AT-NRS | SM-WT |
| 25%<br>12/48 | 39.6%<br>19/48 | 4.2%<br>2/48 | 31.2%<br>15/48 |

**Table 4:** SM classification for the third version of the problem

| Different SM for the version "The broken bamboo that forms a triangle" | | | |
|---|---|---|---|
| SM-RT | SM-AT-RS | SM-AT-NRS | SM-WT |
| 52%<br>26/50 | 26%<br>13/50 | 8%<br>4/50 | 14%<br>7/50 |

**Table 5:** SM classification for the fourth version of the problem

the problem in any of the four versions presented to them. This shows, once again, the importance of the proper construction of the situation model as part of the text comprehension process and as a step towards the development of the mathematical model.

Students' productions in this study included a variety of triangles, and although they do not represent the real situation, we realized that in the problem's version "The broken bamboo that forms a triangle", it was more natural and easier for them to imagine a situation where a triangle is formed than in "The broken tree that forms a triangle" version. One possible explanation is that for students it is more likely to imagine how the wind can break the vertical and thin bamboo structure than the one of the tree, possibly because in the latter case it might be thought that its root is stronger and its trunk is thicker, and therefore more difficult to bend.

It has been observed that in many mathematics textbooks, the text comprehension phase in word problems is presented as if the coherent situation model construction was a trivial process. This phenomenon could be one of the reasons students do not distinguish and are not able to sense its main characteristics. We believe that the findings of this study can be used to conduct further research to help clarify the entire understanding process and to serve as a reference in textbooks design and development, specifically related to word-problem solving.

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