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► **To cite this version:**

Berta Barquero, Marianna Bosch, Avenilde Romo. A study and research path on mathematical modelling for teacher education. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.809-815. hal-01287246

HAL Id: hal-01287246

<https://hal.science/hal-01287246>

Submitted on 12 Mar 2016

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A study and research path on mathematical modelling for teacher education

Berta Barquero¹, Marianna Bosch² and Avenilde Romo³

1 University of Barcelona, Faculty of Education, Barcelona, Spain, bbarquero@ub.edu

2 University Ramon Llull, Barcelona, Spain

3 CICATA-IPN, Ciudad de México, Mexico

Considering the general problem of integrating mathematical modelling into current educational systems, this paper focuses on the ineluctable step of the professional development of teachers. Within the framework of the Anthropological Theory of the Didactic, the use of study and research paths for teacher education (SRP-TE) was recently proposed as a means to combine a constant practical and theoretical questioning of mathematical modelling school activities. After presenting the rationale of our proposal, we will illustrate the phases of the SRP-TE design and some preliminary results with the implementation of an on-line course for in-service secondary school teachers in CICATA-IPN (Mexico).

Keywords: Mathematical modelling, anthropological theory of the didactic, study and research paths, teacher education, on-line course.

INTRODUCTION

There exists an extended agreement about the necessity to foster the teaching of mathematics as a modelling tool and to enrich the study of contents at school through the development of inquiry processes. At the same time, many investigations highlight important objective difficulties that hinder any proposal of implementing modelling and inquiry as normalised activities in current educational systems (Burkhardt, 2008; Kaiser & Maaß, 2007; Doerr, 2007; among others). Many of these constraints are related to what has recently been called the 'monumentalistic' paradigm (Chevallard, 2012), which rules in many of our teaching systems, where mathematical contents tend to appear as 'works to visit' more than tools to provide answers to questions. To approach this far-reaching problem and help move towards the new paradigm of 'questioning the world', recent research carried out in

the framework of the Anthropological Theory of the Didactic (ATD) proposes a new teaching device called *study and research paths* (SRP) based on the long-term inquiry of generating problematic questions (Barquero et al., 2008, 2013). However, designing and locally implementing new devices is not enough to ensure their long-term viability. Among many challenges, an important one is related to teachers' professional knowledge and competences, and furthermore to the mathematical and didactic infrastructures that need to be at their disposal to face this change.

The research project we are presenting starts from some particular cases of SRP that have been designed, locally implemented and analysed in previous research at preschool, primary, secondary and tertiary educational levels. Our purpose is to explore how these SRP could be used in professional development programmes for teachers. The aim is twofold: on the one hand, to identify teachers' professional problems in their day-to-day activities, and establish possible ways of approaching them; on the other hand, to enrich teachers' mathematical and didactic experiences with inquiry and modelling processes which will later be used as a source to introduce the didactic tools for the analysis and the questioning of any kind of teaching and learning processes. In this paper we will present the framework, principles and phases for the design of what we call SRP for Teachers Education (SRP-TE), together with some preliminary results of one of its first implementations.

STUDY AND RESEARCH PATHS FOR TEACHER EDUCATION

Previous research on the ATD concerning the problem of teacher education (Cirade, 2006) has made the following contributions (Bosch & Gascón, 2009). First of

all, it is assumed that teacher education programmes should consider the difficulties and quandaries affecting the development of the professional activities of teachers and locate them at the core of the educational proposals. It is clear that the different tasks teachers should carry out are more or less specific to the content to be taught. However, an empirical study on more than 7000 questions posed by in-service teacher-students (Cirade, 2006) shows that many of these questions have an essential mathematical dimension. In other words, many of the problems teachers face are related to mathematics and, particularly, to the didactic transposition process and the specific mathematical knowledge to be taught (Chevallard, 1985). This issue has a clear connection with research on Pedagogical Mathematical Content (Ball & Bass, 2000), which we will not take into consideration here.

Secondly, starting from teachers' professional problems may help introduce didactic knowledge as a tool to approach them in a motivated way. Didactics of mathematics thus appears as a tool to solve problems instead of a set of (more or less dogmatic) theoretical developments to be known. At the same time, didactic knowledge should also appear as a tool to pose new problems and particularly to question the prevailing teaching proposals, including the curriculum and the pedagogical organisations.

Last but not least, it is important to note that many of the problems teachers face are still open problems for the mathematics education research community. Said problems need an important process of reformulation before they can be approached from a founded perspective. In fact, the more general question of what kind of knowledge has to be made available to teachers and how to help teachers develop it still is (and will always, to a certain extent, remain) an open question.

In order to take these considerations in teachers' development programmes into account, we propose to implement what we call 'study and research paths for teachers' education' (SRP-TE) as a way to provide teachers with pertinent (theoretical and practical) tools to nourish and sustain their professional development. They consist of the following five stages:

- 1) The starting point is an open question that comes from the teaching profession itself and is related to a given piece of knowledge to be taught: how to teach proportionality, algebra, integers, lin-

ear regression, etc.?' This question is initially approached searching information and documentation available, including results from research, official curriculum guidelines and innovation proposals.

- 2) The second stage consists in presenting a *study and research path* (SRP) similar to what could exist in an ordinary classroom and is related to the professional question approached. The SRP can actually have been implemented in previous investigations or may simply have been designed by researchers for this purpose. Teacher-students have to follow the SRP as if they were students, under the supervision of educators.
- 3) The third stage is devoted to the analysis of the teaching process just followed. Three main phases are distinguished: (a) the *mathematical analysis* of the work done, including the elaboration of a reference epistemological model describing the modelling process involved (Bosch & Gascón, 2006); (b) a didactic analysis of the process, including a description of the differences between the contract established during the SRP to manage the modelling process, compared to the usual school didactic contract centred on the transmission of contents; (c) a more general study of the viability of SRP, including the identification of the institutional conditions and constraints affecting the development of modelling practices in school settings.
- 4) The fourth stage consists in *designing a SRP* based on the one previously followed and analysed, adapted to a given group of students. This design should be based on the analyses of the previous stage: sequence of mathematical questions to be posed to the students; sharing of responsibilities between teacher and students to pursue the questions; teaching devices to ensure the viability of the SRP.
- 5) The final stage of the SRP-TE, if possible, corresponds to the *implementation and a posteriori analysis* of the SRP designed. The same didactic tools made available at stages 3 and 4 are again supposed to play an important role: not only to provide some provisional answers to the question that was at the origin of the whole process ('How to teach ...?'), but also as a means to anal-

use other possible alternative answers, as those found in stage 1.

The hypothesis of our research is that SRP-TE may contribute to the considerations previously presented in the following sense:

- *A tool to question mathematical contents to be taught.* The carrying out of a SRP (stage 2) provides a specific form of epistemological analysis of the content at stake (what we call a *reference epistemological model*) that helps approach the problematic mathematical dimension of the problem: what ‘modelling’ is, how the inquiry process can be described in terms of sequences of questions approached instead of contents used, how this sequence provides possible rationales to the contents at stake, etc. This reference epistemological model is a crucial tool to get rid of the transparency of school mathematical contents and to start questioning it.
- *Release teachers from the usual way of doing and teaching mathematics at school.* The mathematical activity developed in stage 2 is clearly different from (even if partially compatible with) current school activities and does not assume all the constraints of traditional teaching. This raises new questions during stages 3 and 4 to describe the mathematical and didactic activities carried out and to adapt them to real school settings. SRP-TE thus appears to be a good tool for detecting the institutional constraints hindering inquiry and modelling activities at school.
- *A fair contract between teachers and teacher educators.* Since in a SRP the teacher assumes the role of supervisor, there is no problem if the professional question taken as the starting point of the SRP-TE is an open question in research: teacher educators are not supposed to provide definitive answers (which do not exist) but help student-teachers approach the question by critically accessing the materials available.

Our research project wishes to explore to what extent these hypotheses can be confirmed and what changes or adaptations are suggested, using different implementations of SRP-TE as empirical basis. We are here presenting a single case of a SRP-TE for in-service secondary school mathematical teachers.

A SRP-TE ON SALES FORECASTING FOR IN-SERVICE TEACHERS

In Autumn 2013, a SRP-TE was experimented in an on-line course for in-service secondary school teachers coordinated by the CICATA-IPN centre (Legaria, Mexico) as part of a postgraduate programme in Mathematics Education. The course was led by a team of six teachers, three from CICATA-IPN and three from Spain, all of them researchers in mathematics education. The authors of this paper were all part of the team. In this case, the SRP-TE took the problem of teaching mathematical modelling at secondary school as the initial question. It was initially formulated as follows:

- Q₀: How to analyse, adapt, develop and integrate a learning process related to mathematical modelling in our teaching practice? How to institutionally sustain a long-term learning processes based on modelling? What difficulties should be overcome? What teaching tools are needed? What new questions arise?

For four weeks, these issues were approached through a SRP on sales forecasting, considering four activities corresponding to the last four SRP-TE stages introduced in the previous section. There were 15 participants, all of them in-service secondary school teachers. They were supposed to spend 80 hours on the SRP-TE for five weeks: one week for each activity and one week for the final report.

The SRP on sales forecasts that was at the basis of the SRP-TE had previously been designed and implemented at university level and also at upper-secondary level (Serrano et al., 2010). In other words, we took an already experimented SRP, with a previous mathematical and didactic a priori design and some material concerning its implementation and a posteriori analysis. Students were informed of it and were invited to review some published works in the third phase of the SRP-TE. More concretely, the first activity (*Activity 1*) proposed the *Resolution and analysis of ‘Forecast sales of Desigual’* with the main aim of letting participants experiment a SRP similar to the one experimented. Participants were asked to ‘live’ it like mathematical learners or apprentices. They had to act like a team of mathematical consultants and had to provide an answer to a request from Desigual (a Spanish fash-

ion brand), which wanted to have an in-depth study on ‘how to predict the evolution of several variables (see Figure 1): weekly sales in several of their shops, evolution of their benefits or of new national and international shop openings, etc.’

Participants were organised in five teams of three consultants each, combining individual work with group work (using the on-line forums of the CICATA virtual campus and Skype). They first had to act individually and propose their own answer to the question (phase 1). They later had to share and contrast their proposals with their partners (phase 2). Finally, in phase 3, they were asked to prepare and present a final report together, providing some answers to Desigual’s request and defending it as the best proposal for the project. The final answer had to be accompanied by an analysis of the process followed by the team, including the difficulties encountered.

In *Activity 2*, the participants were asked to prepare a ‘lesson plan’ based on the mathematical work previously carried out in *Activity 1*. The situation proposed was that they were supposed secondary school teachers that had planned to implement the activity of ‘Forecasting Desigual sales’ in their classroom. Due to a cultural trip with other students, they had to ask another teacher to replace them. They were asked to write a brief and easy to read lesson plan including all the necessary elements for the substitute teacher to carry out the lesson/s. Like in the previous activity, participants first had to prepare and individual proposal, then share their proposal with the rest of their team and agree on a final common lesson plan. This activity was supposed to provide a first spontaneous answer of the teacher to the question: ‘How to teach a modelling activity based on Activity 1?’ in terms of a teaching proposal designed.

Activity 3 consisted in the experimentation of the participants’ own design of the activity with a group

of students. The participants had to individually assume the role of the teacher and implement the initial phases of the lesson plan proposed in *Activity 2*. With this purpose in mind, they had to elaborate a more detailed design, a more in-depth a priori analysis (phase 1), experiment their proposal (phase 2), finish with the a posteriori analysis (phase 3) and prepare a brief ‘experimentation report’ (phase 4).

Finally, *Activity 4* was devoted to a joint analysis and final revision of the lesson plan with the aim of proposing a new version taking into account both their own experience and the experience of their teammates. In particular, the difficulties found in the implementation of the modelling activity (a posteriori analysis) were supposed to highlight the constraints related to the normal implementation of this kind of teaching proposals and the possible ways to overcome them.

The supervision of the teacher educators during the SRP-TE consisted of the following. By way of feedback to the team discussions in the forum and to the activities (reports, lesson plans, etc.) submitted, the course staff progressively introduced some didactic tools to support the mathematical analysis of activity 1: notions of model and system, criteria and ways to characterise the models provided, ways of comparing them, etc. At the end of activity 2, as a means to carry out the didactic analysis of the spontaneous teaching proposals, some publications about SRP were provided: Serrano and colleagues (2010) and Chevillard (2012). Between activities 3 and 4, the educators prepared a guideline with the main sections of the a posteriori analysis of a SRP, including some examples of its mathematical description, some criteria to describe the didactic organisation and some elements of the conditions produced, the constraints faced and the global evaluation of the teaching process. They also provided an assessment grid for the final report and a questionnaire about the development of the course to be answered individually at the very end of the

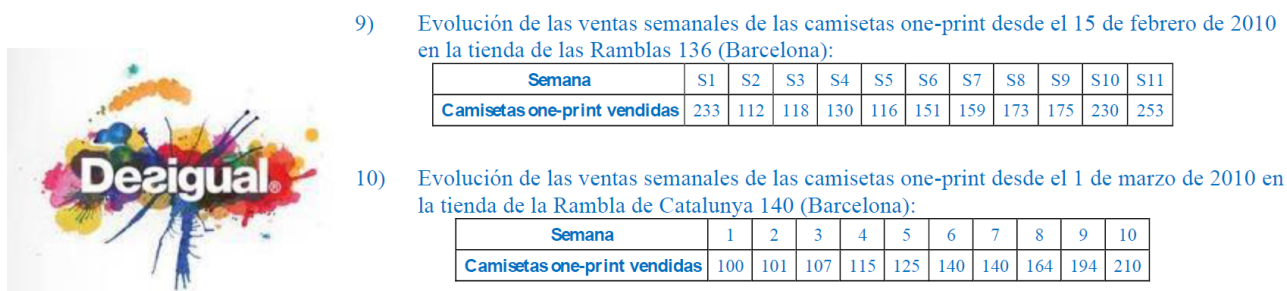


Figure 1: Initial worksheet of Desigual’s request to the consultants

course. All the material produced by the students was gathered during the course, especially the students' discussions in the forums (including the teacher educators' interventions), the students' questions raised (in the forums or by mail), the partial and final reports and their answers to the questionnaire.

THE 'LESSON PLAN' AS A CRUCIAL TOOL

Given the fact that this was the first on-line course based on SRP-TE, the results found are mainly related to the organisation of the course, the weaknesses encountered and the possible way to overcome them in further implementations. We will only present those concerning the function of the 'lesson plan', which appears to be a central element of the SRP-TE.

According to the design of this SRP-TE, activities 1, 2 and 3 were mainly based on the teachers' mathematical and professional knowledge. The function of the lesson plan is to provide shared teaching materials to support the analysis in a triple dimension: (1) as a description of the initial modelling-based activity on forecasting sales; (2) as a teaching proposal spontaneously designed by the teachers according to their professional knowledge and adapted to the usual institutional school conditions; (3) as the support of a real (partial) teaching and learning process. It is mainly in activity 4 where new types of didactic knowledge are needed to provide a critical analysis of the mathematical and teaching processes followed. In this section, we present how the lesson plan was used in the SRP-TE, its productivity and limitations.

In spite of some initial difficulties, the teacher-students quite easily dealt with activity 1 and experienced a specific mathematical 'unguided' work based on modelling and the inquiry of an open question close to the paradigm of questioning the world. In activity 2 (lesson plan), many teachers fell back on the usual didactic contract based on the learning of contents (as opposed to the study of open questions) and searched

a school mathematical subject related to sales forecasting (such as 'linear regression' or 'function graphs') to teach it. They then prepared a list of mathematical techniques needed to answer the question previously provided to the students. For example in the lesson plan of teacher A, the teaching proposal was based on the presentation of different time-series forecast techniques, such as Gompertz (S-shape) curves.

In this lesson plan, the teacher is supposed to play the traditional role of teaching a repertoire of mathematical techniques the students should learn before proposing a forecast, as if the students could do nothing without it. However, other teachers respected the open character of the study process. For example, in the lesson plan proposed by teacher B, the students were asked to find an answer to the forecast question without any previously established strategy:

In the next classes you will be employed by the company to make a short and long term forecast for each of the variables of the file 'Problem of Sales Desigual.pdf'.

Two tasks were proposed to guide the students through this new kind of work:

Activity: Read the information provided by Desigual:

- 1) Which are the variables on which Desigual provided information? What types of variables are they? How does variation can be described for each of these variables?
- 2) Search for the following information: What articles do Desigual shops sell? Where in Spain are Desigual shops located? And outside Spain?

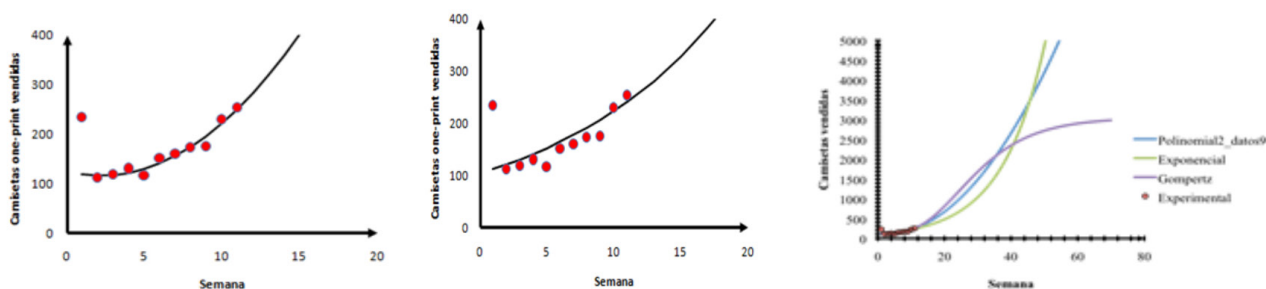


Figure 2: Mathematical models proposed by teacher A

Teachers A and B worked in the same team when preparing the first version of the joint lesson plan. In this case, the team ended up with a proposal similar to the proposal of teacher B (paradigm of questioning the world), but it could also have been the other way round. In any case, the reasons put forward and the discussions carried out in the forum appeared to be highly interesting material for the teachers' initial professional knowledge to be changed (especially the assumptions, reasons and criteria used to support their decisions). This knowledge was related to the school institutional constraints and was enriched with new didactic tools that enabled it to evolve. The a priori elements, progressively made accessible by the educators, consider the lesson plans as an initial empirical basis that was enriched during the teaching implementation in activity 3.

All this work eventually turned into a revision of the initial lesson plan now including the results of the experimentation. At this point, the supervisors proposed a guideline to organise the didactic tools provided in relation with the three previous activities. The mathematical analysis corresponded to activity 1. At this stage, the epistemological elements used to describe the modelling activity were completed with some examples of SRP descriptions in terms of sequences of questions and answers (see Figure 3).

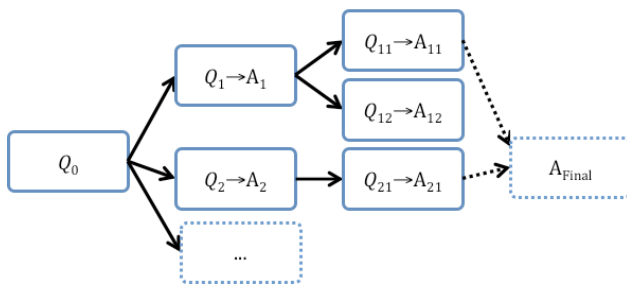


Figure 3: Sequence of questions and answers to describe a study and research path

This description starts with an open question (Q_0) and leads to different ways to formulate new sub-questions (Q_i) and obtain partial answers (A_i) until arriving at an acceptable final answer (A_{Final}) (also partial but provisionally considered as definitive). For instance, the team of teachers A and B proposed a description starting with the following sequence (the questions in italics being added by the authors):

Q_1 : How to forecast the sales of the company, given some time-series real data?

- A_1 : Fit different functions to the real data, choose the best function and evaluate it in the future periods considered using appropriate software.
- $Q_{1.1}$: What software to use: Excel, Geogebra, R?
- $A_{1.1}$: Geogebra and Excel, which are the ones the participants know well.
- $Q_{1.1.1}$: Do both tools provide the same results?
- $A_{1.1.1}$: In the first calculations, the results obtained were different.
- $Q_{1.1.2}$: How to explain the differences? [unapproached question]
- $Q_{1.2}$: Which theoretical tools can be used? Where can they be found?
- $A_{1.2}$: Document "Time-series. Least squares fit" provided by a participant.
- $Q_{1.2.1}$: Should we use this sophisticated material to forecast the sales of an item that has increased from 100 to 500 units in ten weeks' time?
- $A_{1.3}$: Elementary functions, 'trend line' option using Excel, 'fit line' using Geogebra.
- $Q_{1.3.1}$: Which model fits the data best?
- $A_{1.3.1}$: The best model is the one with the fewest errors.
- $Q_{1.3.1.1}$: What types of errors are there? Do they lead to the same results?
- $A_{1.3.2}$: The best model is the one with the highest R^2 .
- $Q_{1.3.2.1}$: What is R^2 and how is it related to the errors?

This schema of questions and answers, called the 'mathematical skeleton' by the educators, help the participants describe the elements of the teaching proposal designed, both those which effectively appeared in the process implemented and those left out. Based on this epistemological analysis, the guideline elaborated by the educators proposed certain elements to carry out a didactic analysis of the teaching and learning process (activities 2 and 3) experimented, using the notion of didactic organisation and focusing on the sharing of responsibilities between teacher and students during the development of the modelling process. We will not describe the SRP-TE any further. To this short description, we will simply add that the documents given to the participants as complementary reading were of crucial importance.

CONCLUSIONS

To sum up, let us stress what we consider at this very initial moment of our research, to be the main contributions of the theoretical framework used, ATD. An important characteristic of a SRP-TE is to locate the questioning of the mathematics content to be taught (here, a modelling activity) and of the traditional didactic organisations prevailing in our current schools at the heart of the teachers education programme. In the case here presented, the first stage of the SRP-TE (searching information and documentation available) was not developed. This clearly appears as a weakness, since it would most certainly give rise to interesting discussions about different ways of interpreting modelling as a school content, the ambiguity of the official guidelines regarding this matter and the variety of proposals existing in different countries and even within the same country. Another characteristic of a SRP-TE is to nourish the questioning of the mathematical content to be taught through the ‘in vivo’ performance of a mathematical activity based on a previously designed SRP. The role-playing technique used was a good choice and it worked well, in spite of some logical difficulties at the beginning for the participants to enter the new contract: they all initially hesitated between acting as secondary school students or as real mathematicians, but ended up playing their role. Of course, another main contribution of the ATD is the kind of methodological tools provided by the educators to help participants describe the content to be taught and the didactic organisation of the teaching and learning process, an aspect of our research that has just been outlined here, especially with respect to what concerns the description of the knowledge to be taught, the new responsibilities assumed by both teachers and students, and the institutional constraints found.

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