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# Six year old pupils' intuitive knowledge about triangles

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*This communication discusses the intuitive knowledge that six years old pupils reveal about triangles. Data is collected through a class group discussion, which was videotaped and transcribed. The results show that pupils articulate visual prototypes with known attributes to recognize triangles. Furthermore they identify non-examples and its features to recognize this shape. Position didn't seem to be a problema to identify triangles, however, some features, as curved lines or topological properties, appear to be a problema on triangles recognition. Pupils used mainly a partitive classification type but, when they observed and discussed known attributes or properties, they can also use hierarchical classification type.*

**Keywords:** Shapes, properties, classification.

## INTRODUCTION

The research presented on this paper reports six years old pupils' intuitive knowledge about triangles when solving tasks, which were based on those presented in (Clements et al., 1999).

This is an exploratory study whose main goal is to get some clues about the intuitive knowledge that Portuguese six years old pupils have about shapes. It is a part of a larger PhD study of the first author, named *Shapes classification: a teaching experience in the early years*. The work reported on this paper intends to discuss what kind of intuitive knowledge Portuguese six years old pupils reveal about triangles.

All participating pupils are six years old and belong to a 1<sup>st</sup> grade class. The data collected addresses to intuitive knowledge about circles, squares, triangles and rectangles but, in this paper, we only focus on triangles.

## CONCEPTUAL FRAMEWORK

When we presented a set of figures in order to identify the shapes which belong it is possible for pupils to reveal knowledge that is related to intuitive character thought, based on visual prototypes, without considering attributes or properties of those forms (Clements et al., 1999). This knowledge can be related to previous experiences and promotes different levels of development (Burger & Shaughnessy, 1986). Also, visual representations, impressions and experiences make up the initial concept image (Vinner & Hershkowitz, 1980).

Intuitive character thought can be related to major theories of concept formation: classical view and the prototypical view (Klausmeier & Sipple 1980; Smith, Shoben, & Rips 1974; Smith & Medin, 1981). According to the classical view, categories are represented by a set of defining features which are shared by all examples. The prototypical view proposes the existence of ideal examples, called prototypes, which are often acquired first and serve as a basis for comparison when categorizing additional examples and non-examples (Attneave, 1957; Posner & Keele, 1968; Reed, 1972; Rosch, 1973).

Initially, the mental construct of a concept includes mostly visual images based on perceptual similarities of examples, also known as characteristic features (Smith et al., 1974). This initial discrimination may lead to only partial concept acquisition. Later on, examples serve as a basis for both perceptible and nonperceptible attributes, ultimately leading to a concept based on its defining features (Tsamir, Tirosh, & Levenson, 2008). Following this idea, some pupils may recognize shapes supported on the recognition of properties of those shapes and others will articulate

visual prototypes with known attributes or properties to identify the same shapes (Clements et al., 1999).

Non-prototypical examples, are often regarded as non-examples (Hershkowitz, 1989; Schwarz & Hershkowitz, 1999; Wilson, 1990) and specifically, “nonexamples serve to clarify boundaries” of a concept (Bills et al., 2006, p. 127). On the other hand, Fisher (1965) admits that topological properties, mental structures that enable shapes abstraction, such as the configuration or appearance, cannot leave some pupils arrive to the identification of a particular shape, because they can't consider specific properties of that shape.

Regarding to classification, Clements and Sarama (2007) mentioned that pupils tend to a partitive type of classification, where various subsets of concepts are disconnected from each other, into opposition to a hierarchy classification, where the most particular concepts integrates the general ones (de Villiers, 1994).

Tsamir and colleagues (2008) claim that concepts often serve as a means by which people may categorize different things, deciding whether or not something belongs to this class. In other words, one of the functions of a concept is to enable a person to identify both examples and nonexamples of the category.

At a partitive classification process pupils can identify the shapes name without ever having been the opportunity to reflect on their names, attributes or properties and only a small part will be able to provide nonexamples (de Villiers, 1994).

In front of a different figures sets, where the goal is to identify circles; squares; triangles and rectangles, Clements and colleagues (1999) and Sandhofer and Smith (1999) claim that, in order of difficulty, children identify the circle; square; rectangle and triangle.

## **METHODOLOGY**

The exploratory study reported in this paper follows a qualitative interpretative approach (Denzin & Lincoln, 1989). Participants were all six years old and belong to the same 1<sup>st</sup> grade class, constituted by 21 pupils, of an elementary private school near Sintra, a small village of Lisbon area. All of them attended the kindergarten at the same school and belong to a socio-economical high level. All participants had had informal contact with different shapes during last

school year and that fact influenced their intuitive knowledge of shapes.

The study started with four clinical interviews, carried out by the first author to four pupils to test a reworking of tasks used by Clements and colleagues (1999), whose goal was to identify all circles, squares, triangles and rectangles of figures sets. So, the task where children must identify triangles followed two different steps: on a first approach four pupils, two boys and two girls, in different four days, were taken out of the classroom and solved the tasks, as we want to experiment the task with pupils before taking it into classroom. Furthermore, the first four clinical interviews intended, on the one hand, identify the first knowledge about triangles of these four children, and, on the other hand, the given answers served as a starting point to all group discussion. The intention was to create a kind of game where pupils not interviewed should guess whose triangles had been chosen by their colleagues. This game was a motivation for a collective selection of triangles and a discussion about it. These interviews were videotaped. After this first step, the researcher took the task for the classroom and promoted a whole class discussion session, which was also videotaped. The researcher played a kind of game where pupils who did not participate in individual interviews tried to guess which shapes were chosen by the interviewed pupils. The data presented in this paper focus on this whole group discussion.

So, data collection was through a group discussion, by videotaping, where pupils could discuss different ideas, complete or disagree with arguments, creating new reasoning and clarify concepts. This group discussion intended to lead pupils to the possibility of inclusive classification through the construction of shape families that display equal or similar attributes or properties.

The work consisted on a chosen triangles task using a manipulative set of figures, placed in the same position as the one presented by Clements and colleagues (1999), each figure printed on a separate card allowed pupils compare, rotate, overlap, among others. On this set of manipulative figures, pupils had to choose all triangles, justifying their choices. During this stage we intended to understand what kind of knowledge pupils used to recognize triangles: visual prototypes; shape attributes or properties; nonexamples; among others. Besides, their justifications and new questions,

related with their answers, we tried to forward them to the construction of shape families to look out to hierarchical classification.

The data were analyzed regarding to the pupils answers, based on visual prototypes, without considering attributes or properties of that forms (Clements et al., 1999), and when they claim that visual representations, impressions and experiences make up the initial concept image (Vinner & Hershkowitz, 1980).

Another aspect we considered on the analysis of data refers to a partitive type of classification influenced by Clements and Sarama (2007) and de Villiers (1994), which mentioned that pupils tend to a partitive type of classification, where various subsets of concepts are disconnected from each other, into opposition to a hierarchical classification, where the most particular concepts integrates the general ones.

Finally, we had in consideration that, probably, triangles, according to Clements and colleagues (1999) and Sandhofer and Smith (1999), will be a difficult shape to identify.

With this kind of methodology we wanted to get some clues about intuitive knowledge of six years old pupils about triangles; what kind of language they use to express this knowledge; understand how large group discussions can lead pupils to a better knowledge of triangles and their properties and, finally, understand if large group discussions facilitate the idea of hierarchical classification.

To preserve the identity of all pupils, on the analysis we used fictitious names initiated by the same capital letter of their real names.

### PUPILS' INTUITIVE KNOWLEDGE ABOUT TRIANGLES

As we wrote before, the task where children must identify triangles followed two different steps: the first one we carried out individual interviews with four pupils out of their classroom. With the data collected on these interviews a group discussion session was organized. The researcher played a kind of game with the pupils who had not participated in individual interviews. They should try to guess which were the shapes chosen by the interviewed children. To start

the researcher asked to all group which were the triangles they thought had been chosen by their colleagues.

Researcher: In front of these set of shapes, António; Geraldo; Marta and Mara should choose only triangles. Which ones do you think they chose?

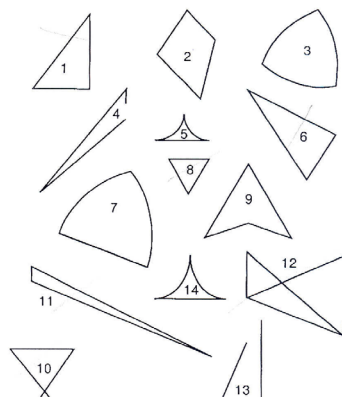


Figure 1: Set of shapes to identify triangles (Burger & Shaughnessy, 1986; Clements & Battista, 1992a)

Augusto: I think they didn't chose number 2. Because it has 4 sides and triangles only have 3 sides.

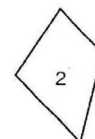


Figure 2: A nonexample of triangle given by Augusto

Some pupils, as Augusto, could recognize triangles identifying a nonexample of them, nominating attributes as the number of sides. Here he seemed regarded non-prototypical examples as nonexamples.

Joel: Picture 2 has 4 sides and 4 angles.

Researcher: So it can belongs to which family?

All group: Rhombus...

Augusto: ... and squares.

Researcher: Why?

Maria Manuela: Because it has 4 sides and 4 angles.

Researcher: So, square and rhombus can be part of which family?

Geraldo: Rectangles.

The group discussions conducted children to observe some attributes or properties and this observation, sometimes, led to comparisons between figures and

their properties and offered a larger possibility to identify common attributes or properties that can produce a primary idea of hierarchical classification. However, last answer needs a different kind of work because all mentioned shapes don't have equal attributes or properties and can't be part of same family.

Researcher: What about you Geraldo... which triangles do you think your colleagues had chosen?

Geraldo: Pictures 1, 3, 4, 6, 7, 8, 11 e 13.

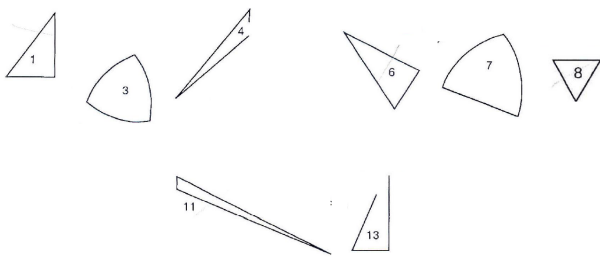


Figure 3: Triangles identified by Geraldo

Analyzing Geraldo's choices we can understand that when he had chosen figures number 3; 4; 7 and 13, his choices were based on topological properties, such configuration or appearance, without consider specific properties of triangles.

However, when he chose figures 6 and 8 he seemed recognize triangles articulating visual prototypes and known attributes. Already, when he identified figure 11 it is possible to say that he had identified some attributes and properties of triangles because the triangle identified by number 11 was a long and narrow scalene triangle which isn't a common representation of triangle represented on an unusual position.

Researcher: Augusto said that picture 10 has 6 sides but, António, you had identified it as a triangle. What do you think now, António?

António: It's a non triangle because it has 6 sides.

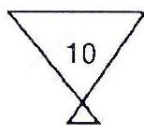


Figure 4: Figure identified as a triangle by António

When children were in group discussion they were encouraged to discuss shapes, attributes or proper-

ties, they could compare and integrate new concepts as the one showed on transcription "non triangle", which meant that these group discussions might become richer in terms of concepts. Moreover, the group discussions allowed all pupils participation, sharing, abstraction and reflection about triangles attributes or properties.

Researcher: António, you said that picture 14 is a triangle, but Geraldo, Mara and Marta said that is a non triangle. Who is right?

António: I think I was wrong..

Researcher: Why?

António: Because, now I can observe it has curved lines .

Researcher: So, do you think triangles shouldn't have curved lines?

[...]

Marta: Triangles only could have straight lines because they have 3 angles.

Lívia: I agree with Marta, triangles have angles because it doesn't have curved lines.

Researcher: So, picture 14 is a triangle or, like Marta and Geraldo said, a non triangle?

All group: A non triangle.

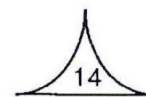


Figure 5: Figure identified as a triangle by António

Once more, group discussion seems to be a place where pupils can reflect about appearance, prototypes, attributes and properties and clarify concepts.

Researcher: António, now, you said that picture 14 is a non triangle because it has curved lines and because of that they can't have any angles.

Can you identify other pictures from the same family?

Mara: Pictures 3 and 7 are not triangles because they don't have any angles.

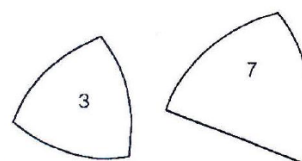


Figure 6: Figures identified as "zero" angles family

Lívia: I think they all can belong to “zero” angles family.

When pupils establish relations between different reasoning and identify necessary shape conditions, inclusive classification may emerge.

Researcher: Marco said that António, Geraldo, Marta and Mara probably had chosen picture 13.

Do you all agree?

Lívia: I didn't because it has open lines. All triangles should have 3 sides, all close. I think picture 13 is a non triangle.



Figure 7: Figure identified as a triangle by Geraldo

During all discussion, Lívia seems to understand what are the necessary conditions to be a triangle and also a necessary condition to be a two dimension shape, when she considered the attribute “close lines”.

Researcher: After all this discussion, who wants to tell me what pictures we are sure that represent triangles on this set?

Large group of pupils: Pictures 1; 6; 8 and 11.

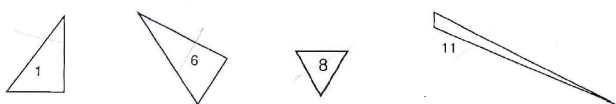


Figure 8: All triangles considered by a large group of pupils at the end of discussion

At the end of group discussion a very large number of pupils seemed to have clarified the concept of triangle and a strong concept image of this shape, while they were observing features of discussed figures and were articulating attributes and properties of triangles.

## DISCUSSION

In this exploratory study the pupils articulated visual prototypes with known attributes, for instance, when they identify examples and nonexamples of triangles

and were able to justify it, which is according to the results obtained by Clements and colleagues (1999).

In line with Rosch (1973) and other authors, few pupils used nonexamples to identify triangles and justify their choices. But in this study nonexamples served to clarify boundaries of a concept (Bills et al., 2006).

A group of pupils seems to have some difficulties to identify triangles because of some topological properties, as appearance or configuration that prevent the recognition of triangle specific properties. These results are according to those mentioned by Fisher (1995).

Contrary to what would be expected, some pupils recognized triangles independently of the figures position, which may be related with their previous experiences. Burger and Shaughnessy (1986) reveal that experiences and informal contacts with different shapes are very important factors at children intuitive knowledge of shapes.

Finally, we think it's possible to say that when pupils of 1<sup>st</sup> grade were encouraged to discuss shapes attributes or properties, in group discussions, they can observe differences and similarities and hierarchical classification could emerge (de Villiers, 1994). Furthermore, they seemed to be able to clarify concepts and construct new ones (Tsamir et al., 2008).

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