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► **To cite this version:**

Ana Henriques, Hélia Oliveira. Students' informal inference when exploring a statistical investigation. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.685-691. hal-01287073

HAL Id: hal-01287073

<https://hal.science/hal-01287073>

Submitted on 11 Mar 2016

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Students' informal inference when exploring a statistical investigation

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This paper reports on preliminary results of a study aiming to identify informal inference aspects that emerge when grade 8 students explore a statistical investigation using the software TinkerPlots for data handling. Examples from students' work on one task in a sequence, designed to engage students in posing statistical questions about meaningful phenomena, in collecting and representing data and finally in making data-based inferences, illustrate how informal statistical inferences emerge. The results provide suggestions for further research and some educational implications are drawn.

Keywords: Informal statistical inference, statistical reasoning, statistical investigation, tasks, TinkerPlots.

INTRODUCTION

Students' ability to reason about data and to use them effectively and critically for prediction and decision-making is now a priority in statistics education, meeting the call for having statistically literate and informed citizens in our data-driven society (Batanero, Burrill, & Reading, 2011). Informal statistical inference (ISI) is widely recognized as an important foundation for statistical reasoning, given its potential to deepen students' understanding of the purpose and utility of data in meaning making of the real-world, aspect that has been neglected in statistics education (Makar, Bakker, & Ben-Zvi, 2011; McPhee & Makar, 2014). Hence, it is important to rethink the nature of students' statistical school experiences in order to consider how to best develop their informal ideas of inference. Reflecting these concerns, this paper reports on preliminary results of a study that aims to identify informal inference aspects that emerge when grade 8 students explore a statistical investigation using the software *TinkerPlots* (Konold & Miller, 2005) for data handling.

INFORMAL STATISTICAL INFERENCE

Informal statistical inference is described as a reasoned but informal process of "making probabilistic generalizations from (evidenced with) data that extend beyond the data collected" (Makar & Rubin, 2009, p. 83). For these authors, three key principles are fundamental to ISI, wherein the first is particular to the process of inference and the latter two are specific to statistics: (i) generalization (predictions, parameter estimates, conclusions) that extends beyond describing the given data; (ii) the use of data as evidence for generalizations; and (iii) the use of probabilistic language for describing the generalization, including reference to levels of certainty about the drawn conclusions. Moreover, deriving conclusions that apply to a universe beyond the data leads naturally to the need of providing persuasive arguments based on data analysis. Accordingly, Ben-Zvi (2006) argues in favour of an integration of both informal inference and informal argumentation when aiming to develop students' statistical reasoning in rich learning contexts.

References to ISI appear in curriculum documents (e.g., Common Core State Standards Initiative, 2010; Franklin et al., 2005; National Council of Teachers of Mathematics [NCTM], 2000), as well as in recent research in statistics education. The research advocates the development of students' informal ideas of statistical inference from the earliest years of schooling as it is a known area of difficulty for older students when formal ideas are later introduced (Ben-Zvi, Makar, Bakker, & Aridor, 2012; Makar, Bakker, & Ben-Zvi, 2011). However, ISI should not be taught to students as an entity in itself but rather it would be preferable to focus the instruction on reasoning processes that lead to inference (Makar & Rubin, 2009). Thus, one possible approach to foster the emergence of students' inferential practices is to embed those processes in a

data analysis cycle, like the PPDAC (Wild & Pfannkuch, 1999). Presenting statistics as an investigative process to solve real-world problems may be quite motivating for students. They can engage in posing their own statistical questions (hypothesis) about a meaningful phenomenon, designing and employing a plan to collect appropriate data, selecting adequate graphical or numerical methods to analyse the data, and finally drawing data-based conclusions and inferences that relate the results interpretation to the original questions (Franklin et al., 2005; McPhee & Makar, 2014). Due to their nature, statistical investigations often provide a distinctive context for observing students' conceptual ideas about statistical reasoning, namely fundamental processes like variation, transnumeration, evaluating statistical models and integrating contextual and statistical features of the problem (Wild & Pfannkuch, 1999). At the same time, they may involve students (even the younger ones) in fundamental components of informal inference, such as decision making and prediction (Makar & Rubin, 2009; Watson, 2008). For teachers, the use of investigations also provide knowledge that can be used in the design, implementation, and assessment of instruction in statistics and data exploration (Henriques & Oliveira, 2013), since they incorporate domain-specific knowledge of students' statistical reasoning. Accordingly, Pratt and Ainley (2008) argue that inferential reasoning and statistical investigations cannot be separated.

Additionally, the huge development of technology has provided teachers and students with new tools to explore the ISI in rich and meaningful contexts, including through the broader process of statistical investigation (Ben-Zvi et al., 2012). The educational community has been leading a number of studies on the affordances provided by dynamic statistics learning environments, such as the *TinkerPlots* (Konold & Miller, 2005), for making inferential reasoning accessible to (even young) students, with very encouraging results (Ben-Zvi et al., 2012). These studies illustrate how the use of such software, in combination with appropriate curricula and instructional settings introducing ISI, may help students to develop a strong conceptual base on which to build later a more formal teaching of inferential statistics.

RESEARCH QUESTION

While the referred perspectives and studies have begun to shed some light on important aspects of ISI,

research results are still insufficient to guide an effective implementation of instructional settings that support students' informal notions of statistical inference. This paper addresses the following research question which has been used to structure the study and the analysis of the collected data: *What aspects of informal inference do grade 8 students reveal when exploring a statistical investigation using the dynamic statistics software TinkerPlots for data handling?*

THE STUDY

Background

This study arose in the context of a developmental research project (DRP), conducted by the two authors, aiming to provide opportunities for professional development in statistics for basic education mathematics teachers in Portugal. The DRP is part of a larger research project – *Developing statistical literacy: Student learning and teacher education* (Oliveira, Henriques, & Ponte, 2012) and strives to understand of how sequences of instructional activities, based on the use of the software *TinkerPlots*, promote students' statistical reasoning. In Portugal, formal statistical inference is reserved for high school courses and, traditionally, grade 8 (13–14 years old) students are not exposed to ISI methods. The eleven basic education mathematics teachers, who accepted the invitation to participate in the DRP, are effective and innovative teachers, in the sense of including inquiry in their mathematics classes and became interested in learning how to use an instructional approach aligned with the recent curricular trends in school statistics (Franklin et al., 2005; NCTM, 2000).

Participants and methods

Research studies involving teaching experiments are especially powerful because they enable researchers and teachers to trace students' individual and collective development in statistical reasoning during instruction. The study followed a teaching experiment design (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) where a sequence of tasks and classroom implementation conditions are improved through iterative cycles. The results reported here come from the study's first cycle which was carried out during the second term of the school year of 2013/2014 in one class of thirty grade 8 students (20 boys and 10 girls, which are referenced by their first names' initial letter). The class teacher was one of the participants in the above referred DRP. Data collection included

students' written work, screen records of their work with *TinkerPlots* (using *AutoScreenRecorder 3.1 Pro*) and audio and video records across the lessons. A qualitative and interpretative analysis of the data, using Makar and Rubin's framework (Makar & Rubin, 2009), provided insights about the characteristics of ISI that emerged in students' work on a statistical investigation.

Lessons context and tasks

A classroom culture which promotes inferential reasoning requires students addressing problems in an environment that encourages collaborative norms, statistical concepts (formal or informal) and one complex problem context where they face conflicts with their knowledge and beliefs about the world (Makar, Bakker, & Ben-Zvi, 2011). In all lessons, students worked both autonomously in pairs and participated in whole class discussions, in order to create a classroom culture that supported questioning and the use of evidence.

Participant teachers in DRP and the authors jointly selected and adapted or produced tasks and materials for a sequence of lessons aligned with the Statistics Reasoning Learning Environment principles defined by Garfield and Ben-Zvi (2008). Since in Portugal, inference is not included in mathematics syllabus for this age group, the sequence of tasks was not explicitly designed to focus on inference. However, these tasks, using meaningful contexts and real data, provided opportunities to engage and simultaneously support students in several aspects of informal inferential practice. *TinkerPlots* software (Konold & Miller, 2005) was used in all tasks, as a tool for data handling, since it is easy to use and provides a dynamic learning environment to support the development of students' statistical reasoning. Students can create and explore, in ways that they choose, the potential of different graphical representations and statistical measures, by using the drag-and-drop facility or a set of informal operators (e.g. separate, stack, and order). Taking advantage of the software's tools the students can also engage in goal-directed activities that may lead to further insights in their data examinations.

Two preliminary tasks (tasks 1 and 2) were applied to build students' skills that they would need for conducting a statistical investigation and for making inferences, since they were not familiar with this kind of activity. Tasks 1 and 2 engaged students in explor-

ing real data using *TinkerPlots*, while developing key statistical concepts of syllabus as well as their reasoning. Since this was the students' first contact with the software, task 1 was structured and oriented by a set of questions leading them to interpreting contexts, exploring data (available from *TinkerPlots* database) in several ways and using various representations (which were discussed regarding their adequacy to give evidence). Finally they used their knowledge of the data to answer one initial question and to make predictions. Task 2 involved students in comparison of distributions using simulations provided by the *TinkerPlots* tools.

This study focus on task 3 – *The human body: a study in school*. Following the previous activity, this task engaged students in a statistical investigation (Wild & Pfannkuch, 1999) to discover more about the students in their school. Students were required to use their previous knowledge about the context and statistical concepts and processes (e.g., understanding the need of data, graphical representations, distribution, variability and sampling) to make informal judgments and predictions about the school population, based on data collected in their class. Finally, students were asked to explain their reasoning, integrating persuasive data-based arguments in their conclusions.

RESULTS: THE EMERGENCE OF STUDENTS' INFORMAL INFERENCE

In this section, a description of students' work in the context of a statistical investigation provides an opportunity to envision the role of this activity in the emergence of students' informal inferential practices.

Making generalizations beyond the data

This component of informal inferential reasoning emerged in students' work in different moments of the task. The teacher introduced the task in the whole class with an initial question aimed at having students collecting data and using them to make predictions and draw conclusions about a population beyond the collected data: "How would you characterize middle school students [grades 7, 8 and 9] of your school regarding some of Vitruvius' measures such as height, foot size and arm span?". In the subsequent whole class discussion, it was clear that students understood the utility of the samples in the inferential process, to draw conclusions, because they showed no intention to survey the entire school population to get the infor-

mation they needed to answer the question. They said: "It is best to conduct a survey" (E) and "To randomly select [students] from grade 7 to grade 9" (R). Although they were not unanimous about how the sample could be collected and used to find out the characteristics of all school students, they stressed randomness as an important condition to ensure the representativeness of the sample. Due to time limitations, students agreed to collect data from their own class, although recognizing the lack of representativeness of the sample.

After data collection, the teacher also asked them to formulate possible interesting questions (hypothesis) about the phenomena in study and to predict how their questions might be answered, first without providing them with data. This was proposed to later confront students' anecdotal answers with data-based evidences, pushing them to understand the need of data. Some students reveal inability to think about the data as an aggregate and to full make sense of the questions' intention to help them to answer the initial question in the task. Some of the emerging questions/hypothesis such as "What is the biggest value for the arm span?" (W&F), "What is the percentage of students with more than 160 cm?" (B&R) or "Does the highest student has the bigger shoe?" (B&R) reveal a deterministic nature and did not prompt students to think beyond their data. As expected, some of these students justified their predictions based on guessing and simple observation: "Based on [the observation of] my colleagues, I think that the average height of the students [boys] is 165cm" (M&S). Although these findings were consistent with the difficulties observed in other studies (e.g., Meletiou-Mavrotheris & Paparistodemou, 2014), students had just collected their own data and this activity might have diverted their attention from using inferentially the data. However, the majority of students seemed to accept the existence of variability and formulated general questions/hypothesis such as "Boys tend to be bigger than girls" (T&D), "Do boys and girls have similar arm spans?" (D&B) or "I think that maybe the arm span has a higher value for boys" (B&B). These statements show students' attempts to find one global description that accounts for variability between the groups. These claims could be considered a progress if compared to the ones in the previous questions, which are mostly local. However, they still focus on describing the class. These predictions also contained elements of uncertainty, which reveal that students were beginning to adopt a statistical perspective of trend.

After the exploration of the collected data (from the class), using the *TinkerPlots*, each group wrote a report predicting what they might find about the height, foot and arm span, more generally, for the whole school, and explaining what evidences they had to support their conclusions. These reports aimed at pushing further the students to see their own data as evidence for making inferences about whole school, building their ability to think "beyond the data". As occurred with young students (Watson, 2008), their initial focus on describing their own class data did not restrict them from creating more global interpretations of data. In fact, students' responses suggest that many of them did see the data collected in their class as useful evidence for their predictions. The excerpts below show that some students assumed that data collected in the class would have very similar properties to the data from the whole school, and generalized the results: "Considering the whole school students, the boys' height and arm span are bigger when compared with the girls, based on our class" (A&E) and "Boys tend to be bigger than girls, based on our class data" (A&N). It is interesting to notice that in their predictions students used probabilistic language and were attentive to the distribution and not to specific measures as they were in the formulation of the initial questions. This finding may suggest that students associated the generalization to uncertainty and that previous work in organizing and describing their own data supported them to go further in the inferential reasoning by helping them to improve their thinking of the data as an aggregate and to consider variability. Even though they have found the average height for their class, their predictions went beyond that to include uncertainty, adopting the statistical perspective of trends that are generally true but still have exceptions: "On average, boys are higher. But we also observe boys with 1,4 [m] who are shorter than girls (...)" (W&F). On contrary, a noteworthy number of students predicted that there would be some differences, as observed in the following claims: "The average height of the school students probably will increase due to students' natural growth" (M&S), "Boys will be higher than girls, so the average will move towards the boys' value because, probably, there are more boys than girls in school" (T&D) or "Although some differences in the height may occur, the average height will be higher, depending on the grade" (J&D).

Using data as evidence for generalizations

Making predictions about the whole school from their data was evident across the groups of students. Initially, students explored the data collected in their class, based on graphical displays created as they use the *TinkerPlots*. Given that students have not been oriented toward a specific graph, it was interesting to observe that they created a variety of graphical displays and adequately used them to represent and interpret their own data to get evidence for their claims, which shows their meta-representational competence (English, 2014). Dot plots were the most popular representations to gather evidence for answering deterministic questions focused on individual features, but most students also created boxplots to compare the two genres regarding the variables in study, as the ones shown in figure 1. These students' graphical displays were accompanied, respectively, by claims such "Students' average height is 162,7" (S&P), "The percentage of students with more than 160 cm is 55%" (B&R) and "Boys are higher than girls. Boys' average is 163 and girls' average is 160 cm" (E&S).

Students started comparing groups by noticing special values within distributions (such as the average or the maximum) and then moved to think about between-group variability. The comparison activity, based on their representations, also supported their claims about the whole school, as they said: "Boys tend to be bigger than girls, based on our class data" (A&N) or "We have based on our class which has more boys than girls and they are higher" (D&B). Moreover, their claims are not based only on data but also on their knowledge about the context which is familiar to them. For example, one group states "I think that for the whole school we will find something similar because our class is the 8th grade and that is the "median" of the middle school [it includes grades 7, 8 and 9]" (R&R). The exploration of a sample (data collected in the class), guided by the initial questions and facilitated by the visualization provided by graphical representations created in *TinkerPlots*, supported the students' involvement in the subsequent generalization of the observed characteristics and the use of such data as evidence.

Using probabilistic language to describe generalizations

Students' responses, presented in previous sections, suggest that they recognized that their predictions were tentative since they included, in their claims,

several elements of uncertainty expressed by the terms "probably", "maybe" or "something similar" instead of equal. They also used the term "tend to be" showing that they started to adopt a statistical perspective of trend. In all these cases, the uncertainty was expressed qualitatively, without confidence levels or margins of error (Dierdorp, Bakker, van Maanen, & Eijkelhof, 2012), as expected taking into account the students' statistical experience. Nevertheless, this component of ISI was the less evident in students' work, suggesting the need of specific work regarding probabilistic language.

DISCUSSION

In this paper, we described some examples of students' ISI that emerged through an informal data-based approach, rooted in the statistical investigation cycle (Wild & Pfannkuch, 1999). Despite the reported findings are limited to one task in one class, those examples give us some insight into the students' capabili-



Figure 1: Students' graphical displays

ties and the challenges they faced in making informal statistical inferences when experiencing statistics as an investigative process.

All students, even in the less successful groups, were able to demonstrate some aspect of informal statistical inference during the lessons. Students drew their conclusions based on the data they had collected from their class and often used the data to make inferences about an unknown population (whole school). Few students used probabilistic language for describing their generalizations but even those who made it had difficulties in including references to levels of uncertainty. These results highlight the need of working on probabilistic language issues, helping students to evolve from a deterministic perspective of inference to include uncertainty in their statements.

There are also some evidences from the study that the software *TinkerPlots* was used by the students as a reasoning tool (Ben-Zvi, 2006) providing opportunities for structuring and displaying data in ways they choose and understand. However, this was not the focus of this study, and although students' interaction with the software in the previous tasks seems to have contributed to their meta-representational competence (English, 2014) the complex role of technology in supporting ISI deserves further discussion. These encouraging findings do suggest that the adoption of a statistical investigation approach in a technological environment has the potential to make ISI accessible to students, as observed in other studies (Ben-Zvi et al, 2012).

Finally, many studies show that several skills are required for the emergence of students' practices of ISI, such as: articulating or predicting from observations, recording and organizing data using invented methods and working with aggregates and variability (e.g., McPhee & Makar, 2014). The proposed sequence of tasks in this DRP appeared to have supported students' development of such skills but they still need to be re-examined before they are used in a new teaching experiment. Further analysis of students' interaction within the groups, which were not considered in this study due to time and space limitations, may convey new and deeper insights on their reasoning, as well as to contribute for the reformulation of the tasks and their implementation conditions.

ACKNOWLEDGMENTS

Study supported by national funds through FCT – Fundação para a Ciência e Tecnologia, Project Developing statistical literacy: student learning and teacher training (contract PTDC/CPE-CED/117933/2010). We thank Ana Isabel Mota who assumed the role of research assistant for her extensive work in data collection.

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