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Working with visually impaired students: Strategies developed in the transition from 2D geometrical objects to 3D geometrical objects

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In this paper, I will present some of the results of research that was carried out, during my master studies, that aimed to examine the strategies that visually impaired students develop while coping with the transition from 2-dimensional (2D) to 3-dimensional (3D) geometrical objects, and also their correlation with the concepts of visualization, haptic perception, gestures and language. A teaching experiment took place in a support unit for visually impaired students. The results showed that the students develop geometrical thinking procedures that are influenced by the concepts of visualization, haptic perception, gestures and language.

Keywords: Geometry, visualization, haptic perception, visual impairment.

INTRODUCTION

The subject of geometry has always been and continues to be a “headache” for most students. For many years researchers in didactics of mathematics, have studied the levels of students’ geometrical thinking (Van Hiele, 1986; Gutierrez, Jaime, & Fortuny, 1991), as well as the processes that students follow in geometry (Battista, 2007; Duval, 2011). But if geometry is one of the most demanding subjects for the “mainstream” students, what happens in the case of students with visual impairments? How does a student who doesn’t have the luxury of vision, cope with the demands of geometry, which besides mathematical abilities also includes visualization and spatial abilities?

Such questions and concerns led us to address these issues and conduct research in order to observe how visually impaired students interact with the objects of geometry and what strategies they develop while solving geometrical problems that connect 2D with

3D geometry. The basic concepts I will use to examine the strategies developed by students are: visualization, gestures and language that students use in order to describe geometrical objects and also haptic perception. Through these concepts, I will observe the processes of reasoning developed by students during their transition from 2D to 3D geometry, in the context of two tasks.

THEORETICAL FRAMEWORK

Important theories on the evolution of geometrical thinking

Many researchers have studied students’ geometrical thinking and here I will present some significant theories of the evolution of geometrical reasoning, as well as some other concepts that relate to it, such as similes. In the field of didactics of mathematics and especially as far as geometry is concerned, much research has been done regarding the levels of students’ geometrical thinking. The Dutch mathematician Van Hiele proposed a widely accepted description of the development of students’ geometrical thinking (Van Hiele, 1986) through five levels, which show the way students think about figures and other geometrical concepts. Gutierrez, Jaime & Fortuny (1991), extended the Van Hiele levels for the case of three-dimensional shapes. They describe four stages concerning students’ thinking levels regarding the geometry of solid objects and their properties. Gutierrez (1992) extended these levels, examining them also from the perspective of visualization.

An important contribution to research concerning students’ geometrical thinking is the research work of Duval. Duval (2011) argues that when observing a figure from a mathematical perspective, we should determine its figural units, depending on its dimen-

		Figural Units (Components)	
Shape	Cube (3D shape)	0D/3D	Vertices (zero-dimensional components)
		1D/3D	Edges (one-dimensional components)
		2D/3D	Faces (two-dimensional components)
		3D/3D	Cube (three-dimensional object)
	Square (2D shape)	0D/2D	Vertices (zero-dimensional components)
		1D/2D	Edges (one-dimensional components)
		2D/2D	Square (two-dimensional object)

Table 1: Figural units of cube and square

sions, i.e. its different dimensional components (see examples in Table 1). This procedure can help us see the same shape as a composition of different figural units. In this way we choose the composition that is useful for solving a specific problem.

Similes, haptic perception, gestures and geometry

Visually impaired students often use, phrases like: “I see it” or “I do not see that yet”. These phrases are usually verbal expressions of their haptic experience. Another phenomenon in language is the great influence of everyday life on visually impaired students’ vocabulary. It is many times that influence that makes it difficult for these students to describe their thoughts in the formal terms of geometry. The results of Potari, Diakogiorgi & Zanni (2003), showed that similes are an important tool for students’ access to geometrical concepts, which were inaccessible only with the use of formal terminology.

According to Figueiras and Arcavi (2012), haptic perception provides us with access to spatial details that would not be easy to observe in another way. This is due to the fact that haptic exploration combines the process of visualizing an object and “action”, since it actually is the translation of a haptic stimulus to a mental representation, without the existence of a visual stimulus. According to Triantafyllidis (1995), when exploring an object by touch, a series of steps is required in order to identify its shape. As a first step, the key characteristics of the shape are identified, though without following any particular strategy. Then a more detailed exploration of the shape begins, which is this time based on some strategies that will reveal in greater detail its characteristics and properties. For students with visual impairments, haptic exploration of shapes or figures plays for them the role of vision. It is the way through which they can “see” the shapes. According to Williams (1983) and Millar (1981),

blind students explore an object in such a way that provides them a comprehensive view of it, thus leading them to a more accurate mental representation of its figure. According to Triantafyllidis (1995), there is a correlation between the strategies a student chooses to use and the level of his/her geometrical thinking (Van Hiele levels). As a consequence, vision is not the only source of mental images in mathematics. To “feel” some physical objects with our hands, without seeing them with our own eyes, is also a source of a rich production of mental images.

Our hands though, are not a tool useful only for our ability to perceive haptic stimuli. They can also be used as mediations of our thoughts, through the gestures they perform while we argue or think. McNeill (1992) categorizes gestures in Iconic, Metaphoric, Deictic and Beat gestures, depending on the role they had in the talk of the speaker. Radford (2009) argues that thinking is not something that occurs only in the mind, but also through language, body and the tools one has at his disposal. He argues that gestures are operative components of thought. This means that gestures do not only make apparent to us what someone thinks, but they are in fact the ones to generate ideas, the ones that incite thinking.

Visualization and geometry

Gutierrez (1996) describes visualization in mathematics, as the kind of reasoning activity based on the use of visual or spatial elements, mental or physical, in order to solve a problem or prove properties. But what happens in the case of students with visual impairments? If someone cannot see, how can (s)he visualize? Just because someone is blind, it does not mean that (s)he cannot “see”. Miller (1987) argues that mathematical visualization also involves intuition through pictures formed in the eyes of the mind. As described by Jackson (2002), visualization may go far beyond the sense of vision, because it implies the kind of under-

standing that comes from the intuition described by Miller. However, in order to achieve this kind of understanding, we should not isolate visualization from a general mathematical context, but instead connect it with other kinds of reasoning.

Lohman (1988) correlates directly the concept of visualization to that of spatial ability, defining the concept of spatial visualization, which refers to the ability to comprehend imaginary movements in a three dimensional space or the ability to manipulate objects in imagination. According to Kospentaris and colleagues (2011), a student's visualization process can influence the strategies (s)he chooses in order to cope with a geometrical task, as also the way in which (s)he implements these strategies. It is therefore apparent that there is a strong correlation between visualization and the formation of geometrical thinking.

RESEARCH QUESTIONS – METHODOLOGY

My research questions in the study were the following:

- 1) What kind of strategies and procedures do the students follow during the transition from the 2D to 3D geometry?
- 2) What is the contribution of gestures, the use of similes, and visualization in the selection and implementation of these procedures?

Participants. The research took place in a support unit for students with visual impairments. Five students participated in the study. The students were chosen according to their age, visual ability and their mental ability (visually impaired students with mental disabilities did not take part in this research as its focus was on students with only visual impairments). The reason why we wanted to have students of different ages and grades is that at different ages, students have gathered different experiences from their everyday

lives and different levels of geometry knowledge from school. The research was conducted within the frame of interviews. There was no classroom situation. During the interviews the researcher was alone with one or two of the participants at a time. Students M and A worked together (group 1) as also students D and T (group 2), while student B worked alone.

Tasks. For the purpose of the study, four tasks were given to the students. Here I will present and analyze the results of only two of these tasks.

Procedure – Aim of Task 1 (Plane figure Rotation). The students were asked to think about what kind of solids will be created by the rotation of a plane figure (Rectangle, Right triangle, Disk), around a vertical axis of their choice. They used 2D objects (plastic plane figures) which they could rotate and manipulate freely. The objects were neither attached nor fixed on the desk or anywhere else. The students did not use any 3D objects. The procedure of the task was that the students touch the plane figures, create a rotation, imagine the solid being “created” and finally identify it. Students' prior knowledge regarding rotation was related mainly to what they knew about the earth's rotation around its axis.

Procedure – Aim of Task 2 (Nets of Solids). The students were asked to identify a solids' nets that were given to them. They started with the net of a cube and I asked of them to construct as many different nets as they could, which when folded would create a cube. Later they also did the same for more solids (Triangular pyramid, Square pyramid). My aim was, through the nets of the solids, to see how students handle the properties of the above mentioned solids, as also if they could identify a solid by its net and the properties its net “reveals” at the levels 0D/3D, 1D/3D and 2D/3D.

The nets were consisted of hard plastic parts (squares and triangles) which were detachable. The connec-

STUDENTS	M	A	B	D	T
GENDER	Male	Female	Male	Female	Female
AGE	18	16	19	14	17
GRADE	12th	9th	12th	7th	11th
VISUAL IMPAIRMENT	Low vision	Congenitally blind	Low vision	Congenitally blind	Congenitally blind

Table 2: Participants' profile

tions between the parts of the nets were flexible and could be folded in order to create the 3D objects. In a discussion with the students, which took place before the beginning of the teaching experiment I asked them some questions regarding their theoretical background in geometry. I learned that they all had knowledge of the basic plane figures and solids and their properties, from the school course of geometry. My goal was to “challenge” the students to identify the solids during their creation, either through the rotation of plane figures or by their nets.

The first step of the analysis was the transcription of the recordings of the interviews. At first, the recordings of the interviews’ discussions were transcribed. Then some critical events in each group were identified and analyzed based on the concepts explicated in the theoretical framework of this study. The critical events were classified as such based on the following criteria: (a) haptic exploration of the figures and students’ haptic perception about figures, (b) students’ geometrical thinking and spatial perception regarding the task they performed, the figures these included and their properties, (c) the way in which students seem to visualize the plane figures and the solid objects and their properties through haptic exploration and geometrical thinking, (d) the students’ gestures while trying to explain what they were thinking/doing or in their attempt to argue about their answers, (e)

the students’ reasoning processes in order to accomplish the transition from 2D to 3D geometry.

RESULTS

Task 1 – Plane figure rotation

During the rotation task, some students (A, M, D and T) chose to rotate the figure for a while and then stop, making the rotation a picture in their minds. Student B chose to rotate the figure nonstop, in order to identify the solid that was being created. The common factor in all cases was the haptic exploration of the invisible solid that was being created. All the students touched the plane figure during its rotation, as well as the trace that was imaginably left of it on the desk, forming the basis of the solid. All of the students recognized the solids created, moving from a part to the whole of the solid, that is starting from the bottom and then moving to other parts of the solid. The different ways of rotation (temporary or nonstop), and hence visualization and spatial perception of the plane figure’s movement, combined with the haptic exploration of the formed solid, led students in different strategies of spatial visualization of the solid created (Table 3). All the visualization processes and the strategies were identified based on students’ verbal and gestural descriptions. During the identification procedure the students were always asked by the researcher to describe loudly their thinking processes and also the

ROTATED PLANE FIGURE	ROTATION AXIS	SOLID	STRATEGIES
Rectangle	One of longer edges of the rectangle	Cylinder	<ul style="list-style-type: none"> – Identification of the base of the solid. Then identification of the whole solid by its net. (students A and M) – Infinite number of identical rectangles in circular array (one behind the other), with common edge the rotation axis. (student B) – Two horizontal disks, joined together by infinite vertical lines or two horizontal disks with height difference. (student D)
Right Triangle	The longer perpendicular edge of the triangle	Cone	<ul style="list-style-type: none"> – Infinite number of identical right triangles in circular array, with common edge the rotation axis. (student B) – A “cylinder” that narrows gradually towards the top. Instead of a base, the top of the solid is formed to a vertex. (students A, M, D and T)
Disk	One of the diameters of the disk	Sphere	<ul style="list-style-type: none"> – Infinite number of identical vertical semicircles in circular array, with common part the vertical diameter. (student B) – Infinite number of horizontal circles of different diameters, one above the other. The circle with the largest diameter is in the “middle”, whereas moving above and below it the circles have gradually reduced diameters. (students A and M)

Table 3: Strategies of visualization process of solids

ways in which the images of the 3D objects were created in their minds.

Through these strategies we can distinguish the different ways in which students apprehend and conceptually manipulate the plane figures and solids. For instance, the strategy in which students visualize a solid as a composition/array of infinite number of identical plane figures is perhaps a result of their choice never to stop rotating the figure until they identify the solid that is being generated.

The students argued about the way of rotation and the figures they felt being created during the rotation, based on the properties of both the rotating plane figure and the created solid. For example, student B's decision regarding the rotation axis of the disk was based on his knowledge of the infinite number of its diameters. In the case of the cone, the students justified its creation by the rotation of a right triangle because of the existence of the hypotenuse, which "forces the solid to have a pointed top instead of a face top". By the end of the interview, all the students had successfully identified all the created solids.

While thinking or arguing, students used gestures and similes. The similes concerned familiar objects from their everyday lives, which had the same shape as the geometrical solids investigated. The cone was described as a funnel or the orange cone used in football workouts, the cylinder as a milk can and the sphere as a ball, the earth or a "round" egg. Student B and the students D and T (group 2) described the solids' shape in their own words. The use of gestures was particularly strong in describing the rotation of the plane figure and the creation of the solid. Students used gestures sometimes in order to describe what they said (iconic gestures, McNeill, 1992) and other times to highlight some specific features of the figures, like a vertex or an edge (deictic gestures, McNeill, 1992).

Task 2 – Nets of solids

In this task the procedure of visualization was based to a great degree on students' haptic perception. They touched and explored the units of a net and identified their shape and through tactile contact they created visual images in their minds, which allowed them to imagine the solid that would be created by the folding of the net. Also here students used a lot of gestures. While thinking and visualizing the folding of the nets, their thoughts were followed by iconic and

beat gestures (ibid.) described the ways in which they were imagining the folding of the net. These gestures helped them get a better understanding of the final shape of the created solid. The students started identifying haptically the figural units of the nets (faces). The basic strategies the students followed to identify the solids, while manipulating their nets, were:

1) Identification based on the properties of the figural units of the solid

At first the students identified the solids based on their figural units, without following the process of the imaginary or real folding of the nets. Thus, the presence of six squares indicated the existence of a cube, while the presence of triangles in the net indicated the creation of a pyramid. For example, in the case of the nets of the pyramids, student D said something very enlightening about her working process: "The pyramids are the only solids having triangles in their nets". This statement emphasizes the importance of knowing the properties of a geometrical shape, which can help a student create connections between geometries of different dimensions, evolving his/her geometrical thinking not only on the level of just 2D or just 3D geometry, but most importantly between them.

2) Identification by the imaginary folding of the net

In this case the students started identifying the figural units of a net, but still that was not the "guide" in finding the solid that corresponded to the net. In order to identify the solid they started to fold the net mentally. At this point one can recognize the important contribution of the process of spatial visualization in geometry and the decisive nature of a student's ability to visualize geometrical shapes dynamically. Students managed to create, in their minds, dynamic images of the net, which they could manipulate and fold mentally thus resulting in the identification of the solid being formed. This is a complex process that requires the student to be familiar with the properties of the shapes and their manipulation, even in the absence of visual contact with the object. In our case, only student B managed to manipulate mentally and in a dynamic way the object that was given to him. After the identification of the net that was given to group 1 of students A and M (cube), I asked them if a change in the net would result again to the creation of a cube. Student M explained that not all nets with six squares result to the creation of a cube, but only

those that do not “leave gaps” somewhere in the solid when being folded.

These two strategies may differ. However, both of them are of great importance since each one shows a different aspect of the transition from 2D to 3D geometry, indicating that not all students perceive the geometrical shapes in the same way.

CONCLUSIONS – DISCUSSION

Through the strategies they followed, the students were able to successfully perform transitions from 2D to 3D geometry. The strategies were formed as a combination of both students’ everyday-life experiences with objects, both plane figures and solid shapes, and their knowledge regarding the geometrical properties of plane figures and solids from the school subject of geometry. Their tactile experiences of plane figures and solids both from everyday life and school also made a very important contribution to students’ visualization processes and reasoning in the transition from 2D to 3D geometrical objects.

The students identified the geometrical solids created, starting from a part and moving to the whole of the solid. This way the students managed to gradually build an image of the object, which was the result of a synthesis of all its individual parts. This procedure was not limited only to the synthesis of an image of a plane figure or a solid, but also continued to the composition of the one through the manipulation of the other. This process is consistent with Duval’s view, which supports the partial identification of a solid by its figural units, but is in contrast to the usual, and more holistic, procedure of solids’ identification by sighted students.

In most cases students used gestures, such as those mentioned by McNeill (1992), in order to describe something they could not easily express with words or the formal mathematical terminology. Such was also the role of similes used by students. It was often difficult for students to verbally express their reasoning through formal mathematical terminology. Thus, they used similes from their everyday lives, in order to describe the shape or form of a geometrical object or even some of its properties.

In all tasks the visualization process of the geometrical objects and their manipulation by the students

was most apparent. The students created images like the ones described by Presmeg (2006), calling them “mental images”. These images were sometimes static while other times students assigned to them a dynamic character, by moving them mentally, creating new images. Thus, the contribution of visualization was particularly important both in manipulating and identifying geometrical objects, as also for the transition of geometrical reasoning between geometries of different dimensions. This bidirectional relationship between visualization and geometrical reasoning can help students develop their geometrical thinking.

In the field of didactics of mathematics, no research has yet been done as to the levels of visual impaired students’ geometrical thinking. Thus, and mainly because of some parameters that do not exist in the case of sighted students (e.g. lack of real visual images for solids), we cannot be sure whether we are allowed to classify the levels of the participating students’ geometrical thinking, using Gutierrez’s levels classification (1992). What we can say though, based on these results, is that the strategies followed by the students, and also the way they chose to implement them and result to their responses, show that the students based their geometrical reasoning on the properties of both the plane figures and solids, as also on the relations between these properties. However, realizing the limitations mentioned above, and also the limitations of our research (e.g. limited time and number of students), we cannot but emphasize the importance of further research in this field of mathematics’ education, so that we are able to draw clear conclusions.

Agreeing with Healy’s view (2012), that if we manage to understand the similarities and differences in practices of blind students in relation to these of sighted students we will then be able to understand even better the correlation between experience and understanding, we think that it would be extremely helpful, not only for visually impaired students but for all students, that the examples these five students gave to us on how to cope with geometrical objects, find their role in the field of didactics of geometry and not be left unexploited.

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