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How much space for communication is there for a low achieving student in a heterogeneous group?

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This paper reports on a case study aiming to deepen our understanding of low achieving students' learning of algebra, in particular when they work with pattern problems. We observed one low achieving student, May, who participated and worked in three different heterogeneous settings. Data were analysed from a multimodal perspective on key and regulating activities in the groups. The analysis revealed that May's contribution varied, depending on the composition of the groups, and that her contributions were influenced by regulating activities by peers and access to physical artefacts. The findings show that a low achieving student is able to generalise beyond her arithmetic knowledge, but the environment in a heterogeneous group did not offer her the space to do so.

Keywords: Regulating activities, low achieving, pattern problems, heterogeneous groups.

INTRODUCTION

Pattern problems have been studied extensively in classroom situations, particularly in group work (Radford, 2009; Ryve, Larsson, & Nilsson, 2013). Pattern problems can be tackled using different strategies, including arithmetic strategies (counting) and grouping strategies, before advancing to generalised explicit formulae (Lannin, Barker, & Townsend, 2006). Radford (2012) has explained that generalising is central in algebraic thinking, not symbolising. With pattern problems, students can reason informally, yet algebraically, they can even generalise their answers, without immediate need to symbolise.

To supplement the studies of students working on pattern problems, we are interested in how to facilitate low achieving students' learning of algebra within het-

erogeneous groups. We define low achieving students as students following regular mathematics lessons, but achieving low on tests compared to their peers. Many studies have demonstrated that these students are able to reason algebraically, but their potential is often not captured by standard tests (Karsenty, Arcavi, & Hadas, 2007; Watson, 2002).

The rationale for our study of low achieving students within heterogeneous groups is that in Norway, the official policy is an inclusive school system: it is not allowed to teach in fixed ability groups over time (The Educational Act, 1998). Research on learning in group settings has identified that heterogeneous groups are beneficial to students' learning and invite all students to become active participants (Dekker & Elshout-Mohr, 1998; Webb, 1991).

In the current paper we present a case study of May, who is considered as a low achieving student by her teacher and based on test results. We observed her work with pattern problems in three different groups.

THEORETICAL FRAMEWORK

Algebra learning is defined to involve both symbolic expressions and reasoning about generalisation (Caspi & Sfard, 2011; Måsøval, 2011). In our study, we focus on the algebraic reasoning and generalising, and to a lesser extent on symbolising. Strømskag (2015) emphasises the importance of natural language as a basis for symbolic expressions which might be even more crucial for low achieving students. According to Karsenty and colleagues (2007), low achieving students do have a potential to think algebraically, yet they are not always capable of working with symbols. Therefore, low achieving students' algebraic poten-

tial might be more visible in natural language than in symbolic expressions.

A relation between students' communication and algebraic thinking is emphasised by Caspi and Sfard (2011) when they define algebra as a discourse. This definition is promising for researching low achieving students' algebra learning because it "transfers algebra from the category of passive tools to that of human activities" (p. 470). In their study, students' communication is analysed according to verbal interaction and written products, not focusing on gestures. However, research on low achieving students has identified gestures to be crucial for students' active participation in meaning making processes (Simensen, Fuglestad, & Vos, 2014).

Radford (2009) demonstrates how students' written products from generalisation processes can be traced back to their use of speech, gestures and actions (such as using physical artefacts) during the solution process. He also argues that gestures in isolation do not tell much; it is when all three actions work together that observing communication can give insight into students' meaning making. Radford refers to activities where students combine speech, gestures, and actions as *multimodal activities* (2009, p. 120). Multimodal activities are not only local products of the situation in which they take place; they have roots in previous situations and contribute to future communication (Civil & Planas, 2004). Therefore, facilitating all students to become active participants is not only about an individual's empowerment to become an active contributor by offering appropriate tasks and physical artefacts; it is also about previous communication, and mutual encouragement of communication.

Dekker and Elshout-Mohr (1998) have defined two categories that can be used to analyse multimodal activities: *regulating activities* and *key activities*. Regulating activities are activities that encourage students to communicate about their work, namely by questioning or criticising it, for instance: "*What are you doing? Why are you doing that?*" (Pijls, Dekker, & van Hout-Wolters, 2007, p. 312). Regulating activities can be practiced by other students, the teacher, or by the student herself.

Regulating activities do not only encourage communication, they can also be used to define students' *key activities*. Four key activities have been identified as

indicators for level raising in mathematics (Dekker & Elshout-Mohr, 1998) and they can be observed in students' communication. The four key activities are: showing one's work; explaining one's work; justifying one's work; and, reconstructing one's work.

Pijls and colleagues (2007) have carefully demonstrated how to classify utterances according to regulating and key activities. They emphasise particularly the key activities' dependence on regulating activities, because the regulating activities are crucial to categorise key activities. For instance, Pijls and colleagues (2007) claim that it can be difficult to decide whether a student is showing or justifying her work. The decision should then be based on the regulating activity that initiated the actual key activity. For example, justifying one's work is always a response to critique, while showing is not (p. 313). For further explanations of regulating and key activities, we refer to work by Dekker and Elshout-Mohr (1998) and Pijls and colleagues (2007). Heterogeneous groups in which all four key activities were identified showed to be the most beneficial for attaining mathematical level raising (Pijls et al., 2007). Nevertheless, students who demonstrate fewer key activities, like showing and justifying, can attain a certain degree of level raising. We build on these studies, but extend the analysis beyond utterances, and also analyse other multimodal activities.

Despite the large amount of research concerning algebraic thinking and communication, not much of it relates to low achieving students' communication and their algebraic learning potential. For this reason, we want to investigate: *How do regulating activities encourage or hinder a low achieving student's communication about pattern problems in heterogeneous groups?*

METHODS

To answer our research question, we designed tasks on hexagon patterns in such a way that the questions should be easy to understand and such that they could be solved by multiple methods. Motivated by Radford's (2009) claim that students' use of gestures and artefacts is crucial for communication on pattern problems, we made hexagon tiles and sheets with hexagon patterns available for the students. We assumed that these factors (questions that were easy to understand, multiple methods for solving, availability of physical artefacts) would invite all students to become

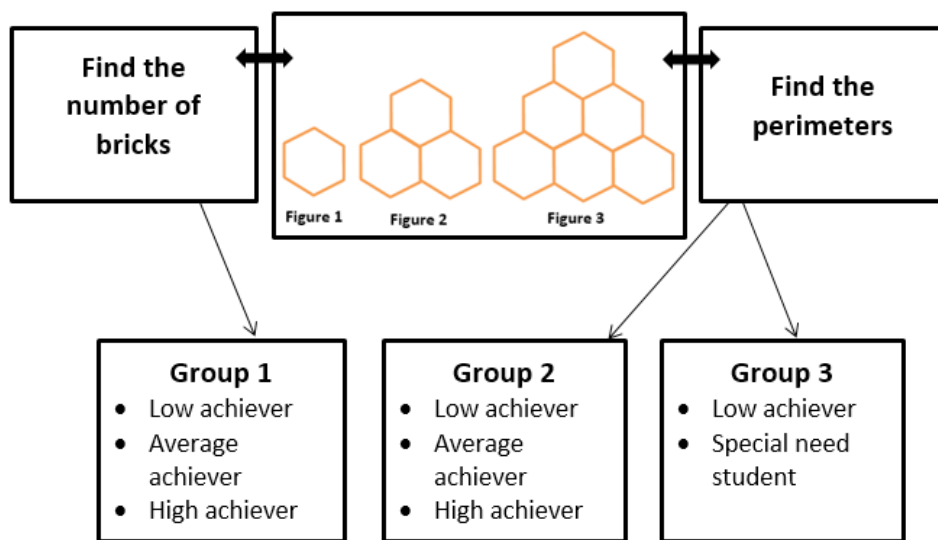


Figure 1: Tasks and organisation of heterogeneous groups

active participants. Based on these problems, we carried out a task-based intervention on three different days during one week in a Grade 8 class.

For this case study, we will focus on one student, May, who performed low on a National Test for Grade 8. Before the intervention, we had communicated with the teacher, and created a group in which we assumed that May would become an active participant: May (low achieving), Tom (average achieving) and Bo (high achieving). But Bo was sick when our intervention started and we had to reorganise the groups, replacing Bo with Eva (high achieving). After that first session, Bo was back and May could work with Bo and Tom in the ensuing session. This accidental change in our study enabled us to observe May in two heterogeneous settings, which turned out to be very different. Because of the different outcomes, we became interested in learning more about what encouraged or hindered May's communication in heterogeneous settings. Therefore, we carried out an additional, third session in which we asked May to work together with Siv. Siv is not only May's friend, but she has special educational needs as well. We assumed that these two factors, working with a friend and being the most knowledgeable, might empower May to take an active role in the work. A third choice we made to strengthen May's role as the most knowledgeable peer, was to give May and Siv a task with which May was familiar from the work in Group 2 (see Figure 1).

All the sessions were video recorded and transcribed. We then counted the number of utterances made by May in each group, and this gave us a quantification of

her participation in the communication between the group members. However, counting the number of utterances does not give insight into the nature of utterances nor the chain of activities that the utterances are part of. We therefore analysed the communication according to key and regulating activities, as developed by Dekker and Elshout-Mohr (1998). In this analysis, the four key activities can be identified independently, one activity is not dependent on others, and they are incremental in the sense that we can see attempts to carry out key activities, partly or strongly. Because of this, we consider key and regulating activities as suitable to identify both students' actual level raising and their potential level raising. In previous studies (Dekker & Elshout-Mohr, 1998; Pijls et al., 2007), key activities and regulating activities are only observed in students' speech, and not in their gestures and actions. However, we decided to analyse communication according to key and regulating activities from a multimodal perspective, following Radford (2009).

RESULTS

We carried out a frequency count of May's utterances in the three groups. In the first group she hardly said a word (3 % of the group's utterances are from May), in the second group her contribution raised to 16% of the groups' utterances, while in the third group she contributed with 53% of the utterances. Below we will report exemplar episodes from the three groups.

Group 1

On the first day of the intervention, May sat with Tom and Eva. They discussed pattern problems on finding



Figure 2: Organisation of the three groups

the number of hexagon tiles required to build various figures. In their discussions, Tom and Eva agreed about how to find this number, reasoning from a recursive approach, namely by adding the next figure number to the number of tiles in the present figure. May said little and her utterances were in general short, consisting of one or two words.

Tom asked May several times if she understood how he and Eva had found the number of hexagon tiles in the figures without counting. Every time May nodded, but except for the nodding, she did not move her arms or body. As can be observed in Figure 2 she was sitting further away from the other group participants, as if she was not part of the group (see Figure 2). There were a few moments, when May's nodding was followed by Tom asking if she really understood. This could indicate that he did not accept May's nodding as sufficient confirmation that she understood.

Eventually, Tom asked May to explain it to him:

- 122 Tom: Ok, explain it for me. I need an explanation. [Looks at May and moves the hexagons in her direction]
- 123 Eva: You know the explanation. [Looks at Tom and gathers the hexagons]
- 124 Tom: Yes, but I want her to understand. [Points to May]
- 125 Eva: To get the next figure number, you have to add the next figure number. [Looks at Tom]

In utterance 122 Tom extended his attempt to invite May to contribute. Here he asked her explicitly to explain her understanding and he offered her artefacts (hexagon tiles) that could have supported her explanation. However, the invitation was not answered by May but by Eva, both verbally and physically (123). May did not give any explanation.

We interpret Tom's question about May's understanding to be a regulating activity, aiming at inviting May to contribute. However, we do not interpret the nodding to be a key activity, because we cannot learn about May's ideas from the nodding. In utterance 122, Tom makes a verbal regulating activity, and he strengthens it by physically offering the hexagon tiles to May. Eva's subsequent contribution (123) can be interpreted as a barrier to May's acceptance of the invitation. Eva is hindering at two levels: by taking the hexagons physically and by verbally answering. We therefore consider both contributions from Tom as examples of regulating activities, asking May to explain. However, they do not result in May using key activities.

Group 2

On the second day of the intervention the groups were reorganised and May's group was joined by Bo (high achieving) instead of Eva. Therefore, Group 2 consisted of May, Tom, and Bo. Their work was still related to the hexagon patterns, but now the question was to find the perimeter of the figures.

The discussion started similarly to what happened on the previous day, with Bo asking May if she understood. She responded in exactly the same way as she did in Group 1, by nodding. However, Bo did not repeat the question like Tom did in Group 1. Bo rephrased it into "May, do you want to count how many sides the second figure has?", at the same time as he pointed at the illustrations on the worksheet. Later on Bo initiated an invitation to May to give an explanation:

- 691 Bo: Why is it multiplied by 6?
- 692 May: Because it is 6 more each time.
- ...
- 722 May: You take the number of sides on one brick. [Teacher nods]
- 723 May: And multiply by the figure number.

...

738 Bo: Instead of writing it in words, can you write it like you would have written it in your notebook? What do you write if you want to do a calculation?

739 May: N multiplied by 6.

The excerpt shows how Bo acted like a teacher, guiding May to make explanations without using artefacts. In utterance 691 he asked May to justify *why* the perimeters can be found by multiplying by six. In utterance 692 May indicates that this is about the multiplication table of six continuing by adding 6. Further, in utterances 722 and 723 May explained in words how to find the number of tiles in a random figure, and in utterance 739 she showed how it can be generalised with symbols.

We interpret Bo's explicit questions at the beginning of the discussion in Group 2 as regulating activities, inviting May to contribute with speech, gestures, and use of artefacts. May shows her ideas by pointing at the physical artefacts and we interpret it as key activities demonstrated with multimodal expressions. In the episodes given above, we identified key activities such as: justify (692), explain (722 and 723), and show (739) one's ideas.

To sum up, in the heterogeneous setting of Group 2 we first observed how Bo asked May to explain her understanding, which did not evoke May's use of key activities. When he asked her explicitly to show her ideas ("do you want to count?"), this was followed by May demonstrating the key activities of showing her ideas. Finally, when Bo asked May again to explain her ideas (691), she demonstrated the key activity explanation by offering the generalised formula "N multiplied by 6".

Group 3

The third setting in which May participated was different from the previous ones in several ways: (1) only two students were involved, Siv and May; (2) Siv is identified to be in need of special education and, therefore, May was the relatively high achieving student in this setting; (3) the students were given the task from Group 2, so May was knowledgeable about the task, while Siv was not.

First May demonstrated that the perimeter could be found by counting the physical sides one by one. We

interpret this as the key activity 'showing one's idea'. May then carried out a regulating activity by asking Siv to count the sides. Thereafter May initiated other strategies to find the perimeters:

51 May: Do you know what we can do instead of counting one by one?

52 Siv: Hm?

53 May: We can, or it is fine to just count. We only have to add these. 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37. [Points on the "new" sides in Figure 5] Then we know that it is 37 sides around. Do you see a pattern?

54 Siv: Where?

55 May: Here. [Points on the diagram in the task]

56 Siv: No.

57 May: It is the times six table. 6, 12, 18, 23, 37. For example: 6 plus 6 equals 12. It is 6 twice. If we have 6 three times, then we have 18. 12, 13, 14, 15, 16, 17, 18. [Counts on her fingers]

In the above episode (53) May suggested that the perimeter can be found by counting on from the previous figure's perimeter, which is a recursive strategy. Thereafter, she claimed that the perimeters are equal to numbers in the "times six table" (57), which is an explicit formula for the perimeter of the triangular hexagon pattern.

We interpret the utterance "Do you know what we can do instead of counting..." (51) as a regulating activity. This regulating activity is followed by May showing her idea (53). Within that very same utterance (53), we also observe a regulating activity: asking for a pattern. Finally, when May showed her idea (57), she demonstrated another key activity. In utterances 53 and 57 she scrutinises the counting process, which we interpret as searching for generalisation. She did not detect that the numbers she found were incorrect, which illustrates how her willingness and potential to generalise goes beyond her skills in mathematics.

May's contributions in Group 3 differed from the other groups, because her contributions did not depend much on others' regulating activities directed at her. May's contributions in Group 3 demonstrated both regulating activities and key activities. In Group 3 we observe that May's contributions relate little to

the other participant (Siv gives little feedback), and more to the nature of the tasks, use of gestures, and access to physical artefacts.

DISCUSSION AND CONCLUSION

The empirical material presented in this paper shows how differently a low achieving student can contribute in various heterogeneous settings. When working with higher achieving students, May's communication was strongly affected by the regulating activities from the other students. She was mostly awaiting and only answering when others invited her to contribute. In contrast, when working with a special needs student, May was the leader; she was the one initiating and regulating the communication. Our findings are consistent with work by Civil and Planas (2004), and it indicates that being categorised as a low achieving student might position a student into a similar position as students who are marginalised because of social inequality. Therefore, we assume that regulating activities are crucial for the lowest achieving student's contribution in heterogeneous groups.

Although May's contributions in both Group 1 and Group 2 were strongly related to the regulating activities by the other students, her contributions in these two groups were different. In the first group we observed how regulating activities can fail to encourage low achieving students' contribution. Tom asked May several times about her ideas but May just nodded and did not answer in words. Pijls and colleagues (2007) reported similar observations and claim that two main factors are crucial for students' use of key activities. First, the use of key activities depends on whether someone asks you about your work. Second, students should think their contribution makes sense to other students. In our study we observed that there was no communication space for May's response because the regulating activities by Tom were blocked by Eva.

Further, in Group 2 we observed how Bo's attempts to encourage May to carry out key activities, did not become successful until May had access to artefacts. This suggests that access to artefacts might be another important factor for low achieving students to be able to carry out key activities. Therefore, our findings indicate that regulating activities can encourage low achieving students' communication about pattern problems, but it is crucial that they have access to combine speech, gestures, and use of artefacts in or-

der to communicate their ideas. A new finding is that regulating activities can also hinder low achieving students' active participation in communication. We observed how artefacts were moved away, and speech blocked the regulating activities that were originally aiming to invite for contributions. This is in line with Radford (2009) who advocates that thinking happens not only as a mental process, but in and through the use of speech, gestures, and actions (like use of artefacts). In our study we observed that these multimodal expressions were important for May in showing her ideas, specifically when she used counting (relying on arithmetic) to communicate about the pattern problem. Later on, when she demonstrated more abstract strategies like recursive reasoning and how to express the problem explicitly in a formula, in Group 2, she did not use gestures and artefacts. This may indicate a development: in the first session gestures and artefacts were important for the key activities, and May needed them to express ideas that were not yet completely her own. But at the end of the session in Group 2 she expressed her ideas verbally without use of gestures and artefacts, which was possible because Bo encouraged her. In Group 3 she could confidently claim that the pattern of the perimeters is equal to the multiplication table of six. This indication of development needs more research addressing low achieving students' use of multimodal expressions when generalising beyond their arithmetic knowledge.

Our results show the complexity of how to empower low achieving students' in order to become active contributors in heterogeneous groups. We observed in Group 2 that a regulating activity for a low achieving student can be an explicit invitation to combine speech, gestures, and artefacts for showing one's ideas. The key activity of 'showing one's ideas' is a prerequisite which enables low achieving students to demonstrate other key activities. Therefore, teachers can empower the students to use key activities by giving access to artefacts and by designing tasks inviting all students to show their ideas. While further work is required to explore low achieving students' learning and their ability to generalise, our findings also indicate that further research is needed on low achieving students in heterogeneous settings where the low achieving student is relatively high achieving, for instance when they work with younger students.

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