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Fostering students' independence in modelling activities

Helen M. Doerr¹ and Jonas B. Årlebäck²

1 Syracuse University, Syracuse, USA, hmdoerr@syr.edu

2 Linköping University, Linköping, Sweden

Over the last twenty years, research on the teaching of mathematical modelling has recognized the difficulties that students encounter in becoming independent modellers. In this paper, we put forward the notion that supporting and preserving student independence should be a central principle in guiding teaching practices that support students' modelling activities. This potentially provides new ways to address the tensions, dilemmas and in the moment decision making that occurs when teaching mathematical modelling. We provide empirical evidence of teaching practices that encourage students' self-evaluation of their modelling activities in ways that foster their independence as learners and modellers.

Keywords: Modelling, teaching practices, student independence, self-evaluation.

If we want students to be productive independent individuals and problem solvers with the abilities to apply mathematical reasoning in various situations, they need to have a range of possibilities for acting mathematically readily available when faced with a problem. Mason and Davis (2013) argue that this readiness is not fostered by pedagogies that keep students dependent on their teachers. Rather, teachers need to support students to act in ways that can become part of their own repertoire of mathematical ways of thinking. Students' mathematical learning needs include developing productive dispositions, flexible strategies, persistence and independent thinking (National Research Council, 2001). Mathematical modelling problems would appear to be a particularly rich site for developing these dispositions while learning mathematics in realistic problem situations. However, this poses many challenges for the teacher, especially how to: tackle classroom discussions, structure group interactions, and provide effective feedback

to students (Brodie, 2011; Lobato, Clarke, & Ellis, 2005; Magiera & Zawojewski, 2011). In this paper, we provide empirical evidence of teaching practices that appear to support and encourage students' independence in their modelling activities and we identify dilemmas that present challenges for teachers in their pedagogical decision making.

THEORETICAL BACKGROUND

Adequately describing and theorizing about what teachers actually do in and around their classrooms is a complex task – one in which research has been slow and sometimes elusive in providing a holistic and comprehensive picture of teaching practices (Even & Ball, 2009). One important strand of research has emphasized and seriously acknowledged the important role of giving voice to and using students' own work and ideas in the teaching and learning of mathematics. Recently, Stein and colleagues (2008) proposed a model of five practices (*anticipating, monitoring, selecting, sequencing, and making connections between student responses*) that can be taken up by novice K-12 teachers, as they learn to orchestrate productive mathematical discussions by simultaneously building on students' ideas and important mathematics. Stein et al. argue that this model gives guidance to teachers so that “the teacher remains in control of which students will present their strategies, and therefore what the mathematical content of the discussion will likely be” (p. 328). However, by centering the control of the mathematical discussion with the teacher, these novice teaching strategies may not help students become more independent learners and, as such, offer little guidance for experienced teachers in managing more complex learning situations, such as modelling tasks. Such situations occur when the teacher has to respond to unanticipated student ideas and manage the emergence of student interactions that cannot

be fully anticipated ahead of time. Mason and Davis (2013) refer to these situations as requiring teachers to make “in the moment” pedagogical decisions, something that presents a dilemma and tension for experienced teachers.

A common response to this dilemma is for the teacher to engage in telling or explaining to the students the intended content. Lobato, Clark and Ellis (2005) suggest a reconceptualization of the telling or not-telling dilemma by distinguishing between the teacher action of *telling as initiating* and the action of *eliciting*. According to Lobato et al., *telling as initiating* refers to teacher actions “that serve the function of stimulating students’ mathematical constructions via the introduction of new mathematical ideas into a classroom conversation” (p. 110). The teacher’s intention is to promote student sense making and is often followed by eliciting students’ ideas. Brodie (2010; 2011) elaborates the dilemmas faced by experienced teachers when, having elicited students’ ideas, they must make in the moment decisions about when and how long to press individual students for making meaning and giving justifications. As teachers engage with model eliciting activities (Lesh & Doerr, 2003), they are faced with pedagogical dilemmas and in the moment decision making about how to move students’ mathematical learning forward. In this paper, we want to put forward the notion that student independence should be one of the most central principles in teaching practices, guiding both planning and in the moment pedagogical decisions and actions. The focus of this paper is on elaborating teaching practices that foster student independence.

Our research is situated in the models and modelling perspective on teaching and learning mathematics (Lesh & Doerr, 2003). Within this framework the notions of eliciting student thinking and developing their emerging models are central to learning mathematics. In this work, we take models to be externally represented conceptual systems that consist of objects, operations, relations, and interaction-governing rules used to predict, explain, describe, or understand some other system (Lesh & Doerr, 2003). By engaging in a sequence of model development activities, students’ models are repeatedly developed, modified, extended and revised through “multiple cycles of interpretations, descriptions, conjectures, explanations and justifications that are iteratively refined and reconstructed by the learner” (Doerr & English, 2003,

p. 112). From this perspective, learning is equated with model development. The ability to develop, apply, and adapt a generalized model to be used in a range of contexts is the essence of what it means for students to be independent learners and problem solvers.

METHODOLOGY AND SETTING

This study took place in a six week summer mathematics course for beginning engineering students, designed around a model development sequence centred on the concept of average rate of change (Ärlebäck, Doerr, & O’Neil, 2013). Model development sequences are sequences of structurally related activities that are intended to engage students in multiple opportunities to describe, interpret, make conjectures, explain, develop and iteratively refine their models while interacting with other students (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). A model development sequence begins with a model eliciting activity where students’ ideas about a problem situation are elicited and made explicit in forms that can be tested, revised and refined. Following the model eliciting activity are one or more model exploration activities and model application activities. In the model exploration activities, the underlying mathematical structure and representations (such as tables, interactive graphics, diagrams, or animations) of the elicited model are further investigated and developed. In the model application activities, the students apply and adapt their previously elicited and explored models in new contexts and situations.

The model development sequence in this study consisted of a model eliciting activity (MEA) using the context of bodily motion along a straight path; a model exploration activity (MXA) using a computer simulated walking-world (cf., Kaput & Roschelle, 1996); and, two model application activities (MAA). In the first model application activity the students developed a model for how light intensity varies with the distance from a light source, and in the second model application activity, the students modelled how the voltage changes over a fully charged discharging capacitor in a simple circuit (Ärlebäck et al., 2013). In this paper, we report on our analysis of the video recordings of the lessons focusing on the teacher moves (or actions) identified in the data as either *independence preserving* or *neutral* or *non-independence preserving* moves. We also analysed debriefing interviews with the teachers to understand her intentions. The data

presented in this paper are from the MEA and the first MAA.

An *independence preserving move*, is a teacher move whose intention and consequence is that it preserves students' independent on-going work. A teacher move that provides students with a tool or strategy to facilitate the students' independence in a future activity/situation would be considered independence preserving. Encouraging student persistence (“keep thinking”, “keep talking”, “keep working”) and collaborative work (“talk to your partner”) is considered to be independence preserving. As well as being independence preserving, a teacher move can be *neutral* or *non-independence preserving*. A non-independence preserving move typically implicitly suggests that meeting external expectations (“label your columns”) is valued rather than independent thinking and work. A neutral teacher move would be the teacher's observations as the students are working on a task.

We identified one particular sub-category of independence preserving moves that occurred regularly within our data set: *encouraging self-evaluation moves*. Teacher moves that either implicitly or explicitly encouraged students to be evaluators of the correctness, appropriateness, usefulness or goodness of their work are called *encouraging self-evaluation moves*. Examples of encouraging self-evaluation include moves that support students in having confidence in their solutions, in validating solutions with a partner, and in giving reasons or justifications about why their solution has to be correct. Encouraging self-evaluation also occurs when the teacher simply responds to the student “think more” about a particular result, representation, relationship, object or idea. We take encouraging self-evaluation to be a particular category within the more general category of independence preserving moves.

RESULTS

We give examples that occurred in two parts of the model development sequence: during the model eliciting activity and during a model application activity. In these examples, the teacher encouraged students' self-evaluation within the larger goal of fostering students' independence in their modelling activities. In the first example, over three days of the MEA, the teacher engaged in a range of moves (described below) that fostered aspects of student independence at the

individual, small group and whole class level. In the second example, the teacher engaged students in an extended argument among the students and led by the students about the rate at which light intensity changes with respect to distance. Rather than resolve the argument for the students, the teacher preserved student independence by engaging them in collecting empirical data that they could use to resolve the question.

Examples from a model eliciting activity

The MEA was designed to elicit students' ideas about constant rates, about the distinction between velocity and speed, and about position graphs for which the motion was not physically possible. The students began the task by working in small groups to create a line with negative slope, using their graphing calculators, motions detectors and graphing software on a computer. The design of the task required them to transfer data from their calculators to other group members' calculators and to a computer. Thus, this MEA was comprised of a mathematical task (using bodily motion to create a line with negative slope) and the learning of technology skills. The teacher assumed that many of the students had limited background with the technology from high school and knew that their fluency with the technology would be needed throughout the model development sequence. Throughout the MEA, the teacher made numerous moves that were intended to set expectations for student independent work at both the individual and the group level and expectations for students to self-evaluate the goodness of their work and that of their peers. We will briefly illustrate five teacher moves: establishing the utility of a reference tool; turning back questions; gathering data; encouraging persistence; establishing criteria for self-evaluation.

At the beginning of the MEA, the teacher distributed a “data management reference sheet.” Since one goal of this particular MEA included developing fluency with technology that would be needed throughout the model development sequence, the teacher had created a reference sheet that contained technical details for using the technology. Nearly all of the teacher's interactions with the students were brief as she answered their technology related questions with “do you have your data management reference sheet?” or “look at the data management reference sheet.” She consistently referred to the need for each individual to acquire the necessary technology skills. In refer-

ring to the computer data transfer, she commented, “your whole group should go because everyone should know how to do that.” We take this as the teacher’s intentional moves to support the independence of each individual in gaining fluency with the technology that would be needed throughout the modelling activities.

In a segment that occurred after the students had worked on finding and interpreting equations for a linear position versus time graph with negative slope, the students initially incorrectly referred to speed as the slope of the line. As a discussion of this was about to conclude, a student asked, “what does the motion detector measure?” Rather than answer this question directly, the teacher turned this question back to the students. We take this as a move to engage students in self-evaluation, that is, in answering other students’ questions. Eventually, there was an answer from the students that what the detector measured was distance in feet, recorded every tenth of a second. The segment ended with the teacher inserting the units for speed as feet/second.

One part of this modelling task focused students’ attention on the possibility of creating a U-shaped graph with the motion detector by walking at the same speed the whole time. The students were sharply divided on this issue. The segment ended as one student suggested “why don’t we just try it?” We take this as an example of students suggesting self-evaluation and of the students taking up the teacher’s encouragement of their independence by gathering data and evidence to support their claims.

One aspect of student independence would seem to be persistence in working on tasks and solving problems. An example of this occurred at the end of the first lesson where one student had worked on finding an equation for the line with negative slope that the group had created. The student asked the teacher if his solution was correct and the teacher responded by asking him how he could evaluate the correctness of his answer for himself (“how do you check it?” “figure it out”). Later, when he determined that there was an error in his work, the student again approached the teacher, who responded with “find your mistake. I’m sure you can find it.” Finally, at the end of class, when the student came to her again, she again reassured him that he can find his mistake. We take this as both encouraging student persistence and as a form of encouraging student self evaluation.

In the third lesson, the teacher displayed examples of students’ descriptions of motion from work done in an earlier lesson. The teacher asked the students to choose a good description and provide a reason why the description was good. The students worked on this individually. We take this as an example of students evaluating descriptions created by other students. However, it was unclear what criteria the students had for deciding what constitutes a “good” or “best” description? This dilemma became clearer as the teacher asked the students to discuss in groups which description was the best. The students had difficulty in coming to consensus in their groups. The teacher had a whole class discussion where students gave their reasons for choosing various alternatives. The teacher shifted the goal of the task when she suggested that they consider how to improve a description as some students claimed “none of them are really good.” As this task ended, the teacher asked the students, “could you create the graph if you were given this description?” This final statement by the teacher appears to be a form of establishing the criteria by which one could decide if a description were “good” enough. However, this criteria was not clear to the students from the beginning of the task.

Examples from a model application activity

This example is from a model application activity, where students investigate how the rate at which light intensity changes with the distance to a light source. At the beginning of the activity, the students are asked the following question as part of their homework prior to the lesson to elicit their thinking about changing light intensity:

Imagine the tail light of a car moving at a constant speed away from you. Is the light intensity:

- 1) fading at a constant rate
- 2) fading slowly at first then quickly
- 3) fading quickly at first and then slowly**
- 4) unsure

Although all of the students had taken a prior course in physics in secondary school, where the relationship between light intensity and distance is studied, only one of the students correctly identified the rate at which the light intensity fades: quickly at first and then slowly. The majority of students concluded that either the light faded at a constant rate (60% of the

responses) or slowly at first and then quickly (27% of the responses). One student was unsure.

These responses were displayed for the students via a student response system. The teacher commented that there was “lots and lots to talk about” and that she wanted to know from them “why did you choose the answer you chose?” To accomplish this, she arranged the students in groups and asked them to discuss their answers. After a few minutes, while the teacher was walking around listening to students' reasoning, the teacher pulled the class together for discussion. Each of these teacher moves – eliciting their ideas with the initial question, asking them to engage in peer discussion, and listening to their reasoning – served to encourage the students to self-evaluate their responses to the question on changing light intensity.

The ensuing whole class discussion began as the teacher asked S1, a member of one group, to start the discussion:

S1: My other group members, they voted for number two, but I voted for number one because I explained that the car is going away at constant speed, so I thought like the light would go away at a constant speed too. But then I don't know if that's the same thing. It would be like how you see the light and the intensity of the light, if they drop in the same way.

The teacher re-stated S1's comment and invited students to consider S1's argument or offer their own:

Teacher: Are you guys thinking similarly to S1 or why did you think it was at a constant rate? [several students mumble] S2, what did you say?
 S2: The car is moving away at a constant speed so I think the intensity decreases at a constant speed.
 Teacher: S3
 S3: The light travels at a different speed...
 S4: ...than the car...
 S3: ...than the car...
 S4: So, it would actually be different.
 S5: Isn't the speed of light constant?
 S6: If the speed of light is constant, why...

S4: ...why is the car moving? – because it creates a variable!
 All: [Laughter and many students talking at the same time]
 S7: But yeah [inaudible] the speed of light and speed of...
 S3: ...S7, what do you think the answer is?

The students were actively engaged in arguing whether or not the light was fading at a constant rate. Many of those who thought the rate was constant were arguing that it had to be constant because the speed of light was constant. However, the intensity of light with respect to distance is not related to the speed of light with respect to time, as S1 had suggested in her initial argument. What is striking about the conversation above is that the teacher is not mediating, restating, or directing the discussion. She is listening, off to the side. The argument takes a turn in a new direction as S3 asked S7 what she thought:

S7: Me?
 S3: Yeah!
 S7: I put two [slowly then quickly], but I'm not sure that, but I don't know what the ratio is between light intensity and [S7 starts gesturing]...
 S2: I mean for instance, if the speed of light's constant, and the cars' constant...
 S7: ...Yes, but the speed of light is the travelling speed of light. We're talking light intensity which is what you see
 S2: Right...

In this segment, S7 has made the critical distinction (which was foreshadowed in S1's initial argument) between the speed of light and the intensity of the light. S2 seemed to acknowledge this, with his comment “right.” But it is not entirely clear what he meant by this. But the next response to S7 is from S3, who had asked S7 for her answer in the first place. S3 claimed to have an example, which was then immediately followed by another student's (S4) example. From the teacher's perspective, it was not possible to fully anticipate what these examples might be, what they would mean, and how they would relate to the central question about the rate of change of intensity of light.

S3: Yes, so in 7th grade we were studying the change of motion and stuff and whatever. And so, we found out, like if you're

in, okay, I'm from New York City, we're doing that, like, if you're walking in a train, like running inside of a train, at, you know let's say your running like, 20 feet per second or whatever, whatever, that's unrealistic, but just say you're doing that, right. And the train is going at, like 100 feet per second, then the total would be 120 feet per second. That's how fast it would seem you're running, because the train is like moving, and then you're moving...

S4: ...I think I have a better example, of how you know that light is not travelling at a constant rate. Why do they put the eenie-teenie-tiny [very tiny] lights on the top of buildings that planes aren't gonna be able to see from 50 yards away? [inaudible] explain that to me – why are runway lights so small, you see the guy [starts gesturing as if he was taxiing a plane to its gate]

All: [many students talking at the same time]

The discussion was ended by the teacher, but not by drawing a conclusion for the students. Instead, the teacher continued to engage the students in evaluating their emerging models of light intensity by initiating the next task of collecting real data that would enable them to resolve the question based on empirical evidence.

The results elicited from the initial question showed that 60% of the students thought that the relationship between light intensity and distance from the car was linear. To address this, the teacher encouraged the students to self-evaluate their emerging models of rates of change and of light intensity. As the episode unfolded, the students unpacked their models by themselves, first in small group discussion and then in a student-led and student-driven whole class discussion. S1 raised the core question about the relationship between the constant speed of the car and the rate of change of the light intensity. The discussion then revolved around the speed of light and its constancy, prompting many students to express their ideas on this matter. When S7 tried to focus the discussion back on S1's core question, S3 and S4 both drew on personal experience in trying to understand the situation. S3 remained focused on the role of the constancy of the speed of light, whereas S4 argued about the rate of

change of the light intensity. The teacher functioned as a listener throughout the discussion. The teacher ended the discussion by initiating the next activity where students were to collect real data to resolve the issue. This shift was a key move in encouraging further student self-evaluation of their emerging models for how the light intensity varies with the distance from the light source and in preserving student independence.

DISCUSSION AND CONCLUSION

In this paper, we wanted to further the research on using students' thinking in mathematics lessons and the dilemmas connected with this practice, as reported in the literature, to forefront an argument and empirical evidence of teaching practices that encourage students' self-evaluation of their modelling activities in ways that preserve their independence as learners and problem solvers. This focus presents a shift in emphasis on students' and teachers' roles and responsibilities. For example, the essence of *sequencing* in the Stein and colleagues' (2008) model is for the teacher to be in control of the form and content in a whole class discussion. An independence preserving stance would instead advocate sharing with students the responsibility for the sequencing of contributions in the discussion in order to engage them in the self-evaluation of their ideas and their emerging models. The first example illustrates the role of the teacher in setting expectations for students to use tools such as a data management reference sheet to solve technology related problems, to answer questions from peers, to gather data to support arguments, to persist in finding mistakes, and to collectively establish criteria for evaluating the goodness of written descriptions. Taken together, these moves by the teacher appear to support the students in becoming more independent learners and problem solvers. The second example suggests the benefits of engaging students in peer-discussion and encouraging students' self-evaluation by having them collect data that can be used to self-evaluate the goodness of their emerging models. This practice seemingly resolved the teaching dilemma that would otherwise have confronted the teacher as to how to resolve students' conflicting ideas about changes in light intensity, while at the same time providing the students with peer-discussion as a self-evaluation tool for them to use in modelling activities. We offer these examples as ways of thinking about the notions of *students' independence* and *stu-*

dents' self-evaluation, with the hopes of contributing to the on going discussion of effective teaching practices.

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