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The development of student's early number sense

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The present study adopted a theoretical model, suggesting that number sense consists of elementary number sense, conventional arithmetic and algebraic arithmetic, to trace the development of students' early number sense. Two hundred and four 1st grade students were individually tested on five time-points. Data analysis suggested that number sense follows a linear growth rate and six groups of students were identified that follow different growth patterns. Two of the groups had a low initial value, two had a moderate initial value and two had high initial value. Systematically, one of the two groups with the same initial value exhibited a greater growth rate. The gap of the groups with same initial value in algebraic arithmetic progressively increased across the five time-points of the study.

Keywords: Early number sense, arithmetic, algebraic arithmetic.

INTRODUCTION

The development of students' number sense is considered as an important outcome and key ingredient of school curricula and a foundation for developing formal mathematical concepts and skills in elementary school (Yang, 2005). Research findings support that number sense is a powerful predictor of mathematics outcomes and a vital prerequisite to success in mathematics (Malofeeva, Day, Saco, Young, & Ciancio, 2004). Recent longitudinal studies documented that number sense skills developed in pre-K and kindergarten are not only foundational, but also correlate with first grade mathematics achievement (Jordan et al., 2006). It was found that the mastery of counting principles correlates significantly with arithmetic abilities. Moreover, Jordan, Glutting, Ramineni and Watkins (2010) showed that core number competences in kindergarten have strong predictive validity on

mathematics computation and problem solving even at third grade.

The present study capitalizes on previous studies asserting that early number sense consists of three distinct, but interrelated components (Pittalis, Pitta-Pantazi, & Christou, 2013, 2014). Our adopted model hypothesizes that student's early number sense consists of (a) elementary number sense, (b) conventional arithmetic and (c) algebraic arithmetic. The innovative aspect of this model lies on the inclusion of algebraic arithmetic as a component of number sense and builds on the theoretical premise that the artificial separation of arithmetic and algebra deprives students of powerful ways of thinking about mathematics in early grades and makes it more difficult for them to learn algebra in later grades. Algebraic arithmetic adopts Drijvers and his colleagues (2011) description of algebra as a synthesis of (a) patterns and formulas, (b) restrictions and (c) functions. This conceptualization of algebraic arithmetic leads to a broader, more dynamic and flexible construct that underlies the importance of grasping the relations among numbers and manipulating numbers and number relations with flexibility.

In this study, we extend the work presented in PME 38 and present the results of a longitudinal study that traced student's number sense in grade 1 and at the beginning of grade 2. In particular the aims of the study were to: (a) to trace the development of six year old students' number sense and (b) to identify different growth patterns.

LITERATURE REVIEW

Number sense is one of the most important concepts to be developed in early mathematics (Baroody et al., 2009; McGuire & Wiggins, 2009). Children's number sense is a key predictor of later mathematical success,

both in the short (Aunio & Niemivirta, 2010) and the longer term. For instance, research findings suggest that early number sense development contributes in learning more complex mathematics concepts, it promotes numerical fluency and it is foundational to all aspects of early mathematical skills (Baroody et al., 2009; Jordan et al., 2010). Inadequate development of number sense in early grades may be related to mathematics learning difficulties. In addition, it is widely supported that students who enter school with strong number sense are more likely to benefit from mathematics teaching in the elementary grades and that the effect of weak number sense may be cumulative (Jordan et al., 2010).

Number sense develops gradually and matures with experience and knowledge (Reys & Yang, 1998). It develops as a result of exploring numbers in a variety of contexts and relating them in ways not limited to traditional algorithms (Sood & Jitendra, 2007). Moreover, number sense is highly personal and is a by-product of teaching for understanding. Longitudinal studies provided evidence regarding the growth of students' early number sense in kindergarten and in elementary early grades. For instance, Jordan and her colleagues (2010) traced empirically three growth trajectories: (a) children who started with low number competence and stayed low, (b) children who started with high number competence and stayed high, and (c) those who started with low number sense but made relatively good growth. A growing body of research with converging data (Gersten et al., 2005; Jordan et al., 2010) assert that the identification of growth groups is extremely important to trace students that need early intervention.

A number of researchers proposed that number sense is difficult to define, while different research studies identified different conceptual understandings and components of number sense (Malofeeva, Day, Saco, Young, & Ciancio, 2004). A well-accepted and broad definition of number sense refers to a coherent understanding of what numbers mean, numerical relationships and the ability to handle daily life situations which involve numbers (Yang, 2005). In the present study we define early number sense as a synthesis of elementary number sense, conventional arithmetic and algebraic arithmetic (see Pittalis, Pitta-Pantazi, & Christou, 2013, 2014). Elementary number sense refers to key elements of numbers sense. It consists of counting, number identification, number knowledge,

quantity discrimination, enumeration and non-verbal calculations (Jordan, et al., 2006). Counting relates to grasping one to one correspondence, knowing the count sequence and mastering counting principles, such as stable order and cardinality (Jordan, et al., 2006). Number knowledge refers to discriminating and coordinating quantities and making numerical magnitude comparisons. Enumeration relates to the ability of enumerating sets, irrespectively of the orientation of the objects. Number identification refers to the ability of naming written symbols. Non-verbal calculations refer to the ability of manipulating non-verbal addition-subtraction situations with objects, while quantity discrimination relates to the ability of recognizing the relative magnitude size (Reys, & Yang, 1998). Conventional arithmetic includes students' ability to calculate number combinations, conceptualize the effect of operations (Yang, 2005) and solve story problems. Number combinations relates to students' ability of executing calculations with no object referents (Jordan et al., 2007). Understanding the effect of operations involves students' ability to conceptualize, judge and interpret the meaning of operations. Story problems conceptualize the ability to solve simple word problems, without using manipulative.

Algebraic arithmetic refers to the dimension of early number sense that moves beyond conventional arithmetic. It encompasses the development of a more sustainable and abstract understanding of the relations among numbers. Thus, the inclusion of algebraic arithmetic as a component of number sense could be considered as a move from particular numbers and measures towards relations among sets of numbers and measures. It can be conceived as a generalized arithmetic of numbers and quantities in which the concept of function assumes a major role. The inclusion of algebraic arithmetic as a factor of number sense results in reconceptualising the conception of number sense. The proposed nature of number sense defines a more dynamic and flexible construct that could facilitate students' advancements and transition to a more abstract and relational system of thinking. In particular, algebraic arithmetic encompasses the general cognitive ability of relational thinking, since it focuses on relations than on calculating answers (Carpenter, Levi, Franke, & Zeringue, 2005). In particular, algebraic arithmetic refers to number patterns, functions and restrictions (equations and balance scale restrictions), as proposed by Drijvers and his colleagues (2011). The parameter of number

patterns involves searching for regularity and patterns to recognize a common algebraic structure. The dimension of restrictions describes students' ability in finding what value(s) of the unknown satisfies the required conditions in various situations; balance scale tasks or in more formal setting, such as equations. The function component involves students' ability to investigate arithmetic relations between quantities/variables.

METHODOLOGY

Subjects

The subjects of the study were 204 first grade students from 7 urban primary schools in Cyprus. The school sample is representative of a broad spectrum of socioeconomic background. Students were assessed on the number sense measures four times during the period October to June (approximately one administration per two months). A fifth measure took place at the beginning of the following year, when students attended grade 2. During each measurement students were interviewed in two sessions of approximately 30 minutes each. Students had a time restriction for each type of task (one minute for the majority of tasks). Students were individually tested in all five occasions. The order of the parts was rotated in the five time series.

Measures

The majority of the test items were adopted from the Curriculum Based Measurement (Fernstrom & Powell, 2007) and the rest ones were developed based on the theoretical considerations of the study. Six types of tasks were used to measure elementary number sense: (a) counting tasks, (b) number recognition, (c) quantity discrimination, (d) number knowledge, (e) enumeration, and (f) non-verbal calculation. In the counting tasks, students were asked to enumerate objects, in the number recognition tasks, students had to read numbers, in the quantity discrimination tasks students were asked to decide which was the largest number, in the number knowledge tasks, students were asked to find smaller and bigger numbers of a given one and in the non-verbal calculation tasks students had to add or delete objects in a given set so the number of objects corresponds to a given number. Conventional arithmetic was measured by three types of tasks: (a) story problems, (b) understanding of operations and (b) number combinations. In story problems tasks, students had to select the appropri-

ate number sentence for a list of story problems. In the understanding of operations tasks, students were presented with simple word problems and were asked to select out of 4 mathematical sentences, the one that fitted the problem. In number combinations tasks students had to find mentally the result of addition, subtraction, multiplication and division combinations. Finally, algebraic arithmetic was measured by four types of tasks: (a) number patterns, (b) restrictions-equations, (c) restrictions-balance scale and (d) functions. In number patterns tasks, students had to extend or complete number patterns, such as 5, 8, 11, ... The ability to solve number patterns implies that a student can conceptualize the relations among numbers to fill in or extend a number pattern. For the assessment of students' abilities in number restrictions two types of tasks were used, number equations and balance scales. In the number equations, students were asked to complete the missing terms of equations, such as $3+5=4+\square$. Solving number equations encompasses conceptualizing the equal sign as equity of two quantities and the relations among all its components. In addition, estimating the missing term by analysing the relations among the components of the equation, instead of executing calculations, indicates students' flexibility and it is less time consuming. The other restriction task appeared in the form of a balance scale. Students had to identify the value of two or three shapes which balanced in the balance scale with a given number. Finally, regarding student's abilities with functions, students were presented with function machines and a table which showed the input and output values. Students were requested to provide the input or output numbers which were missing. The task was an adaptation of a task presented by Drijvers and his colleagues (2011). Students had to identify the relation between the input and the output value, identify the effect of change and examine whether their hypothesized rule was valid in all cases.

Data Analysis

To trace the development of number sense we used latent growth models. Growth models examine the development of individuals on one or more outcome variables over time. In order to evaluate model fit, three widely accepted fit indices were computed (Muthén & Muthén, 2007): The chi-square to its degree of freedom ratio (χ^2/df should be <2); the comparative fit index (CFI should be $>.9$); and the root mean-square error of approximation (RMSEA should be close to or low-

er than .08). Latent class analysis was used to trace categories of students reflecting different developmental patterns in number sense; this is a statistical method for finding sub-types of related cases from multivariate data. To make the extraction of an average score for each dimension of number sense feasible, it was thought appropriate to set the highest score in each test across the five time-points as 1 and then make the necessary adaptations. That was necessary because all the tasks were equivalent and the time restriction did not allow the setting of an ultimate maximum score. The same procedure was followed for the extraction of an average number sense score.

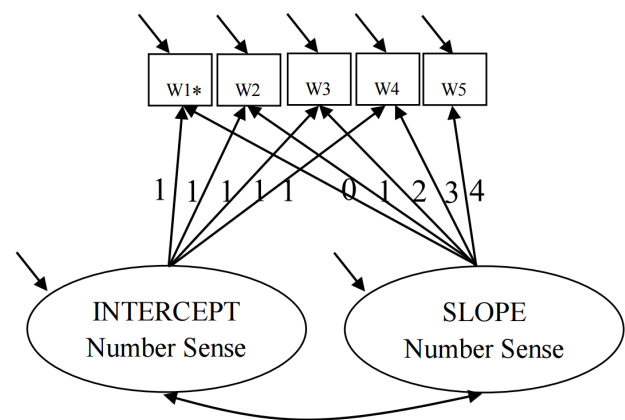
RESULTS

Student's early number sense development

To evaluate change in students' number sense components, the repeated measures longitudinal data were analysed using latent growth curve analysis. The latent intercept of number sense was defined by fixing all loadings at 1, whereas the loadings for the latent slope of number sense were set to the values of each time-point (i.e. t_1 =beginning of Year 1, t_2 =January, t_3 =March, t_4 =June, t_5 =beginning of Year 2). It was assumed that $t_1=0$, indicating that the intercept of each component can be interpreted as the individual initial value and the slope as the individual rate of change over time. The hypothesized latent growth model assumed that student's early number sense followed a linear growth rate across the five time-points. Alternative models hypothesizing (a) a quadratic trend in the slope and (b) a logarithmic trend were examined. Figure 1 presents the way in which the factors of the model relate to each other.

The results of the study showed that the hypothesized linear growth model had an excellent fit to the data ($\chi^2/df=1.94$, CFI=.99, RMSEA=.07). The fit of the two alternative models was extremely poor ($\chi^2/df>10$, CFI<.70, RMSEA>1). Thus, it was concluded that student's early number sense followed a linear growth rate. Two latent factors were estimated for number sense, the intercept (or initial value) of the latent number sense and the slope (or growth rate). Each contains a latent growth factor mean and variance, with the covariance being allowed between them. Both the mean intercept and mean slope of number sense were significantly greater than zero. The results of the model fit and significant slope mean and variance supported the proposed growth model regarding the linear develop-

ment of the parameters of number sense. The analysis showed that the mean value of student's number sense intercept was 0.20 and the mean value of the slope across the five time-points was 0.07. The $M_{N.S.}$ value of 0.07 can be interpreted as the increase in performance per unit of time (i.e., $0.07 \times 4 = 0.28$), which corresponds to the actual increase of the mean value of number sense from time-point 1 to time-point 5. The amounts of variation of the intercept factor and slope factor were significant larger than zero. This finding is extremely important because we could conclude that there is variability. The correlation between the intercept and the slope was positively strong ($r=.55$, $z>1.96$), indicating that students entering first grade with a strong number sense may exhibit greater growth rate.



* w corresponds to time wave

Figure 1: The hypothesized growth model

Tracing growth patterns

To trace categories of students that follow different growth behaviour, latent class analysis was used. To do so, we first examined whether students varied at entering grade 1. Thus, we applied a stepwise method to validate the model under the assumption that there were one, two, three, four or five categories of students at time 1 (first measurement). The best fitting model with the smallest AIC and BIC indices and the largest entropy was the one involving three categories of students. The mean value of the three categories were 0.13 ($n_1=62$), 0.20 ($n_2=99$) and 0.30 ($n_3=36$), respectively. Then, we repeated the same procedure for each category of students, using the data of the four remaining measurements. The results of the three latent class analysis showed that each of the three initial categories of students could be more accurately modelled by two sub-categories of students. Thus, in total we traced six groups of students that varied across the five time-points. Figure 2 presents the development rate of the six groups.

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Total
N.S.T_1	0.12	0.13	0.20	0.22	0.29	0.31	0.20
N.S.T_2	0.18	0.21	0.27	0.30	0.36	0.40	0.27
N.S.T_3	0.22	0.27	0.31	0.38	0.41	0.49	0.32
N.S.T_4	0.28	0.35	0.40	0.52	0.48	0.59	0.40
N.S.T_5	0.35	0.41	0.46	0.62	0.56	0.68	0.47

Table 1: Means of the six groups

Groups 1 ($n_{Gr.1}=32$) and 2 ($n_{Gr.2}=30$) represent the two subgroups of the first initial category. Groups 1 and 2 had almost the same initial value, but progressively Group 2 adopted a greater growth rate. Groups 3 ($n_{Gr.3}=70$) and 4 ($n_{Gr.4}=21$) represent students entering grade 1 with moderate initial value. However, Group 4 exhibited a remarkable growth rate, overtaking after the third measurement a group that had a greater initial value (Group 5). Groups 5 ($n_{Gr.5}=19$) and 6 ($n_{Gr.6}=17$) exhibited the greatest initial value. Overall, the mean value of Group 5 increased from 0.29 to 0.56 (see Table 1), while the mean value of Group 6 increased from 0.31 to 0.68, indicating a significantly greater growth rate.

Summing up, we traced six groups of students that followed different growth patterns. Two groups with low initial value, with the one following a moderate growth rate and the other one adopting a remarkable growth rate, two groups with moderate initial value, with the first adopting a moderate growth rate and the second exhibiting an amazing progress, and two groups with high initial value, with the one adopting a constant greater increasing rate. To get a better insight of the reasons that may explain the different

developmental trends of the six groups of students, we examined their growth rate in the three dimensions of number sense.

Figure 3 presents the development of the six groups of students in the three dimensions of number sense. Comparing groups 1 and 2 (with low intercept), it could be concluded that in the first four time-points the gap between the two groups in elementary arithmetic got constantly bigger and it was reduced in the last measurement (beginning of grade 2). The gap between the two groups in conventional arithmetic increased until the third measurement and then it stabilized. However, the gap between the two groups in algebraic arithmetic increased systematically, especially after the third measurement. It should be noted that the gap between the two groups with moderate initial value (Groups 3 and 4) got impressively larger every successive time-point in all number sense components. On the contrary, the gaps of the three components between the two groups with high initial number sense (Groups 5 and 6) followed three different patterns. Their gap in elementary number sense increased until the third time point and then

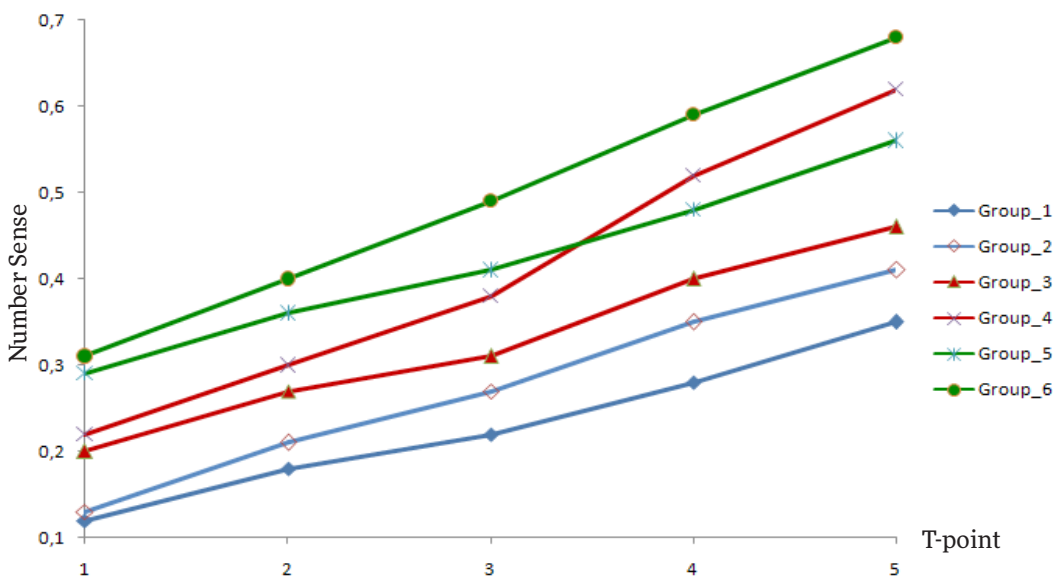


Figure 2: The development rate of the six groups

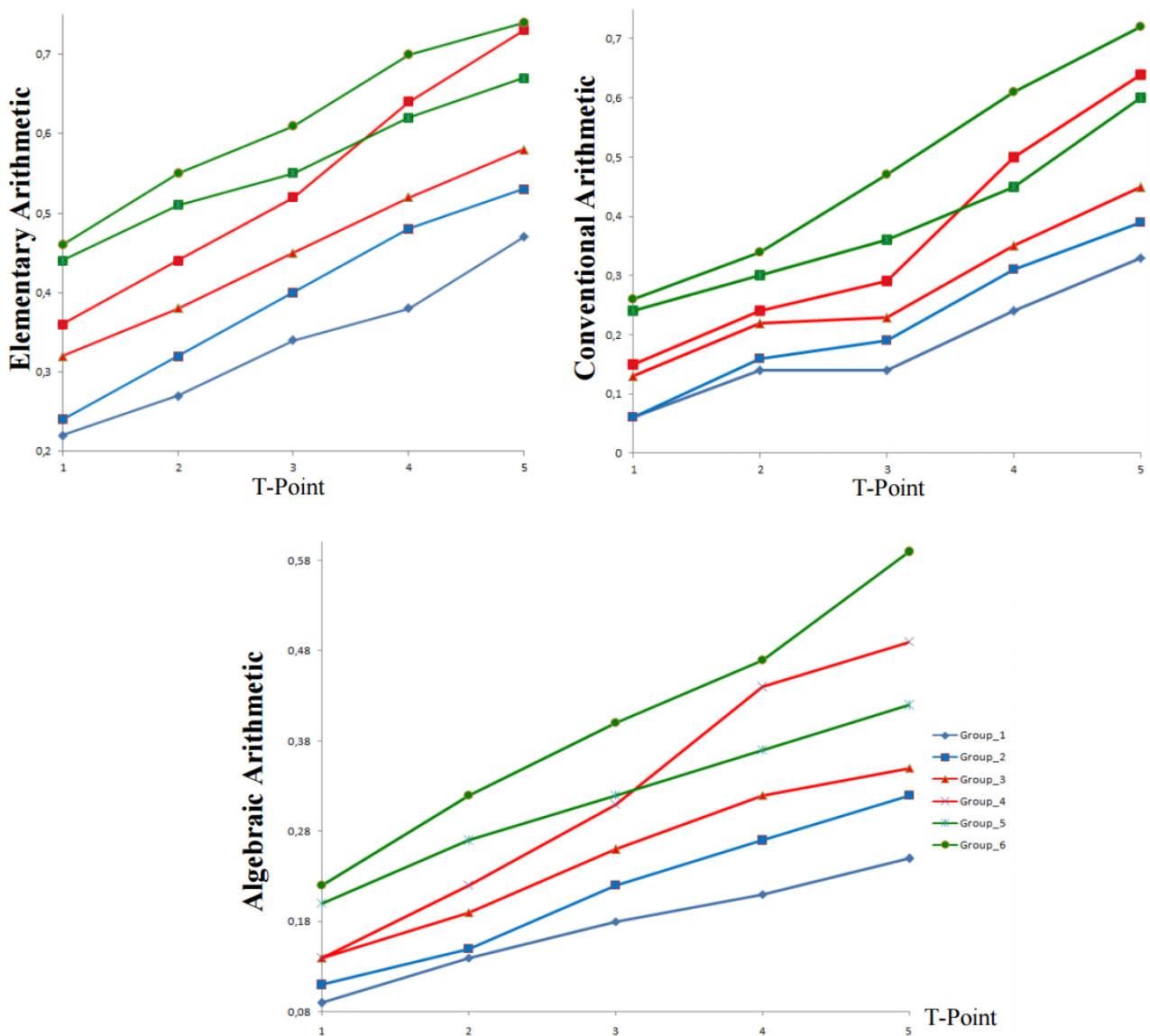


Figure 3: Development of the six groups in the dimensions of number sense

stabilized. The gap in conventional arithmetic got systematically bigger until the fourth time point and then reduced, while their gap in algebraic arithmetic progressively increased.

DISCUSSION

The present study utilized the model proposed by Pittalis, Pitta-Pantazi and Christou (2013, 2014), suggesting that students' early number sense consists of elementary, conventional and algebraic arithmetic, to trace six year old student's development in number sense. The results of the study showed that student's early number sense followed a linear growth rate from the beginning of grade 1 to the beginning of grade 2, suggesting a constant increase across the five time-points. However, the variation in the intercept and slope factors suggest that there are individual

trajectories in the development of number sense. In addition, mixture growth analysis showed that six groups of students could model more accurately student's variances in the five time-points. We identified two groups of students with low initial number sense, two groups with moderate initial value and two groups with high initial value. Systematically, the one of the two groups with the same initial value adopted a greater growth rate in all the dimensions of number sense.

We might suggest that the gap between the groups with same initial value could be explained by their progress in algebraic arithmetic. This could be concluded by the fact that the gap between the groups with the same initial value progressively increased across all the five time-points only for algebraic arithmetic. Thus, we might suggest that algebraic arithmetic

tic might be the source of large individual differences in number sense development. In addition, it might be assumed that the development of students' algebraic arithmetic might positively affect the development of the other number sense components. For instance, Group 4 every major improvement in algebraic arithmetic was followed by a corresponding huge improvement in conventional arithmetic in the following time point. These findings highlight the dynamic nature of algebraic arithmetic and underlie the significance of studying (a) more extensively the factors that may enhance or prohibit the development of algebraic arithmetic and consequently the development of student's early number sense, (b) the interaction among the growth of the three components of number sense and (c) the effect of instruction and teaching material on their growth rate.

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