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► **To cite this version:**

Brikena Djepaxhija, Pauline Vos, Anne Berit Fuglestad. Exploring grade 9 students' assumption making when mathematizing. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.848-854. hal-01287253

**HAL Id: hal-01287253**

**<https://hal.science/hal-01287253>**

Submitted on 12 Mar 2016

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# Exploring grade 9 students' assumption making when mathematizing

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*Making assumptions is a key activity in modelling. The present study aims to explore the variety of assumptions that lower secondary students make in this process. As theoretical basis for the data analysis, we used the modelling cycle by Blum and Leiss (2007) and framed a definition of assumptions. The study was carried out with grade 9 students. The results show three categories of assumptions: (1) parameter assumptions, (2) assumptions for the choice of the mathematical model, and (3) assumptions about task expectations. Assumptions from the first two categories assist students to use extra-mathematical knowledge to construct a mathematical model, while the third category of assumptions can assist but also hinder them. Our study shows, that students were well aware of their assumption making.*

**Keywords:** Assumption making, task expectations, modelling, mathematizing, lower secondary students.

## INTRODUCTION

In mathematics education there is an increasing emphasis on applications and mathematical modelling (Vorhölter, Kaiser, & Borromeo Ferri, 2014). This international development is also reflected in the intended curriculum of Albania, the country in which the current study is based. In the curriculum framework for primary and lower secondary schools (grades 1-9), the Ministry of Education and Sports highlights that students need to understand the role of mathematics in everyday life and use mathematics adequately to solve problems from everyday life (IED, 2013).

However, Albanian students have displayed a rather poor performance on modelling problems as used in the Programme for International Student Assessment (PISA), in which Albania now has participated three times (Harizaj, 2011; OECD, 2014). The Albanian re-

sults on PISA urge for a closer investigation of the process when students deal with modelling problems. Therefore, we have started a study on the first phases of this process, when students are facing a problem situation and have to make a translation to a mathematical model in order to reach a solution of the problem. In particular, we aim to explore the various assumptions that students make in this start-up phase of the modelling process.

## THEORETICAL FRAMEWORK

Blum and Leiss (2007) have framed mathematical modelling as a process, which consists of subsequent activities. See Figure 1. The process begins with understanding 'the real situation' given in the task, which leads to the construction of a 'situation model' (e.g., a rough drawing of the problem situation). Then, this 'situation model' is idealised into a 'real model' through relevant structuring, and by making assumptions and simplifications (e.g., the drawing is made more specific).

In the next step this 'real model' is translated into a 'mathematical model' (e.g., an algebraic formula). The 'mathematical model' is then used to obtain 'mathematical results'. These results are interpreted into 'real results' and then validated in light of the given problem. In the case the results are considered inadequate for the real situation the entire modelling process is run through again.

The above description of the mathematical modelling process is an idealized one, because in practice the modelling process is more complex and often non-linear (Borromeo Ferri, 2006; Galbraith & Stillman, 2001). Moreover, Borromeo Ferri (2006) empirically showed that some phases in the modelling process can overlap and that the problem structure affects the process.

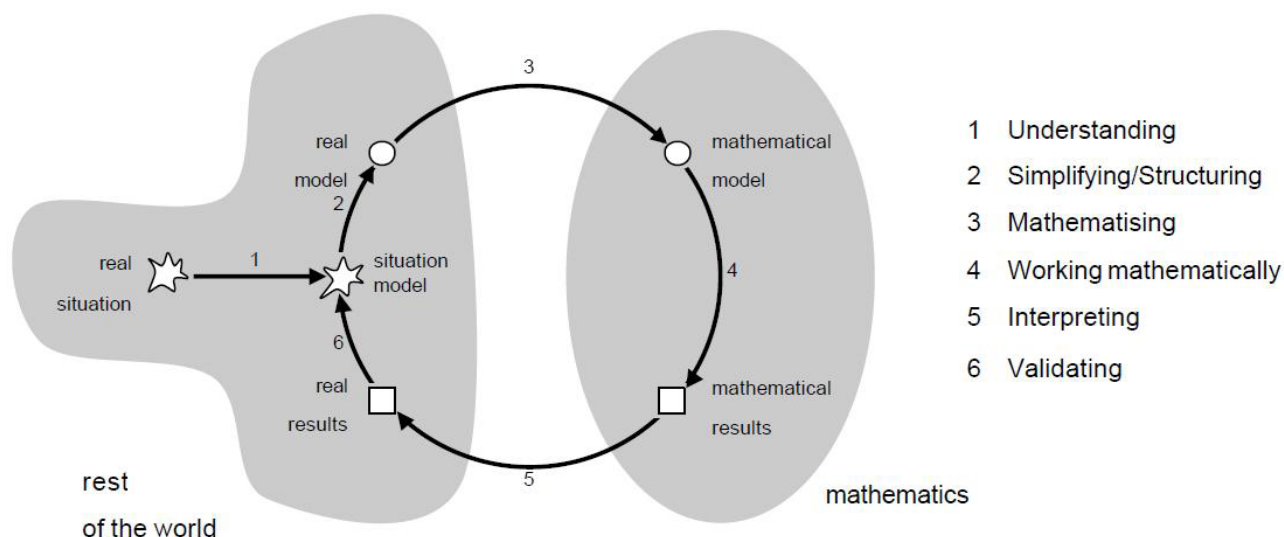


Figure 1: Modelling cycle from Blum and Leiss (2007)

An important term in the modelling process is *mathematizing*. Blum and Leiss (2007) use this term to describe the translation activity from the 'real model' to the 'mathematical model'. However, Borromeo Ferri (2006) has argued that it can be difficult to distinguish between 'situation model', 'real model', and 'mathematical model', because these differ between students and tasks. For example, many tasks with problems from the real world are already structured and de-authenticated by the task authors, to communicate the problem situation unambiguously to the students (Vos, 2011). Therefore, in the present paper the term *mathematizing* will be used to describe holistically all activities from 'problem situation' to 'mathematical model'. In this frame, *mathematizing* comprises all activities before a student starts on the purely mathematical work, such as: understanding; simplifying; scouting the problem (Schaap, Vos, & Goedhart, 2011); assuming; structuring; idealising; and finally translating into mathematics. Results of several studies in the field of mathematical modelling (Schaap et al., 2011; Stillman & Brown, 2012; Stillman, Brown, & Galbraith, 2010) display that the mathematization process is complex and that students face various obstacles within this process.

Making assumptions is one of the activities in the mathematization process (Edwards, 1989; Galbraith & Stillman, 2001). Maaß (2006) empirically showed that inadequate assumptions (unrealistic or oversimplified) lead to an inadequate real model, which further leads to an inadequate mathematical model for the problem situation.

Goldin (2002) defines assumptions as being:

[...] often propositionally encoded, taken as a basis for exploration or discussion but (at least temporarily or provisionally) without attribution of truth, validity or applicability. (p. 65)

This definition converges with the definition in the Oxford English dictionary ([www.oed.com](http://www.oed.com)), which says that an assumption is:

The taking of anything for granted as the basis of argument or action.

An assumption can be formulated as "*let's take <statement A>*", proposing statement *A* to temporarily be true without further argumentation, but statement *A* offers a productive opening towards subsequent activities. Assumptions made during the mathematization process "provide building materials from the real world to bridge the divide between a descriptive problem statement and its representation in mathematical terms" (Galbraith & Stillman, 2001, p. 304). Assumptions are practical, enabling students to continue their work; assumptions may even assist in overcoming cognitive blockages. The present study aims to explore the variation of assumptions during the mathematization process. The research question is: what assumptions do students at lower secondary school level make while mathematizing?

## METHODS

The study was operationalized by studying Albanian students while working on modelling items. The participants in the study were four grade 9 students from a lower secondary school in Albania. The four students were average achievers in mathematics and they were selected by their teacher. They had no particular experience in modelling. The students participated voluntarily and they will be identified by pseudonyms.

A three step design was used to capture a holistic view about students' mathematization process, consisting of (1) observation, (2) stimulated recall and (3) interview (Busse & Borromeo Ferri, 2003). The combination of these three methods draws on strengths of each and makes it possible to capture the connection between internal and external processes which occurred when mathematizing. The first author was the interviewer, being able to communicate with the students in their language. The present paper will report only on the data associated with the assumptions that students made while mathematizing.

In the first phase (observation), students worked in pairs on solving three modelling items collaboratively. To enhance their thinking aloud, just one pen was made available to both students. In this way they could not write both at the same time. The second and third phases (stimulated recall and interview) took place in another session (later on the same day or on the next day), whereby each student was individually invited to watch excerpts of the video recordings from the first session. The students were asked to comment and reflect on their activities in the video. In this way stimulated recall helped students to be "close to the process of working on the task without interfering [with] the process itself" (Busse & Borromeo Ferri, 2003, p. 257). In the third phase (interview), the interviewer cited some statements made during the solution process and asked them to comment on these statements. By using statements of the students, the risk of making students speak with the interviewer's words was avoided.

All the three phases of data collection were video recorded and then transcribed. First, transcriptions were used to identify all assumptions students made in the complete modelling process. Then, based on Blum and Leiss (2007) modelling cycle, we identified assumptions that the students made in the mathe-

matization process. Assumptions associated with the mathematization process were investigated for their purpose (why), their emergence (when), and the awareness that the students displayed on the making of these assumptions.

The modelling items for the present study were taken from PISA. The PISA items for mathematics are designed as problems with a real world origin and they have been designed in a careful process (OECD, 2013). Moreover, their globally widespread use should enable us to compare, validate and generalize research results internationally. We selected PISA items on several criteria: (1) they should require students to integrate the offered task information with extra-mathematical knowledge in order to construct a mathematical model; (2) the cognitive demand should match with students' abilities and therefore the mathematical content was taken from the curriculum of at least one year below the students' level; (3) the real-life situations presented in the tasks should align with Albanian students preferences on the relevance of mathematics, as studied by Kacerja (2012).

## FINDINGS

Because of the page limitations of this paper, we report only on the data from one pair of students, the girls Joni and Megi, on two of the PISA items used in the study. The first item is named Rock Concert.

For a rock concert, a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert?

- |         |          |           |
|---------|----------|-----------|
| a) 2000 | c) 20000 | e) 100000 |
| b) 5000 | d) 50000 |           |

Joni and Megi started to calculate the area of the field (100x50) and then started a discussion on how to connect this to the other information in the problem statement.

- Megi: Since the concert was completely sold out then there will be 5000 fans because the field was full.
- Joni: But one fan per one square meter will be...here it says that all fans are stand-

ing, therefore they are more than 5000... there will not stand one fan per square meter because it has no meaning for a concert...how many fans can stand in one square meter?

Megi: But since nothing else is given, it will be 5000 fans... Just option b) has to do with it. Because we have the numbers 50 and 100 in the problem statement, we do not have any other numbers... The total number of fans according to me is 5000 because we cannot solve a problem by supposing.

In the above episode Megi equals the area with the number of fans, and then Joni translates this into the assumption: *one fan per m<sup>2</sup>*. However, she thinks that this is not realistic for a rock concert, considering the information that all the fans are standing, so she implies the assumption that *there are more than one fan per m<sup>2</sup>*, and she asks explicitly the clarifying question on the density of fans standing in a concert: “*how many fans can stand in one square meter?*”

Her partner, Megi hesitates to connect the problem statement to the real world. She bases her argumentation on two other assumptions that we reformulate as: *only given numbers should be used to build a mathematical model* and *one cannot make assumptions in these kinds of tasks*. However, these assumptions are not accepted and they continue the discussion on the density of fans. This leads to another assumption:

Megi: The rock concert is attended only by adults, because rock concerts cannot be attended by children... It is attended only by adults and adults use more space.

Here, Megi has changed her perception of what the task is asking her, namely that she can use extra-mathematical knowledge. So, she makes the assumption *only adults attend rock concerts*, which is relevant to define the density of fans.

Thereafter, Joni and Megi are carried away by the context of the task, talking about whether the rock players are popular, or not and there may not be too many fans coming. They also talk about whether there can be chairs in the field. In these discussions we did not identify assumptions, until the next utterance:

Megi: It depends on how fans stand...but we can divide the field in rows and columns... it depends on how fans stand because in a rock concert the audience tries to be as near as possible to the stage, but we can consider the same everywhere.

This utterance shows that Megi recognizes the complexity of defining the density of fans, and that one needs the assumption: *fans are uniformly distributed in the field*.

The above findings from the students' work on the Rock Concert item show that they made different assumptions in the mathematization process. Some of these assumptions can help students to incorporate new information into the mathematical model that is missing in the problem statement, while other assumptions are about their perception of task expectations. Also, we observe that students display an awareness of their assumption making.

The other PISA item, on which we report here, is the Pizza item:

A pizzeria serves two round pizzas of the same thickness in different sizes. The small one has a diameter of 30 cm and costs 300 ALL. The larger one has a diameter of 40 cm and costs 400 ALL. Which pizza is better value for money? Show your reasoning.

After reading the problem, Joni and Megi make a straightforward mathematization of the situation by taking the ratio between the diameter of the circle and the price. Implicitly, they take for granted that *the size of the pizza is linear to its diameter*, which fits the definition of an assumption. After calculating these ratios they conclude: “*both pizzas have the same value for money*”. However, they do not find this result satisfying – they validate the obtained answer. To do this, they return to the mathematization process and start to ask clarifying questions on the meaning of “better value for money”:

Megi: In this case, it means which one is bigger, which one makes you full.

Joni: But it also can be: which one is cheaper, which makes you save money...or which one is smaller, which helps to keep your body in a nice shape.



In this short episode the students make both a dietary assumption (“*makes you full*” and “*keep your body in a nice shape*”) and a financial assumption (“*makes you save money*”), which both give a basis for further calculations. After some discussion, the students select one assumption: *better value for money means it makes you full*, because, according to Megi, “*this is what we do in our everyday life*”. Then she uses this assumption to carry on:

Megi: Since they have same ratio, the same thickness, but different size, it is the big pizza because it is bigger...therefore it has more calories ...offers more opportunities to make you full.

So, the mathematical model consists of selecting the maximum. However, Joni is uncertain about this approach/choice:

Joni: It is a mathematical problem and we cannot solve it supposing what we do in our everyday life... We involved our individual opinions from our life into the solution and the solution was not a fixed one, we got it by supposing, while in the problems of our mathematical textbook...there are given more numbers and information and the solutions are fixed.

In this utterance Joni expresses uncertainty about the use of familiar, everyday assumptions into the mathematization process. This ‘assumption on the making of assumptions’ was also expressed in the Rock Concert item: *one cannot make assumptions in these kinds of tasks*. Interestingly, in that case it was expressed by the other student, Megi.

The above findings from the students’ work on the Pizzas item show again that they purposefully made different kinds of assumptions. With this Pizza item the assumptions are made either during the mathematizing or the validating process. Some assumptions are made by using extra-mathematical knowledge to choose a mathematical model. Other assumptions limit the students to focus only on the given information in the task.

To summarize, we have categorised the assumptions identified in the above episodes into three broad categories:

### **Parameter assumptions**

Parameter assumptions are assumptions that students make to refine a mathematical model, such as refining the formula in the Rock Concert item: *density of fans  $\times$  area*. These assumptions incorporate extra-mathematical knowledge into a parameter of the mathematical model, such as in *density of fans*. These assumptions emerge when students understand that the given information in the problem statement is not enough for mathematizing.

### **Assumption for the choice of the mathematical model**

Assumptions for the choice of the mathematical model are assumptions made to select a mathematical model, such as taking either the ratio of diameter and price, or taking the maximum of the size. These assumptions are made in the mathematizing process (for creating a model) or validating process (for critiquing the model and refining it).

### *Assumptions about task expectations*

Assumptions about task expectations emerge from the learning environment of mathematics education, for example when students have been trained on so-called *word problems*. These word problems consist of short texts, generally describing inauthentic situations and irrelevant questions, and students have to deduct some numbers and an operation from the text to find a number answer. Word problems are part of a school culture with unwritten rules, also known as ‘didactical contract’ (Brousseau, 1997). Assumptions such as *only given numbers should be used* and *making assumptions is not allowed* make students hesitate to consider the situation context and to generate assumptions based on their extra mathematical knowledge. When students’ perceptions do not align with the task, their assumptions about task expectations may hinder them. However, not all assumptions about task expectations will be counter-productive. For example, an assumption such as *we should use our extra-mathematical knowledge* is also about task expectations, but it is a productive opening in modelling.

Assumptions about task expectations can be statements on whether or not extra-mathematical knowl-

edge can be used, and these assumptions can assist or hinder students to make modelling assumptions. Assumptions about task expectations emerge in the mathematizing or validating process.

## CONCLUSION AND DISCUSSION

In the present study, we explored the various assumptions that grade 9 Albanian students make while mathematizing, that is while constructing a mathematical model from a problem situation. From the data analysis we identified three categories of assumptions:

- *parameter assumptions*,
- *assumptions for the choice of the mathematical model*, and
- *assumptions about task expectations*.

The first category, *parameter assumptions* are assumptions made to refine a mathematical model by using an additional parameter. The second category, *assumptions for the choice of the mathematical model* are assumptions made to choose, or even justify the mathematical model. These two types of assumptions emerged in different phases of the modelling process, such as during mathematizing or validating activities. Their need arose, when the students were constructing or criticising the mathematical model and they used extra-mathematical knowledge for choosing operations, and as such they used these assumptions for creating or refining the mathematical model.

We observed that the students were well aware of their assumptions. This is evident in the words they used: “we can consider”, “we can suppose”. Moreover, students were explicitly commenting on whether it was allowed to make assumptions based on extra-mathematical knowledge. This led to the third category of assumptions: *assumptions about task expectations*, such as *only given numbers should be used* and *making assumptions is not allowed*. These are assumptions made by students to deal with the norms and values of a mathematical culture. These assumptions derive from students' beliefs on how they should work, in particular in relation to work on inauthentic word problems. Beliefs “are abstracted from one's experiences and from the culture in which one is embedded” (Schoenfeld, 1992, p. 74). Other researchers have termed the educational culture, in which students

behave according to certain norms as a ‘didactical contract’ (Brousseau, 1997).

The above three categories also were observed in other episodes of our study, but space does not allow us to report this here. We do not claim that the three above named assumption categories are the only assumptions that students make while mathematizing. Our study was a pilot study done with only a few participants (sample of convenience) and a limited number of problems. Empirical findings from other studies (e.g., see Borromeo Ferri, 2006) show that mathematization activities differ between tasks. Therefore, further research with the same focus on assumptions is recommended. If items are used that require students to define variables or constants, it is possible that they will make assumptions on those variables and constants.

Our study revealed that *assumptions about task expectations* increased students' insecurity in the mathematization process. Therefore, we suggest as teaching implication, that assumption making should receive emphasis in mathematics lessons, as also said by Seino (2005). By discussing assumptions on the parameters or on the choice of the model, both teachers and students can learn about their *assumptions about task expectations*. They can then better understand the role of extra-mathematical knowledge in the mathematization process and in the entire modelling process. Moreover, such a focus can advance their awareness on validating their assumptions.

Our study displays that students make assumptions, which can assist in the mathematizing process. Some authors have described assumptions as “building materials” (Galbraith & Stillman, 2001, p. 304) or “the cement” (Edwards, 1989, p. 95). We support these metaphors, but at the same time we observed some assumption being made, which held back students from the situation context. Thus, some assumptions can be metaphorically described as “*blocked doors*”. Our study shows that if students can open these doors, they can make meaningful assumptions in mathematizing, which will then lead to successful modelling.

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