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Taxonomy of modelling tasks

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When teaching mathematical modelling it becomes essential to be able to construct modelling tasks of a similar difficulty for projects and exams. In order to be able to compare these tasks, an evaluation scheme concerning the difficulty becomes necessary. In this paper, we introduce an extension of a model developed by Bock, W., Bracke, M., Götz, T. & Siller, H.-S. (2014) which is based on a model from Eyerer & Krause (2012) for comparing difficulties of projects between industry and school. This is used to evaluate the difficulty of a modelling task. The theoretical model of Bock and colleagues (2014) is made applicable by means of a software tool: Taking into account all relevant dimensions of modelling, a measure of difficulty is calculated based on the normalized covered area in the constructed multi-dimensional model. This allows a comparison of the difficulty of several modelling tasks.

Keywords: Taxonomy, authentic problem, modelling.

THEORETICAL BACKGROUND

Mathematical modelling is part of the educational standards in many different countries; moreover it is named by the German Education minister Conference (KMK, 2012) as one of the general mathematical competencies that need to be taught and learnt. "By mathematical modelling competence we mean being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context" (Blomhøj & Jensen, 2003). A modelling process can be described as a modelling cycle, see, for example, (Blum & Leiß, 2007) or (Kaiser, 1995); compare also (Ackoff, Arnoff, & Churchman, 1957) for similar cycles in Operations Research. In order to teach mathematical modelling, both concepts as well as good modelling tasks need to be developed. Following a definition of Blomhøj & Kjeldsen (2006), a good modelling task must fulfill the following seven criteria. A good modelling task should

- ... be understandable and reasonable,
- ... give an appropriate challenge for an independent work,
- ... be authentic and include authentic data,
- ... be open for interesting modelling results,
- ... be open for critics to the model,
- ... lead to representative modelling activities and
- ... challenge the students appropriately to work with concepts and methods that are relevant for their mathematical learning.

In most cases mathematical modelling in school, at university or in industry is group work. To be able to compare student results regarding different modelling tasks during class or in exams, an evaluation scheme for their level of difficulty becomes necessary.

Cohors-Fresenborg, Sjuts, and Sommer (2004) have developed a model to determine the level of difficulty of PISA-tasks. The focus of their model lay on the cognitive processes necessary when solving the tasks. The four criteria *linguistic complexity*, *cognitive complexity*, *formalization of knowledge* and the *handling of formulas* were defined to be the criteria affecting the difficulty of tasks. Each criterion was divided into three levels of difficulty 0, 1 and 2. The complexity of a task was then defined to be the sum over those levels achieved in the four criteria.

The difficulty of modelling tasks is much harder to determine as many different dimensions play a decisive role. Reit (2014) has developed a model determining the difficulty of modelling tasks based on thought structures of different solution approaches. The model is based on the assumption that different

mathematical models and solutions to the same task require different knowledge and mental activities. Based on cognitive load theory, the approach acts on the assumption that parallel thought structures are more difficult than sequential thoughts. The level of difficulty is then described as the sum of the factorial of the single levels.

The problem of the above model is that student solutions have to be available a priori. In many problem settings this is not the case. Especially for authentic or real world problems (see, e.g., Bock & Bracke, 2013) possible student solutions are hardly predictable. These problems are classified by a client structure and are of full generality.

Definition: An authentic problem is a problem posed by a client, who wants to obtain a solution, which is applicable in the issues of the client. The problem is not filtered or reduced and has the full generality without any manipulations, i.e. it is posed as it is seen. A real-world or realistic problem, is an authentic problem, which involves ingredients, which can be accessed by the students in real life.

With these problem settings another aspect makes a taxonomy of the tasks hard. The problem can be posed to students from primary school as well as to students from university, since every learning group is using their individual methods. This makes a general classification dependent on competences and the respective pre-knowledge. Since general industrial projects for mathematical modelling are of a high dimensionality concerning the use of competencies, Eyerer and Krause (2012) developed the spider web method to illustrate difficulty of tasks in the TheoPrax method. TheoPrax is a method which focusses on project teaching with real industrial projects. Here, industrial part-

ners give tasks to students who then have to write an offer to the industry to obtain the job. The project is then worked out by the students. The industrial partner is obliged to cover the costs and also to finance workshops for the students if the offer of the project is acceptable for them. The financing part also has to be planned by the students.

The idea of the Eyerer model is to use a spider web diagram, comparable to Figure 1, to grade a project. This model was adapted by Bock, Bracke, Götz, & Siller (2014) to measure how teachers rate the difficulty of certain modelling tasks. For this purpose eight dimensions and corresponding levels of complexity for each of the dimensions were chosen. Using this system teachers and supervisors rated several modelling problems and their ratings were illustrated by spider web diagrams. If we want to compare the difficulty of two modelling tasks we have to compare two spider web diagrams: Let us assume that we get diagrams A and B as in Figure 1 (the diagrams are for the modified nine-dimensional spider webs introduced later in this paper).

We would now like to compare the two tasks A and B regarding their difficulty – of course relative to our special situation (time frame, learning group, ...) and needs. This seems to be nontrivial because of the multidimensional nature of the data and different weights we may have for each of the dimensions. In the following paragraph we first extend the model developed by Bock and colleagues (2014) by one dimension which is relevant when dealing with authentic modelling tasks. In order to compare the difficulty of modelling tasks we propose a *measure of difficulty* which can be easily calculated using a software tool. Moreover, the rating of modelling tasks as well as the computation of the

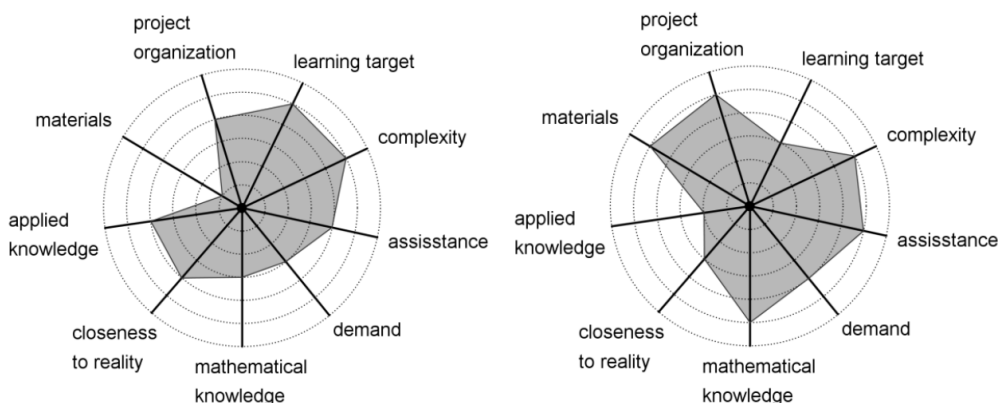


Figure 1: Comparison of two star diagrams for modelling tasks A and B

new measure are made easily applicable by a newly implemented tool.

THE MODEL

The aim of the following model is to describe and compare the difficulty of modelling tasks from the viewpoint of one individual teacher or supervisor. Note that the measure of difficulty for different teachers using the model to evaluate the difficulty may vary. This is due to the different background and experience teachers have. Also with growing experience and competencies the value may vary in time. However on a small time scale, in the opinion of the authors, the score value is stable.

Assume that the difficulty of a modelling task depends on many different dimensions. These dimensions also could be adapted, extended or reduced according to which aspects the score will focus on. *To be able to apply the model it is essential that the person using it can account for practical experiences with mathematical modelling tasks.* This is at first necessary to be able to estimate the different scales of the dimensions and secondly to obtain an intuition for their interplay. In a small pilot study a group of teachers participating in a modelling week was asked to rate different modelling tasks using the following model. For some teachers this turned out to be quiet difficult as they were missing some essential knowledge and experience concerning modelling.

Bock, Bracke, Götz, and Siller (2014) identified the following eight dimensions affecting the complexity of modelling tasks:

- 1) Project organization
- 2) Learning target
- 3) Complexity
- 4) Assistance
- 5) Demand
- 6) Mathematical knowledge
- 7) Closeness to reality
- 8) Applied knowledge

Each of these dimensions was divided into six levels of complexity, where 1 describes the easiest and 6 the most difficult level. For each dimension the possible answers are categorized, where the numbers are assigned to the respective competence levels. An example is given for the dimension *complexity*:

- 1) solution approach is clear
- 2) one-sided methods (e.g. only programming, geometry...)
- 3) alternative solution approaches possible
- 4) data set is too big or insufficient
- 5) solution requires variety of methods
- 6) alternative solution approaches in combination with many methods necessary

All eight dimensions, each consisting of six complexity levels, are illustrated in a diagram in the shape of a star (compare Figure 2). Each dimension is pictured as a ray emitted from the centre of the star. The length of each ray is divided into six equal parts. The first ring defines the easiest complexity level 1, the most complex level 6 is reached at the end of each ray.

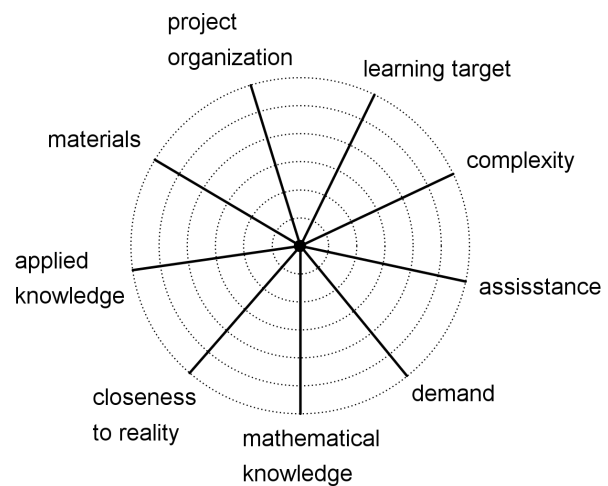


Figure 2: Extended model: nine-dimensional star

Considering a given modelling task, its complexity is rated for each dimension separately and marked on the correspondent ray. The marks of all rays can then be connected and the enclosed area calculated (compare Figure 3). The size of the enclosed area symbolizes the difficulty of the task and therefore gives a possibility to compare the difficulty of modelling tasks. The larger the area, the more difficult the task. Note that the diagram shows also the directional weights of the individual dimensions, which can be used to test the tasks according to the pre-knowledge and competencies of the students.

Due to the authors experiences the model has been developed further by incorporating the dimension *materials* to the model with the following score levels:

- 1) Computer/Laptops with internet connection are available for research the whole time

- 2) Computer/Laptops with internet connection are available for research temporarily only
- 3) A subject-specific library is available for research
- 4) Selected books and journals are available for research
- 5) Some information selected by the teacher is available for research
- 6) There is no possibility for research

This dimension takes into account that the complexity of a modelling task does also depend on the amount of research that is possible during the process of finding a solution. An extra ray for the dimension *materials* was added to the model (see Figure 2). The model was then implemented in a new software tool such that ratings of modelling tasks can easily be evaluated and compared. Of course, rating a modelling task according to the named dimensions always depends on the specific target group and the individual project settings.

With the help of the implemented tool the area which is formed by connecting the neighboring score levels is computed and normalized by the maximal area, i.e., if all dimensions have maximal score. Thus the output can be interpreted as the percentage of the task compared to a task of maximal difficulty and delivers a number between 0 and 1 indicating the difficulty of the investigated task. We will call this number the *measure of difficulty (MOD)*. The closer MOD is to 1, the more difficult the task.

Definition: (i) Let M be the area of the convex hull spanned by the complexity ranking of 6 in each dimension. Let I be the measured area of the enclosed area of a rated modelling task. The *measure of difficulty (MOD)* of the modelling task is then equal to: $MOD = I/M$ and takes values in the interval $(0,1]$.

(ii) Two modelling tasks $T1$ and $T2$ are said to be of equal complexity of fineness e if

$$|MOD(T1)-MOD(T2)| < e.$$

A difficulty that arises when calculating the MOD is the fact, that the area differs for the same rating with different arrangements in the sequence of the dimensions. This problem was solved by calculating not only the area but the mean value of the areas over all permutations of arrangements.

EXAMPLE

As an example consider the following setting: A group of students from 11th grade of a German secondary school is supposed to work in small groups on the authentic *Airline Problem*. During the time of the modelling activity (4 h) the students have access to computers and internet.

Airline Problem: The time a plane is on the ground is time in which the airline is making no money! Therefore the airline is interested in a system for the boarding of a plane such that the time the plane is on the ground is minimized.

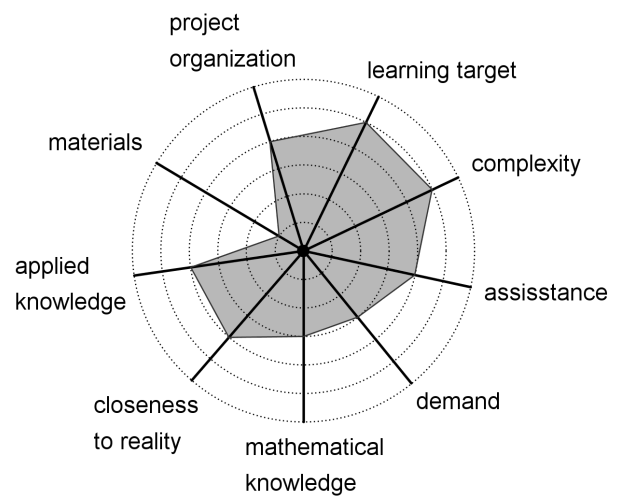


Figure 3: Example: Rating a modelling task to calculate the MOD

The authors would rate this problem according to the model as in Figure 3. This rating was developed by taking the average value of the individual ratings of the authors. The precise categories apart from the material dimension can be found in Bock and colleagues (2014). For the *Airline Problem* the MOD is calculated to be 0.3688. This rating can be retraced in the following way.

The dimension *materials* is rated depending on the individual setting set for the project work planned. In our example, level 1 “computers/laptops with internet connection are available for research the whole time” describes the situation planned by the teacher.

Project organization is rated with complexity level 4 “with great difficulties, risk to fail is controllable.” This is justified by the fact, that the *Airline Problem* has a very open formulation which leaves several decisions and estimations to the students. Still, these difficulties

can be overcome and a solution can be obtained by a simple simulation with chairs and a stopwatch.

The *learning target* is described in level 5, “*new combinations of different techniques*”, which is set as the desired outcome while working on the problem. Hence, the dimension *complexity* is rated with 5, “*solution requires variety of methods*”.

The *assistance* given in the above setting can be described within the meaning of level 4 “*teacher or external tutor supporting in wide steps*”. The rating of this dimension is depended on the individual support teachers are planning to give their students and can be varied for each realization of a modelling project.

The *demand* on the Airline Problem can be rated by level 3, saying that the “*demand (is) alternately increasing*”. This marks a medium level of difficulty to the task. The *mathematical knowledge* in this example is based on the “*recognizing (of) missing knowledge in detail*” (level 3) while the *applied knowledge* requires the “*researching and arranging (of) missing information and correlations*” (level 4).

Finally, the Airline Problem has a “*high correspondence to reality*” which leads to complexity level 4 in dimension *closeness to reality*.

POSSIBLE EXTENSIONS

If one wants to lay more focus to certain dimensions, the model can be extended to a weighted model. This can be dependent on the background and formulation of an individual task. Depending on the situation, some of the nine dimensions carry a greater weight than others and therefore influence the complexity of a modelling task more than others.

For example, in the described setting of the *Airline Problem* computers and internet are accessible but play a minor role for the finding of a good solution. In practice, the authors noted that students for example simulated themselves the time to sit down on a seat while blocked with the help of chairs in the room. With a very limited time horizon there is no gain of having access to computers or internet. Therefore, the dimension of material can be weighted less than the other dimensions in this case.

Of course, also the model of maximal difficulty has to be adapted. This can be implemented by modifying the length of the individual rays. A dimension which is considered to be less important in its effect on the difficulty of a modelling task is assigned a shorter ray than more important dimensions. For this, an exact assignment of weights and the proportional change in the length of the corresponding rays still needs to be formulated.

Possible modifications are also the adding and deleting of certain dimensions from the diagram. But this has to be done carefully since too few dimensions are not reflecting the whole difficulty of the modelling task while also too many dimensions make the tool inconvenient and unclear.

SUMMARY AND OUTLOOK

In this paper, we presented the extension of a model developed by Bock, Bracke, Götz, & Siller (2014) based on (Eyerer & Krause, 2012) evaluating the difficulty of modelling tasks. The extended model considers nine dimensions with six complexity levels each affecting the difficulty and is illustrated as a star with nine rays (see Figure 2). To calculate the newly defined measure of difficulty (MOD) of a modelling task, the task is rated with respect to its complexity level in each of the nine dimensions. The ratings are marked in the nine-dimensional star and connected to calculate the enclosed surface area which leads to the definition of the MOD (see Figure 3). A new tool was implemented such that ratings of various modelling tasks can easily be compared by their value of MOD. The closer the value of MOD is to 1, the more difficult the modelling task. In further research the validity of the model should be analyzed and an exact ranking for the values of MOD defined. This could be done by comparing the MOD values with student solutions and the correlation between those. Up to now, this model represents a subjective rating. It is therefore necessary to undertake an empirical study to validate the model by comparing theoretical ratings with the feedback given by students working on the respective modelling projects.

The model of a multi-dimensional star with rays of equal or weighted length can also be used to evaluate self-assessments of participating students or to assess the expectations of students regarding modelling or other activities. For this, different dimensions and

items need to be defined. Bock and colleagues (2014) developed a questionnaire to investigate the self-assessment of students. In the context of modelling days at the University of Kaiserslautern, Germany, Kreckler developed item formulations to compare expectations and conclusions of students concerning the project days. A small sample of students was tested and evaluated. The answers for expectations and conclusions were marked on the rays and compared for each student. With this an evaluation regarding the categories *exceeded expectations*, *fulfilled expectations* and *expectations not fulfilled* for the single dimensions was made possible.

Of course, it may not be correct to assume that every dimension is of equal weight. To overcome this, practical empirical studies are planned, in which the weights of the individual dimensions are obtained via a fitting. For this, also a survey for the students is in preparation. The aim is to find a benchmark model for the measure of difficulty.

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