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Derek Pilous, David Janda

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Prototypes in secondary and university mathematical education

Derek Pilous and David Janda

Charles University, Faculty of Education, Department of Mathematics and Mathematical Education, Prague, derek.pilous@seznam.cz

The aim of presentation was to demonstrate our extension of previous theoretical framework of prototypes – a term of cognitive psychology – in mathematics education, illustrated by examples of students' outcomes and sets out a research project based on this framework. The study was supported by Charles University in Prague, project GA UK No 227-364.

Keywords: Prototype, function, example, mathematical concept, classification.

THE THEORY OF PROTOTYPES

The theoretical framework of the concept of prototypes lying in cognitive psychology is presented in two ways. First, traditionally, by a description of historical evolution from first research studies carried out by Eleanor Rosch in the 70'th and introduction

of prototypes to mathematics education by Rina Hershkowitz and others in the 80'th of the twentieth century. Second, we will demonstrate inherent usage of prototypes in mental representation of mathematical, well-defined concepts through introspection of poster readers.

Schwarz and Hershkowitz (1999) describe prototypes as construct of probabilistic approach to concept learning – “special examples, that are more central to learning than others”. Hadjidemetriou and Williams (2010) offer strong example, linearity, ascribing it to schooling. Our goal is to define and identify prototypes of mathematical concepts and also observe long-term development of prototypes. In this poster, we concentrate on two questions:

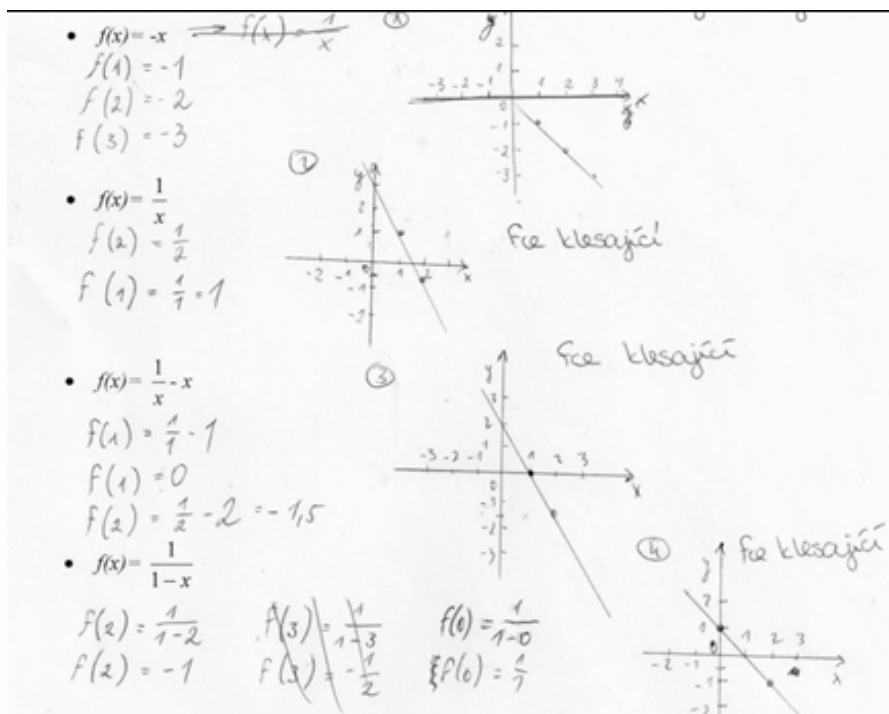


Figure 1: A student's solution

- 1) How to characterize prototypes of mathematical concepts in a way which will provide criteria for their detection?
- 2) Which prototypes of common important mathematical concepts like functions and real numbers do students use?

In the theoretical part of poster we pose two conditions for useful definition of prototypes in context of mathematical education and present our working definition: *A prototype is any instance of concept, a set of instances or any property functionally connected to concept, which is statistically preferred by an individual in usage of that concept, if this preference cannot be explained by a mathematical essence of that concept and informational complexity of the would-be prototype.*

Theoretical framework is followed by examples of prototypes from our observations of students' prototypes in ordinary lessons of mathematics and from our research not specifically focused on prototypes (Janda, 2013). One such example is given in Figure 1. A group of 40 secondary school students solved the problem "How does $f(x)$ change if we replace x by various integers? Describe or represent it." for four functions.

As we can see, substituting two or three numbers for x satisfied the student enough to draw supposed graphs of functions. Numbers 1 and 2 were the most often used (hence, we consider small positive integers to be prototypes of real numbers), number 0 was used only once, negative numbers were not used at all (only 14% of the students inserted at least one negative number into any of the rules).

Figure 1 is an example of the tendency to "linearize" graphs of functions which was generally very strong among students; in Figure 1, we can even see that the student omitted the result of the evaluation for $x = 3$ (by crossing out the evaluation itself and the corresponding point in the Cartesian plane) because it did not conform with the linear function passing through points which resulted from previous evaluations.

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