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Evaluating the effectiveness of a framework for measuring students' engagement with problem solving episodes

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The purpose of this study is to investigate the problem solving episodes that a group of post-primary students in Ireland engaged with during the problem solving process and to determine an effective assessment technique for measuring their problem solving abilities. A framework developed by Artzt and Armour-Thomas was implemented and evaluated within the classroom over a nine week period to assist students in developing their problem solving skills. Testing was conducted on the students to be able to differentiate between their abilities prior to the introduction of the framework. It was found that when students utilised the framework the number of episodes engaged with by the students increased and the number of correct answers to the problems also increased.

Keywords: Problem solving, episodes, implementation, assessment.

INTRODUCTION

With the introduction of a new post-primary mathematics syllabus in Ireland in 2008, named Project Maths, increased emphasis has been placed on developing students' problem solving abilities. Around this time worrying findings from reports such as the *Statement on Raising National Mathematical Achievement* (Expert Group on Future Skills Needs [EGFSN], 2008) highlighted concerns about students' capacity to engage with problem solving. When designing Project Maths the National Council for Curriculum and Assessment (NCCA, 2012) identified five key skills that they saw as being central to effective teaching and learning across the new curriculum. These are *information processing, being personally effective, communicating, critical and creative thinking and working with others*. The new syllabus has placed

increased emphasis on teaching “through” problem solving as it affords students the opportunities to develop these key skills although there have been concerns raised about what exactly teaching through problem solving means (Lubienski, 2011). Regardless of this confusion an aim of the new syllabus is to allow students more time to explore mathematics and to move away from an over-reliance on drill and practice techniques as had been evident in the old syllabus (Lyons, Lynch, Close, Sheerin, & Boland, 2003).

Even though problem solving is seen as a means by which “students deepen their understanding of mathematical concepts by analyzing and synthesizing their knowledge” (Erbas & Okur, 2012, p. 89) this is not always the case. Posamentier and Krulik (1998, p. 15) pointed out that “a substantial portion of problem solving is done by rote. Students struggle through one problem in the section, the teacher reveals a model solution and the remainder of the problems in the section are solved in the same manner”. Mimicking a teacher's solution strategy will work for some problems but when presented with unfamiliar or non-standard problems students still struggle (Harskamp & Suhre, 2007). Having a solid knowledge base, good past experience and knowledge of strategies are considered to be important in successful problem solving (Erbas & Okur, 2012). Now that problem solving has been officially cited as a learning outcome on the post-primary mathematics syllabus in Ireland it is imperative that teachers are aware of effective frameworks for teaching and assessing problem solving to guarantee the successful integration of problem solving into the classroom.

The purpose of this study was to implement an existing problem solving framework in a mathematics classroom and to then attempt to use this framework

as a guide for assessing students' problem solving competence. The following research questions guided this study:

- Do teachers feel that this framework could assist in the implementation and assessment of problem solving in the classroom?
- Which problem solving episodes can be observed while students solve mathematical problems?

SELECTION OF A SUITABLE FRAMEWORK

With the increased emphasis placed on problem solving in the new syllabus the Project Maths Development Team [1] set about designing a modular course with the aim of providing teachers with practical advice on how to approach problem solving in the classroom as well as providing them with a holistic rubric for the marking of solutions. The rubric presented by the Project Maths Development Team primarily focuses on three activities of problem solving – choosing a correct strategy; explaining your choice of strategy and getting the correct answer. Unfortunately this rubric makes no attempt to measure all the remaining, and often important, tasks that individuals engage with during the problem-solving process. To this end a more detailed framework for implementing problem solving was sought by the authors with the additional requirement that a rubric for the assessment of problem solving activities could be easily developed around this implementation framework.

Schoenfeld (1985) developed a framework that separated the problem-solving process into a number of "episodes". He defined an episode to be "a period of time during which an individual or a problem-solving group is engaged in one large task" (p. 292). The initial episodes according to Schoenfeld (1985) were *read*, *analyse*, *explore*, *plan*, *implement*, and *verify*. Using Schoenfeld's framework as a foundation, Artzt and Armour-Thomas (1992) adjusted the framework to "delineate explicitly the type and level of cognitive processes individuals use" (p. 141). To this end modifications of the original episodes within Schoenfeld's framework were needed. For example, the original episode of *read* was separated into the episodes of *read* and *understand*. Artzt and Armour-Thomas (1992) finally settled on eight episodes when looking at problem solving in small groups – *read*, *understand*, *analyse*, *plan*, *explore*, *implement*, *verify*, and *watch*

and listen. This framework, with the exception of the 8th episode as this is specific to small groups and our study focused on students working individually, was adopted by the authors for the purpose of this study as it was deemed to address all the major "stages" of the problem-solving process and it was felt that it could be easily implemented in a classroom scenario.

Additionally each of the episodes within the Artzt and Armour-Thomas (1992) framework is sub-divided depending on whether it involves predominantly cognitive or metacognitive processes. This is important as several researchers (e.g., Goos, Galbraith, & Renshaw, 2000; Teong, 2003) have confirmed the importance of the relationship between cognitive and metacognitive processes where in the words of Artzt and Armour-Thomas (1992) "an appropriate interplay between the two is necessary for successful problem solving to occur" (p. 162).

Due to the structure of this framework it was relatively straightforward to develop an assessment rubric where the focus is placed on each of the individual episodes. In this way no single episode would be overlooked and additionally no individual episode would be deemed more "important" within the problem-solving process than any other. Erbas and Okur (2012, p. 97) have correctly noted that "all episodes don't need to occur to find a correct answer" but since this study is more focused on the process of problem-solving rather than the answer the focus remained on the inclusion or omission of the problem-solving episodes from the student solutions. Additionally it should be noted that the students being assessed in this study could be termed as "novice" in terms of their problem solving abilities and so the authors felt it would be more beneficial to the students if they fully engaged with all the episodes of the framework at this stage of their mathematical development.

METHODOLOGY

Participants for this study were selected from a school in the mid-west region of Ireland. A second year (typical student aged 14 years) higher level [2] mathematics class was selected as the study group for the duration of the nine week study. The selected class was a mixed class of 21 students consisting of 12 boys and 9 girls. Seven of the students volunteered to participate in the study (4 boys and 3 girls). Four teachers also agreed to participate in the study by reviewing the frameworks

presented by the authors and taking part in a focus group to garner their opinions on the suitability of the frameworks for implementing and assessing problem solving in the classroom.

Quantitative data was gathered from the study via testing. Students were tested on their problem solving ability prior to being introduced to the framework within their class structure. During the nine-week intervention one of the authors replaced the regular teacher in the classroom and taught the topics outlined by the regular teacher. Additionally the problem solving framework was introduced during this time frame and then regularly revisited to consolidate this new problem solving approach among the students. Testing was conducted at intermittent stages throughout the nine-week intervention to gain a more comprehensive view on whether or not the students were integrating the framework into their daily problem solving routine. A total of five tests were carried out during the nine-week intervention – one pre-test and four tests during the actual intervention.

Each test consisted of one mathematical problem that students had to solve. The new Project Maths syllabus in Ireland places increased emphasis on the development of students' literacy skills and to that end the questions selected were not just purely mathematical but instead were what has typically in Irish circles been described as "word problems". Word problems, according to Verschaffel, Greer, and De Corte (2000), is the term often used to refer to any mathematical task where significant background information on the problem is presented as text rather than in mathematical notation.

The question utilised in each test was carefully selected in an attempt to maintain the validity and reliability of the study. In the end it was decided that the best way to select questions of a similar standard and difficulty, that were relevant to the new syllabus, would be to utilise past examination papers and sample papers from the Junior Cycle [3] examinations. Each question was selected to be slightly more challenging than the questions that the students encountered during class. This was the case as it was hoped that students would need to employ the problem solving framework shown to them during the lessons rather than being able to solve the question immediately upon reading it.

To assist with the assessment of the test in terms of being able to measure students' success at each of the different problem-solving episodes a template was designed around which each question was structured. When designing the template it was necessary to ensure that each episode of the Artzt and Armour-Thomas (1992) framework was addressed. Unfortunately it was not possible to assess each of the episodes in the framework e.g. *Read*. Since certain episodes could not be assessed independently they were instead combined with other episodes, e.g., *Read* was combined with *Understand* as if students demonstrated an understanding of the problem then we assumed that they had successfully read the problem. For this reason some of the tasks in the assessment template are deemed to assess two of the problem solving episodes as outlined in the Artzt and Armour-Thomas (1992) framework.

The six tasks, along with the episode(s) (in brackets) which each task measured, that students were asked to engage with as part of the assessment template were:

- 1) Underline or highlight the important information/facts given in the question (Read and Understand)
- 2) What is the question asking you to do? (Understand and Analyse)
- 3) Is there a process or method involved in solving this question? (i.e., What methods have you used to solve similar questions like this before?) (Plan)
- 4) What is the first step in attempting to solve this question? (Plan and Explore)
- 5) Complete the question and display all your workings in the space provided. (Implement)
- 6) Does your answer satisfy what is being asked in the question? (Verify)

The scoring of the assessment rubric is rather straightforward. If a student displays evident of completing a particular episode then they are given a score of 1. If no evidence is found of a certain episode then a score of 0 is awarded. Therefore in each test a student could score a total of 7 marks depending on whether evidence of all 7 problem solving episodes was present or not. As previously stated it is not always necessary

to carry out every episode when problem solving but since we are dealing with novice problem solvers an absence of evidence of an episode will be viewed as an omission on the part of the student.

To ascertain whether or not teachers of the new Project Maths syllabus felt that this was a suitable framework for implementing and assessing problem solving in the classroom a focus group was conducted with four teachers towards the end of the nine week intervention. A total of twelve questions were put to the teachers such as “Do you think this framework is relevant to the aims of the Project Maths syllabus?”, “What strategies do you currently use to implement problem solving?” and “Do you think this framework would be easy to implement in your classroom?” and their responses and feedback were recorded and then analysed.

FINDINGS

Episode engagement

From the analysis of the results it is clear that overall there was an increase in the total number of episodes that each student engaged with as part of their problem solving process. Table 1 presents a breakdown of the episodes that students successfully completed in the test that they completed prior to being introduced to the Artzt and Armour-Thomas (1992) framework.

From Table 1 we can see that all students showed evidence of reading the problem with a similarly high number showing evidence of understanding the problem. Unfortunately these positive results do not continue as none of the students showed evidence of analysing the problem (episode 3) or even verifying their solution (episode 7). The lack of analysis by the students is worrying as the analysis episode is important as it is at this stage that students examine the relationships between the information provided in the question and what they are required to show. Similarly to the *Analyse* episode zero students showed evidence of completing the *Verify* episode. An additional goal of Project Maths is that students develop the skills to justify/explain/verify their answers. It is clear from these results that these students have

had very little exposure to this methodology so far in their mathematical careers.

The results from the four tests conducted after the introduction of the Artzt and Armour-Thomas (1992) framework are more promising. Immediately upon introduction to the framework all students showed evidence of engaging with the *Analyse* episode of the problem solving process. The number of students completing the *Plan*, *Explore* and *Implement* episode all show improvement from the pre-intervention test although there is some minor fluctuation between tests regarding the number of students engaging with each episode. Regarding the *Verify* episode it can be seen in Figure 1 that it took longer for student to habituate this episode into their problem solving process but promisingly by the end of the nine week intervention all seven students were displaying evidence of evaluating the outcome of their work. Overall the trend appears to be positive with regards to student engagement with the problem solving episodes.

Correct solutions to problems

Irrespective of the number of episodes that students engage with during their problem solving process it is still necessary to consider the number of correct answers in the tests. Even if a student completed all the episodes it is still possible for them to arrive at an incorrect answer. You would hope that upon reviewing their answer at the *Verify* episode and realising they are incorrect students would persevere and return to earlier episodes and attempt to determine where

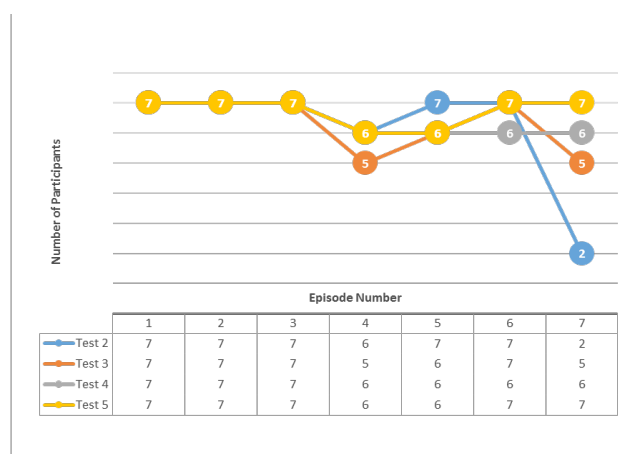


Figure 1: Results of episode inclusion during the intervention

	Read	Understand	Analyse	Plan	Explore	Implement	Verify
No. of students who completed each episode	7	6	0	4	3	3	0

Table 1: Results of episode inclusion in pre-intervention test

they have made a mistake along the way e.g. misunderstood the question in the *Understand* episode or overlooked a vital piece of information at the *Analyse* episode stage.

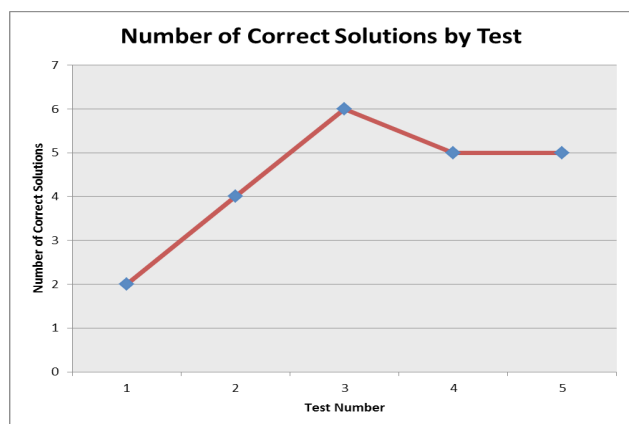


Figure 2: Results of Correct Solutions from Tests

Figure 2 highlights the trend regarding the number of correct answers across all 5 tests. The trend is positive, in general, showing that more students answered the questions correctly once the intervention commenced. There is a minor dip in the number of correct answers in the final two tests but even with this dip the number of correct answers is still higher than the results from test 1. The results regarding the total number of correct answers across the 5 tests offer additional justification regarding the effectiveness of the problem solving framework implemented but also highlight the need, in general, to introduce novice problem solvers to a structured approach to problem solving.

Feedback from teachers

Overall the teachers expressed satisfaction with the framework utilised by the authors to implement problem solving in the classroom. The teachers expressed their views that a framework like this would prove beneficial when attempting to familiarise students with the problem solving process. When asked if they had previously received or utilised any frameworks designed to assist in the teaching of problem solving the teachers stated:

Teacher2: Well I would say I haven't. Have you?

Teacher4: Yes I have, but I ignored it! Well we were given something like that (referring to Project Maths problem solving strategies poster)

Teacher2: Yes that's the only thing, but I wouldn't say there was any time given in terms

of training for teachers with the new Project Maths. In my opinion anyway, maybe it went over my head.

One teacher did go on to state that although she didn't utilise a "structured" framework she felt that within her class, and the classes of her colleagues, they embedded some of the problem solving episodes outlined in the framework.

Teacher3: But I think to some extent that we do a lot of this. I mean I don't feel like this is something I've never seen before. We do say, well how would you solve this? What do we do now? What are you being asked? What are we looking for? Let's see, what are we missing? Do we have this? I think I do this, I'm sure everyone else does too. But I think that a structure where the kids know to put their information down, I think that this would help me.

This final comment about the students having a structure where they know to put their information down is referring to the assessment template structure outlined by the authors. This structure was also commented on by a fellow teacher who felt that it would be beneficial to students to have a structure, especially in an exam situation.

Teacher1: But I think it would be useful maybe in an exam situation where all they see is a jumble of words and they just don't know where to start. At least they have a framework.

On a negative note one teacher commented that she felt it would take significant time to implement a framework of this type. Additionally she commented that trying to get students to focus on a problem for a long period of time might be an issue and that this could lead to discipline problems within the classroom.

Interviewer: Just in terms of what you see there (Framework and Sample Question) do you think it would be worthwhile maybe trying to implement or trying to use this in a lesson?

Teacher4: I do think it would be, but what I do see this as is very time consuming. And in a classroom situation having each student reading and going through all of this a discipline issue probably would arise.

CONCLUSIONS

The results of this study showed that when teaching novice problem solvers it is important to offer appropriate instruction on problem solving so that these students can properly develop their problem solving abilities. This finding confirms Hembree (1992) and Higgins (1997) who observed similar results in their classroom studies. Once students engaged with the episodes of the problem solving framework the overall number of correct solutions to the attempted problems increased.

The result from the first test conducted as part of this study highlighted that very few students engaged with the *Verify* episode once they have reached a solution although almost every problem solving framework highlights the importance of this episode (Polya, 1973). This finding is also consistent with what other researchers such as Erbas and Okur (2012) have found. Encouragingly as the students became more familiar with the framework their results show that more and more of them conducted this episode and made some attempt to check their solutions with regards to the original problem statement.

Overall the teachers interviewed were positive regarding the structure and layout of the framework. Some issues around the time needed to problem solve within a class scenario were mentioned and other issues regarding students going off task when completing certain episodes were raised but neither of these concerns detracted from the overall positive comments from the teachers. The benefit of the framework as a resource within examinations was highlighted by the teachers as once students are familiar with the framework it should help them by scaffolding their problem solving efforts.

Finally it is worth mentioning as a discussion point the “age” of the original Artzt and Armour-Thomas framework upon which this work, and the work of Erbas and Okur (2012), was based. Does a more modern framework exist which would be more suitable

for this purpose or is there a more up-to-date/adapted version of this Artzt and Armour-Thomas framework which might better aid teachers to implement problem solving in the classroom?

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ENDNOTES

1. The Project Maths Development Team is a team of experienced teachers of mathematics who have been recruited to provide professional development support to post-primary teachers of mathematics.
2. All subjects studied at post-primary level in Ireland can be studied at either ordinary or higher level with higher level being the more challenging.
3. The Junior Cycle is the first three years of post-primary education in Ireland. A state-wide examination takes place at the end of the Junior Cycle. Students normally sit for the examination at the age of 14 or 15.