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► **To cite this version:**

Jasmina Milinković. Intuition about concept of chance in elementary school children. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.722-726. hal-01287108

HAL Id: hal-01287108

<https://hal.science/hal-01287108>

Submitted on 11 Mar 2016

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Intuition about concept of chance in elementary school children

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The paper deals with initial development of stochastic reasoning in children. After short overview of the theories of development of stochastic reasoning, we focus our attention on intuition about probability. In the second part of the paper we report results of an empirical research on intuition about the concept of chance across grades 4th to 7th. The analysis of findings provides evidence of preconceptions about probability of undetermined events. These preconceptions are in conflict with formal theory but support Siegler's rule based model. We argue that we should take into account pupils' intuition about the concept of chance as well as other prerequisite mathematics concepts (fractions, proportions, etc.).

Keywords: Chance, intuition, Siegler's rules, stochastic reasoning, misconceptions.

INTRODUCTION

Probability is an area of mathematics contrary to math logic, paradoxical and counterintuitive (Kapadia & Borovcnik, 1991). How do we gain our initial understanding of events with uncertain result? How do we transform our initial conception of chance? Can we expect development of concept of chance despite of lacking opportunities to learn? We endeavoured to address these questions in order to find when the right moment is to introduce these topics in school curriculum. The early years of schooling in Serbian school system are considered as preliminary for empirical introduction of fundamental (and essentially abstract) concepts in mathematics such as relations, functions, equivalence, probability, and the like. The decision when we start to deal with a new class of concepts is associated with the decision on how we want to deal with them.

Our knowledge begins with experience. Experience provides basis for intuitive knowledge. Generally, in-

tuition is expected to support (formal) learning. Kant notices that "... even our empirical knowledge is made up of what we receive through impressions and what our faculty of knowledge...supplies for itself (1929, pp. 41–42). Fishbein (1975) defines intuition as an autonomous cognitive activity based on individual experience. Intuition is general and immediate, which allows direct impact on the reasoning about situation. We can speak about two natures of intuitive reasoning in mathematics learning: the intuitive acceptability and intuitive anticipation. Intuition eligibility gives a sense of the sufficiency of the evidence of an assertion or obvious truth of certain facts.

The main aspects of intuitive knowledge about the probability of events are belief in the regularity of occurrence in the environment and (consequently) their predictability. Intuition is developed naturally as a consequence of experiencing the stochastic nature of the environment. Fischbein (1975) points out that humans need to rationalize and to predict leads us to search for regularities in phenomena. He noted that acceptance of intuition as a subjective conviction carries the conviction of self-proved (obviousness) and inner coherency. Chernoff (2008) discusses the dual classification of probabilities "belief-type" and "frequency-type". We may say that probability estimates are an expression of intuition about the relative frequency. In the case of intuition about the frequency of events, people tend to make judgments on the basis of time-limited past experience, which they estimate to be adequate.

Key theories about development of stochastic reasoning give us clues about appropriate time to start dealing with the concept of uncertainty. To begin with, Piaget and Inhelder (1951) claimed that at the operational level child finally becomes able to detect causal relationships that lead to the differentiation of unpredictable events. They recognize two sources of

reasoning about probability: previous experience and mathematical constructions. Fischbein, on the other hand, believed that without formal education children cannot reach the operating concept of probability. He believed that intuition about probability is formed by the age of 14 or 15. Fischbein and his associates' study (Fischbein, Pampa, & Manzato, 1970) indicated a developmental leap of sixth grade pupils, but also the possibility of achieving a similar level of skills in younger trained pupils. Finally, Shaughnessy (1992) identifies four levels of conceptual development of stochastic reasoning linked to levels of formal mathematics education: (1) non-statistical level, (2) naive statistical level, (3) "on the horizon" statistical reasoning, and (4) pragmatic level. On the first level, reasoning is based on beliefs, deterministic understanding of phenomena, on causal inference or generalizations based on one case. The second level is characterized by conclusions using various heuristics such as representativeness, availability as well as by shifts in thinking about randomness and chance. The next level of reasoning is achieved when person has ability to apply normative models on simple problems, identify differences between reasoning based on intuition and on the basis of a mathematical model as well as knowledge of different mathematical representations of the concept of chance. Finally, the pragmatic level implies deep understanding of the mathematical model, the possibility of comparing different representations, the ability to apply a normative model and knowledge of the limitations of individual stochastic models.

Along the line of Piaget's theory comes Siegler's "rule-assessment approach" to cognitive development. His research encompasses the development of stochastic reasoning. (Yet, unlike Piaget, he does not advocate across domains cognitive developmental levels.) Siegler claims that cognitive development may be described as "acquisition of increasingly powerful rules for solving problems" (Siegler, 1981, p. 3). According to him, children first generate a series of alternative rules based on rational task analysis, previous empirical work and similar activities. Next, a set of problem types yields to patterns of correct answers and errors for children following each of the rules. Finally, if there is a theoretical prediction for a certain type of comparable problems, the asynchrony may be identified and cause changes in reasoning. Scholz (1996, pp. 301–302) reflects on Siegler's study which included variants of card games and drawing an object from the urn. He reports on Siegler's de-

scription of the pathway of cognitive development through process of decision making primarily based on implementation of (new) rules. There were four rules: 1) always choose urn with a larger number of favourable outcomes, 2) if the number of favourable and unfavourable outcomes are the same, elect the urn with fewer unfavourable outcomes, 3) the difference between the number of favourable and unfavourable outcome is calculated for each urn and the one with the greater difference is selected, and 4) the ratio of favourable and unfavourable outcomes is the election criteria (Scholz, 1991, p. 246). The rules were determined in relation to the dominant dimension of a favourable outcome and subordinate dimension of adverse outcomes. The researchers noticed that pre-schoolers only applied the first rule while the children in the lower grades applied the fourth rule and less the third rule while completely ignoring the second rule. Generally, the predictions of the respondents from 3 to 20 years, agreed with the first and fourth rule.

Initial research in the area of development of probabilistic reasoning dealt mainly with intuition, including misconceptions that we have about uncertain events as a result of growing up (not education). Here, we mention the research of Piaget and Inhelder (1951), Tversky and Kahneman (1982), Fischbein (1983), Green (1982), Hawkins and Kapadia (1984), and Nisbet and Williams (2009). Unlike, Kazak and Confrey (2006) for example, who claim that the results of their study conducted among 9 year old children supports the idea that confronted with various tasks with "chance settings" children could develop a quantitative perception of probability. In recent review of the research in probabilistic reasoning Schlottmann and Wilkening conclude that the contemporary research move boundaries for understanding concept of probability for earlier ages prior to instruction but does not provide understanding of the implications of these preconceptions (Schlottmann & Wilkening, 2011). Our paper offers a small contribution in this matter.

METHODOLOGY AND RESEARCH FINDINGS

The study was conducted among pupils in Serbia, where no probability and statistics topics has been a part of the state curriculum for elementary schools. Our research sample consisted of 392 children, Grade 4 to 7 (11 to 14 year old). We have observed lessons in 16 classes in 3 different school. All three schools are

located in the centre of a large city (with about 2 million inhabitants) and a short distance away. Schools were selected based on the principle of similarity of children population, to eliminate factors of education or lifestyle that may have an effect on the intuitive understanding of statistical concepts of interest to us. The fact is that this population may be described as prone to reading the daily newspapers or watching shows on TV (which often use statistical data). The study was performed in regular classes and respected regular composition of classes. The teachers used lesson plans developed by researcher. The researcher made field notes during the lessons. The researcher and the teacher had meetings to discuss what was happening in the classroom.

Students observed and analysed through class discussion different situations that might provoke thinking about the concept of chance. Here, children were prompted to express their beliefs about simple game like situation with undetermined result. In the activity which is in focus of this paper, pupils discussed how likely is to get a red cube out of the boxes containing red and white cubes.

The teachers were expected to probe pupils' intuitive reasoning about chance. Their role were of a moderator. The teachers were not supposed to provide theoretical background, give correct answers or leading clues. They asked pupils either to conduct an experiment and analyse the data or to study a scenario describing results of this experiment.

There were three boxes with different numbers of white and red cubes. In the first one, there was 1 white and 1 red cube. In the second box, there were 9 white and 1 red cubes, and in the third, 2 white and 2 red cubes (Fig 1).

Box	Content
First box	□ ■
Second box	□□□□□□□□ ■
Third box	□□ ■■

Figure 1: Three boxes

We quote and analyse pupils' logic during class discussion. The discussion started with the experiment of pulling a cube from the first box containing 1 red and 1 white cube.

Teacher: What do you think, which colour you are going to get out of the first box?

Milos: I'll get red because that colour is in charge.

Olga: I'll get red because I have to get it.

Angela: Red, it's a beautiful colour.

Goran: I think I'll randomly draw a white.

Jovan: We're sure to a draw some.

Zoran: I think I'll pull the red one, although the chances are fifty-fifty.

Milos, Olga and Angela considered that red was supposed to happen because of their desire to get a red cube. It could be discerned that those children believed that they somehow could affect the result of the pull. We could identify non-statistical level of reasoning among substantial number of children. Zoran and Goran apparently were aware of concepts such as "randomness" and "(equal) chance" but it did not prevent them from having "non-statistical" judgments. Significantly, we could observe confusion between what they believed and what they thought they were supposed to say. Successively, the same question was posed regarding the second and the third boxes. Children's judgments about the second box with 9 white and 1 red cube showed more sophisticated statistical views of children.

Ksenia: White, because there are more whites.

Luka: I'll get white. There are many more (white cubes), and they are more likely.

Obrad: I will pull both colours. Maybe I'll draw some more, but you never know that.

Danko: If we pull a cube for fifty times from the first box we could get, for example, 3 red and 47 white cubes, because anything is possible.

Ksenia and Luke ground their expectations in the principle of "the more favourable outcomes, the better chance." (Siegler's Rule 1). Unlike them, Obrad reasoned that the result of next drawing could not be predicted. Even more, he stretched his conclusion as he stated that "we cannot talk about any predictions what so ever." He was not the only one who believed in that. Danko, had similar thoughts about impossibility to predict results. Pupils who conducted the experiment, by the time of dealing with the third box, have already begun to change their opinion about the predictability of the outcomes on the basis of their experience with two prior cases. Some pupils simply

concluded that they could not predict what they were going to get in the next trial.

Finally, teacher asked pupils to compare the chances to draw a white cube out of different boxes. Most children thought that the chance to get out red cube from the first box is bigger than from the second box (as if using Siegler's rule 2, comparing the number of unfavourable outcomes). But, then came the challenge of comparing chances to pull a red cube from the first box and the third boxes.

- Marina: It's the same thing.
 Mitar: Fifty, fifty. (No explanation what this means).
 Goca: It is the same as 1 to 1 and 2 to 2.
 Janko: Similar.
 Jovan: They are approximate (numbers).
 Milan: It is more likely if there is two plus two cubes.
 Uros: The number of cubes in the box is important.

Note that pupils learned about equivalent fractions and about proportions in the 5th Grade, prior to this study. Goca was a 7th grade pupil (age 14). Similarly to Marina and Mitar, half of 7th grade class agreed with Goca that 1:1 and 2:2 are the same. But, the rest of children have not been convinced that the chances were the same. Milan's and Uros's answers prompted us to continue discussion. Younger pupils, from the 4th grade, were prone to offer incorrect answers. The teacher was provoked by these answers to test pupils' belief.

- Teacher: Suppose you are offered a reward if you pull a red cube. Which box would you like to be given, the first one or the third one?
 Janko: It is easier to draw a red cube from the third box because it had two cubes. And to me it would be easier.
 Igor: I would like (to pull out of) the third one. Chances are bigger in the third. Because, there are twice more.
 Dragana: The chance to win as well to lose are doubled when pulling from the third box.
 Janko: It is easier to pull out a red cube from the first box because there are only two cubes in the box.

Only a small number of pupils thought that it does not matter whether it is drawn from the first box or from the third box. In the process of making judgment, the most common way of reasoning was similar to Igor's response, favouring the box with more favourable outcomes which is in accordance with Siegler's rule 1. But Janko used modified Siegler's rule 2 to pick a case with smaller number of unfavorable outcomes. We should mention that the teacher extended discussion to better understand pupils' perception. Somewhat surprisingly, pupils did not change initially opinion after prolonged discussion. For example, after Igor's response, the teacher inquired delicate questions.

- Teacher: But there are also two white cubes in the third box. Does it matter to you?
 Igor: No.
 Teacher: Ok. If you are promised to get a reward if you pull a white cube, would you like to pull a cube from the first or from the third box?
 Igor: From the third one. There are more whites in it, too.

When comparing the first and third case, most pupils agreed with Igor. Again, they followed up the Siegler's rule 1, comparing the number of favourable outcomes.

CONCLUSION

We have acknowledged in this study the existence of intuition about concept of chance in elementary school children. The episodes we have chosen to present demonstrated that pupils from age 11 to 14, have had formed certain preconceptions (and misconceptions) about chance, prior to any instruction. We remarked that pupil's answers indicated different levels of primary intuition of phenomena with an uncertain outcome. Our study provides evidence that such intuition develops regardless of lacking content matter in formal learning. The preconceptions about chance are in conflict with formal theory but partially support the Siegler's model of intuitive reasoning about chance. Our analysis unveiled that most of the time children displayed non-statistical level of reasoning or tended to rely only on Siegler's rules 1 and 2. Their preconceptions led them toward the simplest and naïve analysis of the "chance situations". The findings allow us to conjecture that grasping other mathematics concepts (such as fractions, proportions, etc.) are considerably important for understanding

probability. The mastery of these concepts could influence children's advancements in formal stochastic reasoning and therefore should happen before learning about stochastic reasoning. The design of our study did not provide conditions for deeper analysis of pupil's preconceptions (as would e.g. one-to-one interview with a particular child). But we believe that pupils' preconceptions should be accounted for when planning initial formal learning of the concept of chance. Our proposal should be examined further.

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