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# Enactive metaphoric approaches to randomness

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*Our work aims at developing means to facilitating the access to stochastic thinking, especially for non-mathematically oriented learners. To this end, we draw on metaphoric and enactive approaches to the teaching and learning of randomness. More precisely, we report on a challenging didactical situation implemented in various classrooms, with students and prospective and practicing teachers, concerning problem posing and solving in the context of randomness that is approached through enactive metaphoring. The findings suggest that this sort of approach allows non-mathematically oriented learners to make sense of and abduct otherwise inaccessible mathematical notions and facts.*

**Keywords:** Enaction, metaphor, randomness, embodiment, waiting time.

## INTRODUCTION

We are concerned about facilitating the access to stochastic thinking and its practice and appreciation by the learners as one way to make sense of the world. We are especially interested in approaches that might be meaningful and helpful for “general” non-mathematically oriented students in school, college and university.

The approaches we develop relay on metaphoring, enaction, embodied and situated cognition. Our main hypothesis is that most students – especially those with no special mathematical skills – can think mathematically if they *enact* first suitable didactical situations, involving problem posing and solving. Here enacting is meant in the most literal sense, as when enacting a role, bodily, on stage. In this way they may notice and “see” facts or relations that they may have trouble seeing in an abstract or symbolic setting. We put this into practice here in the context of probabilistic and statistical thinking, with challenging tasks like figuring out the expected waiting time for success in a dichotomic success-failure random experiment. Of course even

learners with no probabilistic background can tackle this sort of task in a pure experimental statistical way, calculating the average waiting time for an increasingly larger number of repetitions of the experiment. Many of them may become sensitive nevertheless to the fact that they are calculating blindly without being able to anticipate, i.e., to “see” beforehand, what value the experimental average will be close to. We also aim at fostering the development of this sensitivity as an antidote to the common misconception of mathematics as just rote formula applying and calculating. These approaches have been tested with students and teachers with various backgrounds, ranging from elementary and secondary school students to university students majoring in science and humanities and to prospective and in service elementary and secondary school teachers.

## THEORETICAL FRAMEWORK: METAPHORS, DIDACTICAL SITUATIONS AND ENACTION

### Metaphors in cognitive science and mathematics education

Widespread agreement has been reached in cognitive sciences that metaphor serves as the often unknown foundation for human thought (Gibbs 2008; Soto-Andrade 2014) since our ordinary conceptual system, in terms of which we both think and act, is fundamentally metaphorical in nature (Johnson & Lakoff, 2003)

We find remarkably theory-constitutive metaphors that do not “worn out” like literary metaphors and provide us with heuristics and guide our research (Boyd, 1993; Lakoff & Núñez, 1997). We might even claim metaphorically that a theory is just the unfolding of a metaphor! Recall the “tree of life” metaphor in Darwin’s theory of evolution or the “encapsulation metaphor” in Dubinsky’s APOS theory (Soto-Andrade, 2014). In what follows we will use the metaphorical approach as a meta-theory to describe the other theoretical frameworks we will use.

In mathematics education proper it has been progressively recognized during the last decade (Araya, 2000; Chiu, 2000; English, 1997; Johnson & Lakoff, 2003; Lakoff & Núñez, 2000; Presmeg, 1997; Sfard, 2009, Soto-Andrade, 2006, 2007, 2013 and many others) that metaphors are not just rhetorical devices, but powerful cognitive tools, that help us in building or grasping new concepts, as well as in solving problems in an efficient and friendly way. See also Soto-Andrade (2014) for a recent survey. We make use of conceptual metaphors (Lakoff & Núñez, 2000), that appear as mappings from a “source domain” into a “target domain”, carrying the inferential structure of the source into the one of the target, enabling us to understand the latter, usually more abstract and opaque, in terms of the former, more down-to-earth and transparent. Our metaphoric approach to the learning of mathematics emphasizes their “poietic” role, that brings concepts into existence (“reification” in the terms of Sfard, 2009). For instance we may bring the concept of probability into existence when, studying a symmetric random walk on the integers, we see the walker *splitting* into 2 equal halves instead of going equally likely right or left (Soto-Andrade, 2013).

### Didactical situations

The theory of didactical situations (Brousseau, 1998) might be described as an unfolding of the *emergence metaphor* for mathematical content: mathematical concepts or procedures we intend to teach should emerge in a suitable challenging situation the learner is enmeshed in, as the only means to “save his life”. No real learning is possible if mathematical concepts “come out of the blue” or are “airborne” from Olympus.

Metaphors play an important role in didactical situations, that we describe with the help of a “voltaic metaphor”: Key metaphors are likely to emerge, as sparking voltaic arcs, in and among the learners, when enough “didactical tension” builds up in a didactical situation for them. Typically this happens when students try hard and long enough, interacting with each other, to solve a challenging problematic situation. Suddenly a key metaphor to solving the problem emerges in one – or several – of them. Of course, to have the students sustain and endure the necessary didactical tension, is not an easy task in most classrooms...

### Enaction

An unfolding metaphor for enaction is Antonio Machado’s poem (Thompson, 2007; Malkemus, 2012):

“Caminante, son tus huellas el camino, y nada más; caminante, no hay camino, se hace camino al andar” [Wanderer, your footsteps are the path, nothing else; there is no path, you lay down a path in walking].

Indeed Varela had already metaphORIZED enaction as the laying down of a path in walking (Varela, 1987, p. 63), when he introduced the enactive approach in cognitive science (Varela, Thompson, & Rosch, 1991). In his own words: “The world is not something that is given to us but something we engage in by moving, touching, breathing, and eating. This is what I call cognition as enaction since enaction connotes this bringing forth by concrete handling” (Varela, 1999, p. 8).

Enaction in mathematics education may be traced back to Bruner (1953), who introduced it as “learning by doing”. In fact he described *enactive representation* of a domain of knowledge (or a problem therein) as a set of actions appropriate for achieving a certain result, in contrast with *iconic representation*, where summary images or graphics are employed, or *symbolic representation*, based on symbols and their syntax. Later Bruner’s ideas were successfully implemented and diffused via Singapore’s CPA (Concrete-Pictorial-Abstract) methodology. For recent significant theoretical and practical developments of enaction in the field of education, which highlight the role of the teacher as an enactive practitioner acting in situation and prompt us to focus on the ways of being that can be fostered in the classroom rather than just monitoring the specific mathematical knowledge generated, see Masciotra, Roth, & Morel (2007) and Proulx & Simmt (2013).

### ENACTIVE METAPHORIC APPROACHES TO MATHEMATICAL NOTIONS: THE EXPECTATION OF A WAITING TIME

We have discussed elsewhere (Soto-Andrade, 2013) an enactive metaphoric approach to the case of a symmetric 2D random walk (Brownie’s walk). Here we will address the case of an important and ubiquitous family of random variables, to wit *waiting times*, and their expected values. The simplest case is that of the waiting time for success in a dichotomic success-fail-

ure random experiment, with success probability  $p$  and failure probability  $q$ . We specialize here to the simplest case  $p = q = \frac{1}{2}$ , that can be modeled, or *metaphorized*, by tossing a coin and waiting for heads.

### Methodology

The task (rather a problematic situation) proposed to the undergraduate students described below, was to figure out how long (how many tosses) they had to wait to get heads when tossing a coin. They had not been exposed to probability and statistics at the University and they were invited not to recall what they had heard about probability and statistics at school (where the subject is badly taught anyway). So they tackled this “impossible question” essentially bare handed. Notice that figuring out a sensible answer for our impossible question with no definite answer, is part of the task. We have then an “open task” that becomes stepwise more precise through the interactive work of the students (see a priori analysis below), accordingly with the enactive approach where problems are not “out there” waiting to be solved but are co-constructed by the cognitive subject and the world (Varela, 1987, 1999). It is our aim that the students learn to explore when tackling a problematic situation and then to conjecture and “see” beforehand ways of solution instead of blindly calculating the sum of an infinite series or computing an average over many repetitions.

Our experimentation regarding this task was carried out in the classroom with:

- a. 1<sup>st</sup> year University of Chile students majoring in social sciences and humanities, from 2011 to 2014 (1 semester mathematics course, averaging 60 students per semester).
- b. 25 University of Chile students enrolled in an optional one semester course in Post Modern Mathematics, majoring in mathematics or in pedagogy in physics and mathematics, in 2012.
- c. 40 University of Chile prospective physics and mathematics secondary school teachers (one semester probability and statistics course) in 2014.

Students were observed by the authors during interactive 90 minute work sessions. They did group work splitting usually into two groups of no more than 20 each.

We describe now the mathematical analysis and the *a priori* and *a posteriori* analyses of this experimentation in the sense of didactical engineering (Artigue, 2009).

### The mathematical situation

Notice that the experiment of flipping a coin until you get heads can be looked upon as a symmetric random walk on a truncated binary tree (you begin at the root and turn right if you get heads and to the left if you get tails, stopping when you get heads the first time), so that the question “How long will I have to wait for heads?” becomes “How long will it take the walker to get to one of the absorbing ends of the tree?” So flipping a coin or walking on the tree, are each one a metaphor for the other.

Since the random variable  $T$  = “waiting time for heads when flipping a coin” takes values  $n$  with probabilities  $1/2^n$ , its expectation  $E(T)$  is given by the series  $\sum_1^n \frac{n}{2^n}$ . Adding terms diagonally one can show that this series coincides with the geometric series  $\sum_0^\infty \frac{1}{2^n}$ , whose sum is 2.

The challenge we address below is whether the students could enactively “see” this result without calculating the corresponding series (symbolic approach, rather unfriendly for many) or averaging over an increasing number of repetitions of the experiment (statistical approach, also unfriendly for many). Notice that most students, and teachers, do not usually see another way to tackle the problem, besides the theoretical-symbolic one and the empirical-statistical one!

### The didactical situation: Tentative script and a priori analysis of the enactment

The teacher flips a coin once and asks for interesting exploratory questions. Very likely some students suggest to flip it again. Various interesting questions may arise. In particular, if the teacher gets, say, 2 or 3 tails in a row, students may begin to wait for heads. Eventually the whole class may get interested in the question: How long has one to wait for heads?

Different answers may come up, the level 0 answer being: “Nobody knows, only Jesus knows!” Other answers are expressed in gestural language. Some students may suggest experimenting. Each of them flips a coin until he or she gets heads. They realize the variability of their waiting times. Some may find that the situation is hopeless. Others, more positively

minded, may suggest to average. They average their waiting times and find, say, 1,7 or 2,6. So what?

This stirs usually a lot of discussion. Some students suggest further experimenting. Other make guesses, like “the average should tend to 3”. One natural question is what average are we likely to get if we entice all students on campus to do the experiment?

Some become tired of experimenting and begin to look for a more theoretical approach. Eventually they draw the corresponding possibility tree (a truncated binary tree) and assign probabilities with the help of a hydraulic or pedestrian metaphor, as in Soto-Andrade (2013). Recall that in the first one we visualize the possibility tree in vertical position, root at the top, pour a litre of water at the root and let it drain evenly downwards. Probabilities are metaphorized as quantities of water. In the second one a suitable number of pedestrians (a power of 2 in this case) starts at the root splitting evenly at each junction. Probabilities appear now as proportions (ratios) of pedestrians reaching a node in the tree. This is easy, but then they realize that to calculate the theoretical average, i.e. the expected waiting time, they have to calculate the infinite sum  $\sum_1^n \frac{n}{2^n}$ , that is rather unwieldy for them. Some students begin to intuit that there should be something like a limiting value for the average (if all Chinese waited for heads...) or either an “ideal average”. It is not clear for them however how to pin it down. The teacher prompts the students to suggest other approaches. If no new ideas arise, the teacher may suggest to *enact* the situation, all together.

More precisely, she suggests that all students stand up in a circle and each waits for heads when flipping a coin. She asks then how they would calculate the *average waiting time*. The students suggest the obvious way that entails asking each one of them how many times he had to flip the coin to obtain heads, add all these waiting times and divide by the number of players. The problem arises then however that this procedure does not allow them to guess or to estimate neither the experimental average waiting time nor the “ideal average waiting time”. It is nevertheless clear for most of them that “ideally” half of them should get heads at the first flip. Then among those who failed, ideally one half will get heads at the second flip. This is an interesting idea that actually comes up from some students, but leads them to an infinite sum all the same (although simpler to evaluate than the sum

above). So the students are still motivated to look for a friendlier approach.

At this point the teacher might suggest that to ask every player how long she had to wait for heads is a bit cumbersome. The question arises as to how could the students proceed in a friendlier and more concrete way, so that they really “see” what has happened to each one of them (notice the switch to a non verbal cognitive style). After some minutes thought at least one student suggests: flip several coins, one after the other, instead of just one! All appreciate this bright idea and begin to flip one coin after another (eventually the teacher has to lend coins to some students). After a while, each of them has a group of coins in front of her. If nothing happens, the teacher may ask them: what do you see? Some may say: “not very much, just a bunch of coins on the floor”. But others remark quickly: “there is just one head in front of each of us”. Other recall that to calculate the average we should count the total number of coins on the floor and divide by the number of players. Said that way however, the result is not easy to estimate beforehand. Then usually a few students realize that they will be dividing the total number of coins by the number of coins showing heads. But they “know” which is the “ideal ratio” here: if I see 17 coins showing heads on the floor, I would have expected “ideally” 34 coins in all. Of course there might be 37 coins instead. But this shows that “ideally” the average of all waiting times should be 2. After this breakthrough, usually the teacher invites the students to keep silent and quiet for a little while (one minute, say), in an introspective attitude, and to visualize the whole picture. After that she may prompt them to draw an image of the whole enactment (the circle of players, each with a bunch of coins in front of him...), so as to enter the iconic register. And then, according to the mathematical profile of the students, she can prompt them to formulate their conclusions in symbolic language.

An interesting fact that we have observed is that for most mathematicians this enactment is a proof while for most secondary school teachers it is not! Putting the whole situation on its head, we could even say that we have found an enactive metaphorical proof that the infinite sum  $\sum_1^n \frac{n}{2^n}$  adds up to 2. We claim that this is closer to real mathematics than the usual purely symbolic, abstract and axiomatic approach, that is just one genre among many possible ones (Manin, 2007; Soto-

Andrade, 2014). Finally we should remark that also the equality  $\sum_{n=1}^{\infty} \frac{n}{2^n} = \sum_{n=0}^{\infty} \frac{1}{2^n}$  may be gleaned from our enactment: when students realize that they should collect all coins lying on the floor, the teacher may prompt them to suggest different ways to do it. Usually some students come up with the idea of the “common pot” (as in a soup-kitchen), like the one squatters occupying a school in protest would organize (a familiar experience for many students). The idea is that to collect all the coins, the teacher should ask first everyone to put one coin into the common pot. All can do that. Then she would ask for a second coin. Ideally only half of the players are able to do that. And so on. This shows immediately that the ideal average of waiting times coincides with the sum  $\sum_{n=0}^{\infty} \frac{1}{2^n}$ , hence the equality of the two infinite sums above.

### The didactical situation: A posteriori analysis

The idea of flipping a new coin after each failure in getting heads came up as easily in students majoring in humanities and social sciences as in students majoring in mathematics or pedagogy in physics and mathematics. The fact that the number of students in the circle coincides with the number of coins showing heads did not come up very quickly, but we saw no significant difference between mathematically oriented students and humanists. Some of them realized this after contemplation of an iconic representation of their enactment, others did it after staring hard at the coins on the floor. A lot of discussion emerges as to how to register their results.

The relationship between the experimental average waiting time and the ideal one emerged rather slowly. Estimates of the experimental average of waiting times for millions of people varied in a significant way between students a and b, c. Recall that the former had previously flipped a coin 100 times and registered what they observed (not just the final result, but the whole process!). So they realized that the simplest avatar of randomness already creates shapes that look like mountain ridges and stock exchange charts. They were less prone than students b and c to think that in we cannot say anything sensible or even approximate about random experiment results. They also “saw” more easily that the ideal average waiting time should be 2, the inverse of the ideal relative frequency of heads when you flip many times a coin.

In all groups of students those who intuited that the ideal average waiting time should be 2 (out of statis-

tical experimentation) related this immediately with the inverse of the ideal relative frequency of heads when flipping a coin many times. All students conjectured quickly after this experience that the ideal average waiting time for “ace” when tossing a dice should be 6. Further work in more advanced symbolic mode was easier with students b and c that had a more intensive mathematical training. Sometimes a game emerged after this enactment, that may suggest another approach to the expected waiting time: the teacher gives each student as many coins as necessary to get heads flipping them one after another. When a student finally gets heads, the game is over and he keeps all flipped coins. The natural question is: How much should the teacher charge for playing this game, so that it becomes a fair game?

### DISCUSSION AND CONCLUSIONS

Crossing a priori and a posteriori analyses we see that several years of traditional mathematical training (students b and c) did not make a significant difference in performance in an enactment like the one we report here: students a, who come directly from high school, usually with a poor relationship to mathematics, did at least as well as students b and c, when trying to figure out enactively the value of the ideal average waiting time for heads. In fact they did even better regarding their intuition of the behaviour of the experimental average waiting time for an increasing number of flippers. We conjecture that this phenomenon is due to the fact that – in contrast to students b and c - they had made the enactive experience of flipping a coin 100 times and registering the whole stochastic process, realizing its relationship with everyday shapes like mountain ridges and stock exchange charts. This suggests that in some sense enactive experiences may concatenate and interact in a feedback loop in the life story of the learners, as suggested in Varela, Thompson and Rosch (1991): “cognition is not the representation of a pre-given world by a pre-given mind but is rather the enactment of a world and a mind on the basis of a *history* of the variety of actions that a being in the world performs” (emphasis is ours).

We also observed that learning enactively is not a one man (or woman) show, it is a collective social undertaking, that may be seen upon in some cases as an avatar of swarm intelligence. We have noticed, especially in group a, that among students reacting remarkably faster and better than the average in

enactive situations, a high percentage of come from alternative Montessori or Waldorf schools.

We should remark that our enactive approach poses some significant challenges: teachers and students need to be able to play tightrope walkers and transit seamlessly between different cognitive styles (from symbolic to enactive particularly). Special support and dedication is needed for those who come from more formal and robotic school systems, so that they can progressively adapt to new ways of working and approaching mathematical experience and knowledge. If this sort of “cognitive therapy” is not provided (by a devoted highly qualified teacher assistant, for instance) the risk arises of a severe stratification in the classroom, that could demotivate many students. We see then that significant situational intelligence, in the sense of Masciotra, Roth, & Morel (2007) is required from teachers as enactive practitioners.

On the other hand, as pluses of our approach we have observed that is very often highly motivating for the students, involves surprise (a key factor for learning, according to Peirce and Freire), knowledge is constructed in a cooperative way in the classroom, space is given for questions to emerge from the students in experiential situations, instead of answers being given to them before they have a chance to ask the questions first (Freire, 2011; Tillich as cited by Brown, 1971; Mason, 2014) and finally, it fosters participation so that students become protagonists of their learning.

As open ends, we may mention, among others: the didactical study of histories of enactive experiences of the learners; the relation between enaction and intuition (first steps in this direction may be found in Díaz-Rojas, 2013); the systematic study of the emergence of enactive metaphoring in suitable didactical situations; the enactive exploration of the real meaning of being mathematical in the classroom in the sense of Mason (2014), research on curriculum reshaping motivated by the enactive approach in mathematics education.

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