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Spatial and geometric structuring – contributions for a collective construction

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This study aims to understand the contributions of collective moments of whole-group discussions for the construction of didactical pathways, based on geometric tasks experimented with children in the 2nd and 3rd years of primary school (6–10 years old). The experiment contributes to the understanding of Battista's model in its two dimensions of spatial and geometric structuring as well as the relationship between them (Battista, 2008). The data we gathered, although its analysis is still in progress, help us to build a framework from which it seems possible to identify a connection between the individual forms of mathematical knowledge for each student, the taken-as-shared mathematical practices of the classroom community and the taken-as-shared mathematical practices of wider society (Cobb, Yackel, & Wood, 1992).

Keywords: Spatial structuring, geometric structuring, whole-group discussions.

INTRODUCTION

In the Portuguese educational context the teaching of elementary geometry is still very poor and has adverse consequences for future teachers (Tempera, 2010). As pre-service and in-service teacher educators we recognize the need to enhance this area, seeking to produce useful materials for teacher education. These requirements are consistent with research interests in this area (Battista, 2007).

This communication is part of a PhD research project of the first author whose purpose is to study the teaching and learning of geometry in the early years. Its main objective was to design, test and evaluate teaching didactical pathways in Geometry and Geometric Measure for the first cycle of basic education (6 to 10 years old children). Our investigation, a design

research (Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006), was guided by a framework in three phases (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). This framework, which arises from Stein, Smith, Henningsen and Silver (2000), includes the installation of a task, *phase 1*, and its implementation, split in two distinct components, *phase 2*, the moment when students work on the task, and *phase 3*, the moment of whole group discussion. Progressively, implementing tasks in different classes *phase 3* has gained greater importance increasing the role of the researcher as a teacher, but always acting together with the class teachers.

The objective of this study is to identify and understand the contributions of collective moments of whole-group discussions for the construction of didactical pathways, based on tasks focused on the spatial and geometric structuring (Battista, 2008). We try to highlight the importance of these moments and some critical aspects that help to decide the orientation of the pathways and to establish the links between the tasks that integrate them.

THEORETICAL CONSIDERATIONS

The research is based on three fundamental axes: the structuring of geometric reasoning (Battista, 2008; Freudenthal, 1991); didactical pathways based on hypothetical learning trajectories (Confrey & Kazak, 2006; Gravemeijer, 1998; Simon, 1995); implementation of mathematical tasks (Jackson et al., 2013; Stein et al., 2000; Yackel & Cobb, 1996).

We adopt the structuring perspective of Battista (2008) involving three items: (a) Spatial structuring; (b) Geometric structuring; and (c) Logical/Axiomatic structuring. At this level of instruction we only face the first two, although we have in mind that the devel-

opment of good formal logical structuring depends on a good geometrical structuring, as the development of the later depends on the quality of spatial structuring. According to Battista:

Spatial structuring determines a person's perception/conception of an object's natures or shape by identifying its spatial components, combining components into spatial composites, and establishing interrelationships between and among components and composites. (p. 138)

For instance, a geoboard is an instrument of spatial structuring through an orthogonal normal structure. We use it to spatially structure rectangles. If the sides of the rectangle coincide with the lines, structuring is immediate. If not, it is necessary some kind of visualization feature to spatial structuring the rectangle. On the other hand, analyzing a number of different rectangles and identifying the existence of four right angles as invariant we are geometric structuring rectangles. This way we are constructing a rectangle model. This conceptual scheme allows the square to be recognized as a rectangle. This example illustrates, as Battista says, how “for a geometric structuring to make sense to a person, it must evoke an appropriate spatial structuring for the person” (p. 138).

In our investigation, we follow the visualization perspective of Presmeg (1997):

Visualization is taken to include processes of constructing and transforming both visual mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics. (p. 304)

In the context of geometry, the need for research on the nature of tasks that develop spatial visualization skills and visualization is recognized for the development of spatial and geometric structuring. We connect this with learning trajectories. We use the name of didactical pathways to identify a set of tasks suitably structured, based on hypothetical learning trajectories, after experimental use in a classroom which allows its refinement from experimentation. In a way, a didactical pathway can be considered as a photograph of a journey of learning and teaching, already performed, including their reflective analysis.

We attend to students different roles and responsibilities, distinguishing thinking responsibility and participation responsibility (Wood, 1999; Wood & Turner-Vorbek, 1999), the sense of orchestrating to promote productive classroom discussions (Smith & Stein, 2011; Stein & Smith, 1998; Stein et al., 2008), the role and kind of questions (clarification, argumentation, confirmation) and their importance to obtain productive exchanges (Boaler & Brodie, 2004; Wood & Turner-Vorbek, 1999).

METHOD

Concerning the method, this research is an *educational design research* according to Van den Akker and colleagues (2006). At the end of the fieldwork conducted by the first author, we managed to get four didactical pathways. Many of the tasks have been tried more than once, in different classes and years of schooling, with particular focus in the 2nd and 3rd years. The goal was to keep improving their implementation, as well as sequencing. Each path operates as a learning cycle (Simon, 1995) and the development of research constitutes itself as a cumulative cyclic process where interpretation of a trajectory provides added value for the planning, experience, reflection and interpretation of the following cycles. So intervention dimensions, iteration and orientation processes were valued. The introduction of new tasks was one of the important aspects in the iteration of cycles of experiences.

The tasks are of an open nature and provide collective discussions based on students' productions. They provide an easy adhesion because their understanding is very simple and students can reason and act in a personally meaningful manner (Gravemeijer & Cobb, 2006). Students worked first individually and the tasks were performed with geoboard or dotted paper as a visual thinking support.

The fieldwork was divided into two periods. The 2nd one is marked by a dual role of the researcher who assumed the leadership of phase 3. This change occurred because during the 1st period collective discussions were very poor or nonexistent. This change was possible due to the established relationship with the four teachers involved as well as with their students. In the second period, which occurred in two consecutive school years, all tasks have been tried with the same students from one of the classes in their

2nd (episode 1) and 3rd years (episode 2) of schooling. These lessons were recorded on video. Subsequently, we identified several relevant episodes of collective discussion as the unit of analysis.

EXAMPLES, COUNTEREXAMPLES, SPATIAL AND GEOMETRIC STRUCTURING

The following two episodes pretend to illustrate the role of examples constructed by students, as well as of counterexamples. They also illustrate various aspects of spatial and geometric structuring present in the tasks and the importance of giving them prominence in geometric figures as the work was going on.

Episode 1 – “Almost equal”

The proposed task consisted of identifying pairs of congruent figures. Two squares raised some controversy. The researcher asked the students if the two last copies were or not congruent (Figure 1). Almost all students said they were equal, although one student said they were not. The researcher asked one student,

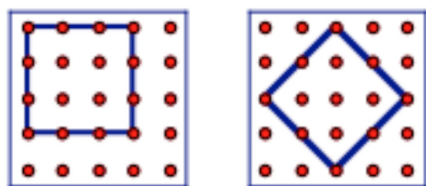


Figure 1: Squares that generated the controversy surrounding the congruence

Beatriz, to show her colleagues why they were equal.

As we expected this could happen, we prepared two acetates with figures and we gave them to Beatriz to experiment, corresponding to the practice of anticipating (Smith & Stein, 2011). She tried to overlap them, at the overhead projector, so they matched. Beatriz said nothing, but went around the acetates to get the match. During this process the other students observed. The researcher asked “who does still think that the two squares are equal?”. Several students were saying “they are not equal”. However, Beatriz remained stubbornly trying to match them. The researcher asked students to argue:

Ana: One of the squares is larger than the other.

Leonor: We can't put one above the other.

Joel: The number of dots inside the square is different. One square has 4 points and the other 5.

We have three different validations and the last one is more elaborate, denoting a more refined visualization. Leonor and Ana still seem to think based on the figure as a whole, while Joel shows the ability to distinguish the underlying structure to the figure to conclude that the two squares can not be congruent. Should we follow the reasoning of this student and share it? What is the contribution of this reasoning to the geometric structuring? This tension between taking advantage of good individual contributions and deciding the orientation of the collective movement was present in several episodes. It is the basis of significant dilemmas that teachers face during practice classroom (Carter & Richards, 1999; Wood & Turner-Vorbek, 1999) and acting as a filter of students' contributions (Sherin, 2002).

Episode 2 – “The rectangle that is not rectangle”

This second episode occurred later. It takes place in another pathway in which quadrilaterals have been worked as composite figures, highlighting its components and seeking to establish simple relationships between these elements, namely the angles. The pathway consisted of six tasks that allowed four moments of collective discussion. This episode occurred at the end of the second task and the discussion was about the students' work conducted in tasks 1 and 2. The task was to find out as much of squares and rectangles on geoboard. Students' works were exhibited with the solutions they found for squares and rectangles.

In the first part of the discussion the researcher and the teacher tried to interest students on the discussion they intended to create, as the appropriation by the students of social rules of whole-group discussion is a slow process and built over several experiments (Wood & Turner-Vorbeck, 1999). The discussion began with an appeal to respect different opinions, highlighting the value of students' opinions, with attention and care to not repeat what others have said which is very common among small children.

The researcher asked the students if they had any different figure, squares or rectangles. Two students, Inês and Beatriz, raised their fingers. They went to the black board to expose their magnified figures. This preparation of figures occurred during *phase 2*, corresponding to the practice of selecting, according to

Smith and Stein (2011), as we already knew that these figures would be important for the discussion. Those students thought that they had made a rectangle, however, when they showed it to their colleagues these at once identified it as a parallelogram.

Two students in chorus:

It is a parallelogram.

The researcher asked students to remain calm and asked Inês to justify why she thought her figure was a rectangle. The episode evolved with a long dialogue with many voices involving several students trying to show why Inês' parallelogram was not a rectangle.

Hugo: It is so because it has two beaks on one side.

Hugo shows with his fingers and surrounds the parallelogram. He points to two opposite sides and says they were inclined compared to the rectangle he considers not to be inclined. Hugo must reinforce the comparison with another figure, a rectangle in prototypical position “on high”, identifying important elements for this decision. Given his difficulty in justifying his reasoning, the researcher decided to ask another student.

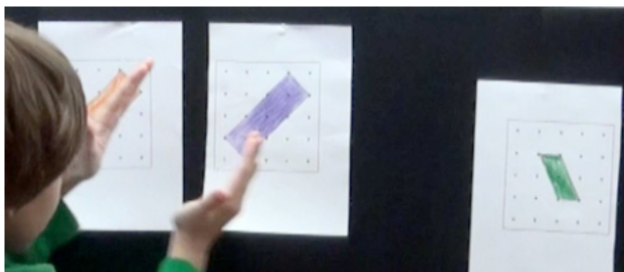


Figure 2: The controversial parallelogram and the rectangle Duarte used to compare

Duarte: It is not a rectangle because it is wry. Instead of being like this, is like this.

Duarte accompanied what he was saying with gestures with his hands (Figure 2). First he made with the hands two parallel segments, $||$, then he made two inclined parallel threads, $//$, and surrounded the parallelogram with his fingers. Moreover, he was able to do with his hands the necessary modification to transform the parallelogram into a rectangle, the angle of two consecutive sides should be a right angle. He compares with another rectangle exposed in a non-prototypical position, that is, “tilted”. Although Duarte showed that

he had very clear ideas about geometric justifications, he was not able to fully verbalize them.

All this discussion focused on the observation and analysis of quadrilaterals, showed the need to work more aspects of spatial structuring of quadrilaterals and the need to give more attention to spatial structuring, required for geometric structuring of quadrilaterals (Battista, 2008). It also emphasizes the requirement to pay attention to the elements that compose a figure, in this case the angles and sides. It also identified difficulties in verbalizing the reasoning, revealing that in most cases the mental images of students are correct and appropriate to their arguments.

This episode revealed the need to enhance students' language and to provide them with a simple tool of visualization to highlight and compare angles. This object, which we call “right angles' detector”, is not more than a corner of an A4 sheet cut so as to be easily manipulated and had already been widely used in previous experiments of these tasks. This episode marked guidance route for the sequence of this didactical pathway, which focused mainly on the spatial structuring. The following tasks were directed to discovering and comparing angles in quadrilaterals. Thus highlights the focus on the right angle as a reference for spatial and geometric structuring, without even resorting to any measurement system to identify acute, obtuse and right angles in plane geometric figures.

Both described episodes, as well as many others experienced, illustrate how students gradually assumed responsibility for discovering examples and counterexamples, incorrect or not complying figures with the established conditions, families of quadrilaterals in hierarchical classification. This was found successful to give students leadership in validation of mathematical knowledge in spatial and geometric structuring tasks.

DISCUSSION

We follow two lines of discussion, the first about geometry and the other on social constructivist aspects of learning.

We consider that the data collected value the potential of the structuring model of Battista (2007, 2008), and help us to better understand spatial and geometric structuring, as well as the relation among them. We

emphasize the importance of visualization in spatial structuring of quadrilaterals and the need to find strategies and objects to support visualization, for example, dot papers, colouring the angles, the “right angles detector”. We saw the importance of structuring quadrilaterals as composed by four angles figures, especially the rectangle as a four right angles composite figure, (Battista, 2008), aimed to structure each of the various types of quadrilaterals in hierarchical classification (de Villiers, 1994). We emphasize the need to incorporate several spatial structuring models to obtain personal’s mental models that determines her or his own way of thinking (Battista, 2008).

With regard to the geometrical structure, it seems to give new contributions for the classification of angles embedded in composite figures (quadrilaterals) and for an understanding of the geometric structure of quadrilateral on several levels (isolated figure, as part of a class, class relation). The examples we worked on showed us it is possible to implement meaningful tasks for young children, demanding in terms of geometric structuring (congruence of figures, understanding of a figure as part of a class, the class understanding, organization of quadrilaterals in hierarchical classes) from didactical pathways. Applying these pathways, with special attention to the collective moments of whole-group discussions proved to be a key factor in design tasks and for taking decisions for tasks sequence.

Second part of discussion is about social constructivist aspects of learning. The data collected makes us aware of the challenges associated with a practice focused on student work when you want them to become providers of mathematical knowledge (Wood & Turner-Vorbeck, 1999), and seeks a balance between

the mathematical value of the work of each student and ensuring that this work is in the mathematical point of view recognized and accepted (Cobb et al., 1992; Yackel & Cobb, 1996). We are in the process of establishing indicators and frameworks based on videotaped episodes. We are trying to construct a framework on learning environment, roles and responsibilities for learning in whole-group discussions from inquiry tasks (Figure 3).

Comparing columns A and B (Figure 3), we face the relation between teacher and students. Teacher action is more like a double mirror reflecting but also allowing seeing through it. As Wood and Turner-Vorbeck (1999) says this is a matter of ways of structuring social interaction and discourse to create contexts for learners’ personal construction of meaning.

The parallel that may exist between both columns and the approximation of the identified practices, highlight the student’s responsibility in the construction of mathematical knowledge. It seems therefore that this work getting closer forms of mathematical knowledge for each individual student, shared mathematical practices of the community classroom and shared mathematical practices of wider society (Cobb et al., 1992).

The lived episodes help us to understand the requirement of the teacher role at moments of whole group discussions. This requirement is expressed in simultaneous attention needed to give individual student’s contributions and the collective movement that teacher has to provide so the whole class advances, ensuring that the ideas and processes at play are widely accepted as having mathematical value and are necessary for future school mathematics learn-

| | Teacher responsibility (A) | Students responsibility (B) | |
|---|--|---|--|
| Taken-as-shared Mathematical practices of the community class- room (Cobb et al., 1992) | <u>Discursive practices</u> Ask question | <u>Discursive practices</u> Validation of own or colleagues knowledge | Responsibility for think- ing (Wood & Turner- Vorbeck, 1999) (Smith & Stein, 2011) |
| | Reflective practices Decisions on the progress of the discussion | Reflective practices (Emerging) | |
| Collective discussion management (Oliveira, Menezes, & Canavarro, 2013) | Interactive practices Student participation management | Interactive practices Engagement in discussion | Responsibility for partic- ipation (Wood & Turner- Vorbeck, 1999) |
| | Reflective practices (Underlie teacher decisions on students’ participation) | Reflective practices (Underlie initiatives and actions of students) | |

Figure 3: Framework for whole-group discussions from inquiry tasks (in construction)

ing. It also illustrates the critical importance of collective moments to construct a challenging learning environment for students, giving them autonomy and authority in the validation of ideas and mathematical knowledge, their own and those of colleagues. Regarding the teacher's role the indicators of orchestration practices are relevant (Smith & Stein, 2011), questions with potential (Boaler & Brodie, 2004) and dilemmas (Carter & Richards, 1999; Wood & Turner-Vorbeck, 1999).

As a final idea for this discussion, we return to the initial purpose of this research to recall the importance this experience has taken to help design and evaluate teaching pathways in geometry, showing as one of the critical aspects of these pathways the links that can be established between the tasks that integrate them.

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