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Student's interpretations of visual models

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Humans perceive and interpret graphical representations of models daily. Student's ability to do accurate interpretations of graphical and symbolical represented information is an important goal for the educational system. A study on 17 student's interpretations of graphical and symbolical representations of linear motion is reported. The collected material consists of about 10 hours video recording divided into 20 sequences about 15–50 minutes each. The tasks used to approach student's interpretation of graphical representation of linear motion where related to distance-time graph and analytical representations of distance function. The theoretical framework used is the theory of conceptual change for learning developed by Chi together with Tall and Vinner's framework on concept image and concept definition.

Keywords: Representations, iconic, every-day and scientific discourse, lateral categories of conceptual allocations.

The research reported here is not aimed at only identifying or describing students' alternative conceptions but instead provide possible explanation of why some students interpret graphical representation differently than other students. Mathematical representations such as diagrams, histograms, functions, graphs, tables and symbols normally makes it easier for us to understand abstract mathematical concepts or phenomenon described in mathematical terms. Representations are, according to Wittmann (2005), usually structured systems with strong connections to theories and may be seen as constructions that links abstract and concrete mathematics together.

Humans of today are facing a world that is shaped by increasingly complex, dynamic, and powerful systems of information that we meet through various different media. Being able to interpret, understand, and work with complex systems involves important

mathematical processes that humans need to understand and be able to address when facing interpretation challenges.

Mathematical representations, structures and constructions are also usual when you study other subjects, such as physics. We wanted to investigate upper secondary school student's understanding and interpretations of mathematical representations of linear motion. Our research question is therefore:

- How do upper secondary school student interpret graphical and symbolic representations of linear motion?

THEORETICAL FRAMEWORK

We humans develop alternative reference systems in order to explain and understand the physical world we meet. We do this through daily life experiences. Several thousands of studies has described students alternative and resistant opinions about mathematics and natural science (Duit, 2004; Pfundt & Duit, 1993), together with reports on how teaching could be altered and developed in order to counter act on students alternative opinions (Slotta & Chi, 2006, p. 262).

Research has also shown that alternative ideas sometimes can be incompatible with scientific models and theories (Hammer, 2000; Shaffer & McDermott, 2005). There are several different labels on individually alternative opinions concerning daily life phenomena such as children's science, alternative ideas, intuitive ideas, and common sense beliefs.

The source of our daily life conceptions is shaped, according to diSessa (1993), by a row of well-developed intuitive ideas which diSessa defines as "Phenomenological primitives" or p-primes. P-primes are generalizable and different than our memories from events we have experienced. P-primes are also

different than learning of scientific theories which generally are studied with a specific purpose, while p-prime are constructed and built rather unconsciously, less formal, intuitively and without a specific aim.

A p-prime can be either right or wrong, it may be compatible with scientific theories or not. To run a bike in hard upwind is more strenuous (p-prime: one has to work harder when the resistance is greater), motion with constant speed demands constant force (p-prime: there is always a force in an objects direction) or gravitation only applies to objects in movement (p-prime: no movement no force). A p-prime is generalizable since it may be used by the individual in different contexts. Even if we never had been closed to an earthquake, we still could determine we would run in a different direction (p-prime: the closer we come to a source, the stronger impact). A p-prime could be correct but used in the wrong context. The fact that the weather (on the northern hemisphere) becomes warmer in the summer is not because we are getting closer to the sun.

A collection of p-primes form the first and most elementary educational system, something diSessa calls the intuitive physical sense of mechanism. The way we humans interpret our surrounding world is the foundation for our learning of basic physics.

The intuitive sense of mechanism contributes substantially to understanding school physics. (diSessa, 1993, p. 105).

diSessa claims that all p-primes are valuable and necessary and not misconceptions that we need to erase. diSessa also uses knowledge in pieces, intuitive conceptions or persistent false intuitions when he talks about p-primes. It seems that a p-prime sometimes needs to be modified, enhanced, or to be generalized.

GRAPHICAL REPRESENTATIONS

Representations, according to Wittmann (2005), are normally from a structured system with clear relations to theories and may be seen as constructions that connects abstract and concrete mathematics. A representation is something that stands for something else (Duval, 2006, p. 103). An object's speed or acceleration described through a speed-time-graph may be determined from the graphs slope, something not explicit obvious from the graphical representation.

Elby (2000) argues for a specific form of p-prime which activates with interaction with graphical artefacts. Elby calls this "What-you-see-is-what-you-get", or WYSIWYG:

In my view, that's because the hill mistake and similar iconic interpretations spring, in part, from the activation of a cognitive structure, specifically, an intuitive knowledge element I call What-you-see-is-what-you-get. (Elby, 2000, p. 483)

Elby (2000) refers to a study where the students are asked to interpret a speed - time graph describing a bicycle path. See Figure 1.

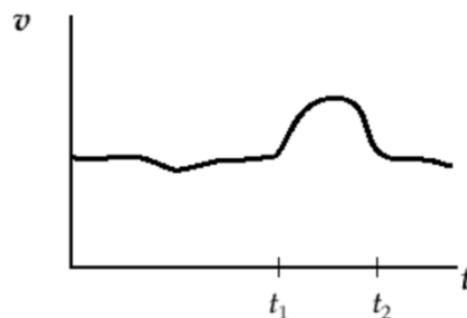


Figure 1: A bicycle path (Elby, 2000, p. 487)

Some students interpret this as evidence that the bicycle path goes over a hill. Some representations include special attributes which quickly catch our attention. Elby call this Compelling visual attributes. It may be all visual qualities and shades that are imbedded in the graphical representation, such as vertex, edges, contours, etcetera.

My claim is that, even though what-you-see-is-what-you-get is not cued strongly in all contexts, it is cued strongly with respect to the compelling visual attribute of a representation. (Elby, 2000, p. 484)

Which one of these visual properties we notice first is strongly context dependent. We need to know what a bicycle is before we can connect its path to a graph.

The over-literal readings call to mind the "what you see is what you get" (WYSIWYG) knowledge element proposed by Elby (2000), where students interpret a representation in the simplest, most literal way possible (a bump on a graph corre-

sponds to a hill). This WYSIWYG element is a representational analog of the phenomenological primitives (or p-prim) described by diSessa (1993) which include such basic reasoning elements. (Kohl, 2001, p. 107)

CONCEPTUAL CHANGE

In order to understand information presented in a graph, we need to understand the underlying concepts of the phenomena presented. Conceptual understanding springs from the idea that knowledge is constructed actively by the individual for instance by testing alternative ideas. All concepts relate to other concepts and most concepts have both definition and image in our mind. Tall and Vinner (1981) said:

We shall use the term concept image to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures.

On the other hand:

The concept definition [is] a form of words used to specify that concept. (p. 152)

Gray and Tall suggest three mathematical worlds to explain development of mathematical understanding. This involves a parallel development of conceptual embodiment, as the complementary use of human

perception and action and proceptual symbolism, involving manipulation of symbols that arise from operations. The term 'procept' address something that can be used flexibly as process or concept, including numbers, fractions, algebraic expressions, derivatives, integrals, and so on (Gray & Tall, 1994).

A key challenge to research in conceptual development and learning is to understand how individual constitute aspects of a scientific understanding of a concept. Knowledge acquisition is viewed as a process that involves actively generating and testing alternative propositions. Conceptual change refers to any change in conceptual understanding.

Three main perspectives on conceptual development are: the epistemological, the metaphysical and the cognitive perspective. The epistemological perspective on conceptions is based on criteria for difference. Our understanding regarding the world around us and how it is constructed relates to three ontologically "lateral" categories; mental state, entities and processes (Chi et. al., 2012). See Figure 2.

When we humans learn concepts, we also learn where the concept belongs in an ontological sense. What is the weight of the world series in soccer sounds like a strange question since the weight of something belongs to "Entities", while the world series in soccer is categorized as an "Event". Since the two concepts belong to different categories they cannot create an understandable context.

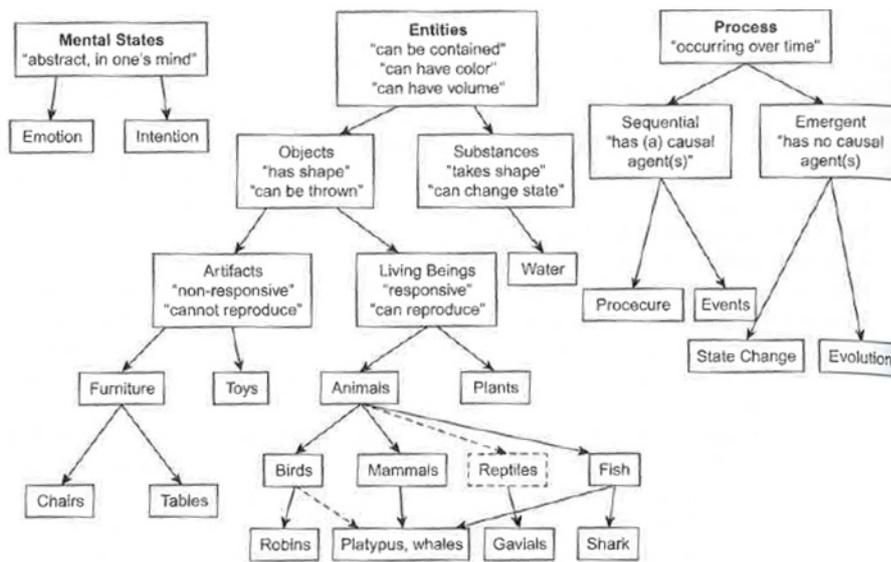


Figure 2: Conceptual change in an ontological structure (Chi et al., 2012)

Even close sub categories under the same branch might be ontological different lateral categories. Most young children know the difference between artefacts and living beings. Conceptual change occurs when we address concepts to different ontological categories. This constantly ongoing process happens inside us and strives for interpretations of new situations and to make them understandable.

Sometimes our interpretation of the world place concepts in the wrong place. One such well known phenomenon is the fact that students might view light or electrical current as material substances. Scientifically theories describe electrical current and light as sequential processes (Chi et. al., 2012, p. 57).

Conceptual understanding from the metaphysical standpoint is described as a restructuring of concepts over ontological lateral (not hierarchical) categories. Sometimes learning of scientific concepts is difficult since the process requires concept has to be re-assigned to an ontologically distinct category (across trees). The cognitive perspective on conceptual understanding is based on nature of misconceptions. Sometimes there are clear qualitative differences between the individual's (alternative interpretations, p-primes) and scientific models or expert's descriptions of a phenomenon.

Conceptual understanding based on a cognitivist's perspective can be seen as a process that tends to obtain a gradual increase in the correlation between alternative ideas (knowledge into pieces) and scientific theories (diSessa, 1993; Elby, 2002, & Hammer, 2000). The cognitive perspective on conceptual change starts in identifiable qualitative differences between an individual's (alternative interpretations, p-prime, and knowledge system) and scientific models or expert's description of systems.

METHODOLOGY

Our cohort consisted of 17 upper secondary school students (9 girls and 8 boys) at the natural science program, all enrolled in year 2. All students' volunteered to take part in the study. The students were grouped in small groups with 2 or 3 students in each group; a video camera was mounted to record the discussion. No observer was present to help the students to be natural in their comments

and attitudes. Students written solutions and notes were collected.

Our intention was to record the individual students responses but we considered that this was easiest done by encouraging group discussions. The students responded to 13 questions all together, this article will present the dialogue in relation to a distance – time graph. The students took note and did some calculations on paper, which we collected. We will only present some short results here, depending on page limitations.

Problem

The graph in Figure 4 illustrates a train that moves during 4 hours. The distance s is in kilometer and the time t is in hours

- 1a) When do the train run at the highest speed?
- 1b) How do you know that?
- 1c) Are there more than one occasion when the train has this high speed?
- 2a) When do the train run at the lowest speed?
- 2b) How do you know that?
- 2c) Are there more than one occasion when the train has this low speed?
- 3) What could be said about the train's direction by interpretation of the graph?
- 4) Sketch a graph to describe the trains speed during these 4 hours.

RESULTS

Student group Arman, Alexander and Gideon – written answers

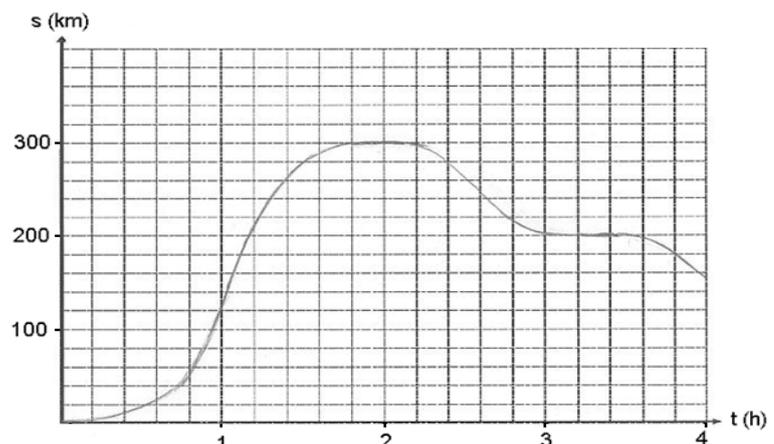


Figure 3: A moving train

Student: 1a. The train has the largest speed at 2 hours.

Student: 1b. Because that is the highest point in the graph.

Student: 1c. Yes, at 1.48 and 2.12. That speed last for 24 minutes.

Student group Arman, Alexander and Gideon – video recording

Student1: The train is fastest up here or ...

Student2: Write when it has the largest speed at 2 hours.

Student1: Here it goes down a hill; here it goes up a hill, up and down...

Student2: I think we should be careful writing about hills...

Student3: I do not think I never have seen a train go up a hill like that

Student1: Well, maybe when it is going up or around a mountain or something...

Student3: Like a rollercoaster? In an amusement park?

Student2: In San Francisco they have cable cars where you can hang outside...

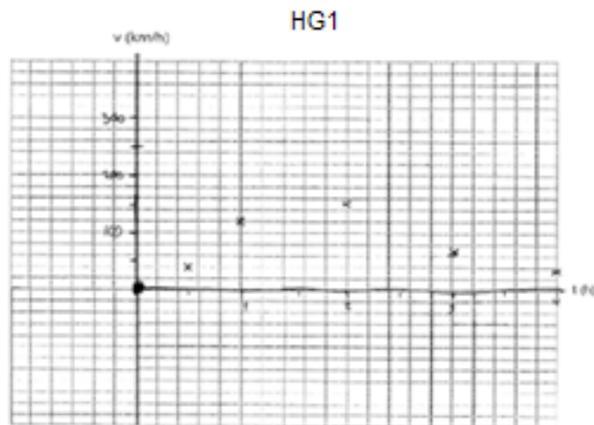


Figure 4: Alexander's response to task 4

Arman claims that the largest speed occur after 2 hours. His interpretation of the graph is built on the opinion that the graph is illustrating a train going “up a hill” and “down a hill”. Arman interpret the graph as a “true image” which is called an iconic interpretation.

Alexander Shares the opinion that the speed is highest “at the top point up here” and that the train is going up and down. Alexander also connects the graph to the streets of San Francisco. Gideon, after some doubts,

associates the graph to a rollercoaster in an amusement park.

Students' alternative iconic interpretations of graphical representations are well known from research and also obvious in this study. Arman's, Alexander's and Gideon's interpretation of the distance – time graph could be seen as an alternative descriptive model. The students talk about different places, refer to different events, and gives examples in different ways. Nevertheless, one thing they do have in common. Their interpretations seem to be iconic and they are all focusing on the wrong attribute.

Alexander's description that “In San Francisco they have cable cars where you can hang outside ...”, or as Arman express it “Here it goes down a hill; here it goes up a hill, up and down...”, are example of intuitive ideas and daily life concepts That seems to be deeply imbedded in the imagination world of these students. In reality it is probably impossible for a train to go up such a steep hill. The interpretation process together with the need to create a context for the train seems to be more important for the students than to think about the models validity and reliability. This in turn keeps their alternative ideas safe and resistant towards change.

Student group Jakob, Maria and Linnea – written answers

Student: 1a. When the curve has the steepest slope.

Student: 1b. Longest length on shortest time.

Student3: No, once.

Student group Jakob, Maria and Linnea – video recording

Student1: One box up in the graph paper is a fifth equal to 20 km, so the train is moving 20 km in short time while over here the train is going 20 km in a long time.

Student2: Yes, it is faster there.

Student3: It is fastest where the slope is steepest since the graph is showing us the real speed, the moment speed, so we have the answer to 1a.

One qualitative aspect of students understanding of a *distance – time graph* is found in Jakob's reasoning.

Jakob illustrates a good technique for determination of the slope of the graph. The discussion also reveals that the group manages to separate the meaning of distance and speed.

Written answers to question 2a-2c

- Student: 2a. When the curve's slope is zero.
- Student: 2b. Distance is not changing although the time is running.
- Student3: Yes, at three occasions.

Quotations from transcribed video material related to 2a-2c

- Student1: When the slope is zero, the train is standing still.
- Student2: The train goes to one point and the back again and then it is standing still for some time.
- Student3: Positive direction upwards and negative direction downwards means that the train is going back but not the whole way.

Through this conversation, the students are validating and develop their reasoning concerning the meaning of the graphs slope. The student's responses show that they are not confused by the graph, instead they are arguing for the relation between the slope of the graph and the movement of the train. The fact that they are not doing any iconic interpretation of the graph, allows them to give a full description of the meaning of the graph. As a result, they arrive to a quite different graph of the speed - time graph asked for in task 4.

CONCLUSIONS

Sometimes may daily life experiences be too strong and to overwhelming when students try to make interpretations of graphical representations. This is obviously the case of Arman, Alexander and Gideon. We have shown you the results of two groups here, results that may be seen as the two extremes in our collection of results.

When students relate to iconic interpretations as cable cars in San Francisco, they do this because such interpretations have some advantages. They are easy to remember, seemingly realistic, easy to identify in a daily life language, easy to compare, and sounds convincing.

Our results indicate that the more a student build on iconic ideas as model for explanation, the more demand he or she will also need for daily life concepts in the arguments.

It is important for us to underline that we do not see the results as misconception, more as misplaced concepts. When Arman, Alexander and Gideon and probably many other students with similar iconic interpretations, are trying to explain the distance - time graph, they seem to relate the meaning to an event. This is important, since a distance - time graph should be seen and interpret as a representation of a sequential process. Event and sequential process are two close sub categories in Figure 2 (Chi, 2013). The fact that the students we studied are so called specialists in mathematics and science indicate that it is perhaps harder to make this restructuring than researcher have thought so far.

As researchers we might marvel over how much material that was derived from these students discussions regarding a situation with a train. But this article is also something teachers in general could read and discuss. The fact that humans in general may misinterpret some of all the graphic information we meet on a daily basis is nothing strange. How could we do otherwise? This article might help teachers to support student's learning and understanding of both mathematical entities and mathematical processes in a more successful way.

In the discussions between students regarding the train's movement we could identify occasions when

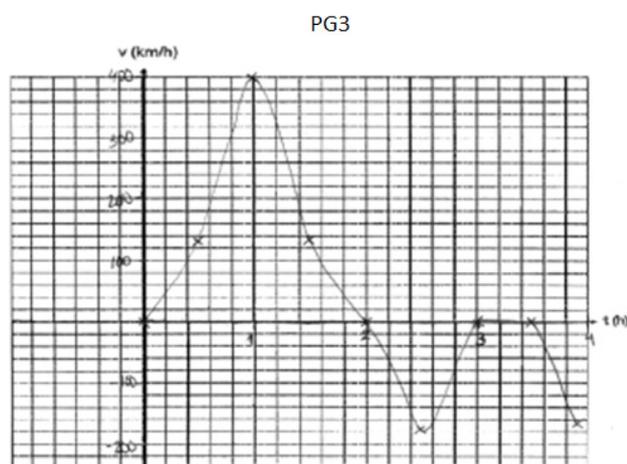


Figure 5: Maria's response to question 4

students learned from each other. Even though this was a research study, it is beneficial when students learn from the organization of the study.

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