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# Nonlinear nature of kinetic undamped waves induced by electrostatic turbulence in stimulated Raman backscattering<sup>\*</sup>

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**Abstract.** The influence of low-frequency waves of kinetic nature induced by electron trapping in backward Stimulated Raman Scattering (SRS) is investigated. Semi-lagrangian Vlasov-Maxwell simulations are carried out not only for periodic boundary conditions but also in the case of an open plasma with parabolic shape, in optical mixing. We provide a numerical example of generation of KEEN (kinetic electrostatic electron nonlinear) waves nonlinearly induced from the SRS through a mechanism we first here elucidate. In particular we identify a process of backward scattering of the SRS probe light from the so generated KEEN waves, which may provide a mechanism for the possible experimental observation and measurement of such nonlinear structures.

## 1 Introduction

The kinetic theory of periodic electron hole equilibria or phase space vortices is a long-standing problem in plasma physics. Since the pioneering work of Bernstein, Greene and Kruskal in 1957 [1], it is well-known that such phase space holes are self-sustained and connected to electrostatic fields that are self-consistent with some manner of trapped particle velocity distribution function. Recently, laser-plasma interaction experiments revealed that such electron trapping structures might conceivably be physically relevant, thus leading to renewed interest in the subject. These experiments [2] were aimed at improving our knowledge of Stimulated Raman Scattering (SRS) by looking at the electron plasma waves (EPWs) generated by a single speckle of laser light, and employed laser scattering as a key diagnostics (in particular in optical mixing experiment).

An extremely surprising result was the presence, in addition to the expected EPW scattering signal, of a second very weak scattering signal which was only a modest fraction ( $\sim 0.37$ ) of the plasma frequency. This unexpected signal was associated with what was termed stimulated electron acoustic scattering (SEAS), a novel scattering involving a so-called electron acoustic wave (EAW), whose phase velocity  $v_\varphi \simeq 1.31v_{th}$  is between an EPW and an ion acoustic wave (IAW) and where  $v_{th}$  is the electron thermal velocity. Such waves are justified in the limit of small amplitude waves, which depends essentially from the

population of trapped electrons, as theoretically predicted by Krapchev [3], Ram [4] and by Holloway and Dornring in [5]. Other studies on waves of this type created by the imposition of an external (electrostatic) driver were published by Valentini et al. [6,7]. We note that in a separate research thread, motivated by Montgomery's experiment in laser-plasma interaction, Johnston et al. [8]<sup>1</sup> carried out numerical experiments using a Vlasov-Poisson model and observed states, referred as kinetic electrostatic electron nonlinear waves (KEEN waves) (see Ref. [11] for more details). Ponderomotively driven (provided that the driver be applied resonantly over many bounce trapping periods), KEEN waves are seen to self-consistently form and

<sup>1</sup> KEEN waves are a non stationary version of Bernstein-Greene-Kruskal (BGK) equilibria and it was shown here that the electron distribution density contours in phase space seem to follow quite closely, the contours of the total particle energy, as should in a BGK equilibrium. On closer examination, however, deviations from the ideal state was observed indicating that the ideal BGK is not necessarily an accessible attractor state. However the works of Dodin and Fisch of references [9,10] show a possible connection between KEEN waves and the negative mass instability (NMI), which is linked to the secondary effects of macroparticles as production of sidebands of the wave field. Thus the asymptotic state is somewhat modified when these sidebands are in an appropriate resonance leading to the formation of an invariant tori in the particle phase space, a solution close to the ideal case. We interpret the two-vortex merging scenario as an example of NMI, which in references [9,10], was suggested to be a necessary ingredient to obtain the self-sustained equilibrium of KEEN wave-type, a property already met in the BGK dynamics where a BGK hole can be obtained as the result of the coalescence of several phase space holes, holes being different of BGK structures.

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persist at nearly constant amplitude long after the driver is turned off. Recently much more light has been shed on these nonlinear trapping structures by Schamel in [12], who points out that linear wave theory is generally non-suited as a starting point for the study of plasmas that are excited by long-living electron hole structures. Plasmas excited by such persistent trapped particle structures are not only kinetic but fundamentally nonlinear. In another development of this kind of wave, semi-lagrangian Vlasov-Maxwell numerical studies by Strozzi et al. [13] or by Albrecht-Marc et al. [14] dealt specifically with the problem of the generation of EAW-type (or KEEN-type in [14]), but now in presence of a strong backward SRS.

To explore the physics of such wave generation and to make contact with a possible laboratory experiment, a Vlasov-Maxwell approach is adopted. We address in this paper the results of numerical experiments leading to the generation of self-sustained low frequency KEEN waves, in a collisionless Maxwellian plasma initialized for the onset of 1D SRS. Then we investigate the interaction of such created KEEN waves with the probe signal. We have studied these phenomena in simulations with two types of boundary conditions: a periodic (and uniform) underdense plasma, and a large-size parabola-shaped plasma. Aiming at pointing out the phase-space vortex structures, rather than using a particle-in-cell (PIC) code, we carried out these computer experiments with a semi-lagrangian Vlasov code (first presented in [15–17]). These simulations have proven to display very low level of noise compared to PIC codes, which results particularly advantageous at phase velocities in correspondance to which the space averaged particle density is low.

## 2 Phase space coherent structures

The spontaneous formation of coherent structures in phase space is commonly observed in experiments and numerical simulations in plasmas. The problem of interest is to explain their physical nature in terms of wave type, their robustness and their interaction features in presence of an electromagnetic wave (pump or idler).

First it is well-known that an electron acoustic wave (EAW) can exist in a two temperature (cold and hot) electron plasma. It is basically a linear acoustic wave in which the inertia is provided by the cold electrons and the restoring force comes from the pressure of the hot electrons. Indeed ions play the role of a neutralizing background and the ion dynamics does not influence EAWs. The wave dynamics of EAW has received a great deal of renewed interest because of its potential relevance in interpreting the electrostatic component of the broadband electrostatic noise observed in the cusp of terrestrial magnetosphere (see Ref. [18]) and more recently in the experimental scattering results by Montgomery et al. [2]. Such linear modes, with a typical dispersion relation of  $\omega_{EAW} = 1.31kv_{th}$  (where  $k$  is the wave vector), are expected to be excited when the plasma is driven externally at frequency  $\omega \simeq \omega_{EAW}$  for a long enough time. Of particular interest is the modification of the wave nature in

the nonlinear regime, in particular when a population of trapped electrons is formed. Because the electron plasma wave (EPW, the Langmuir wave) undergoes a stronger frequency decrease in presence of trapped electrons, according to the basic idea of Morales and O’Neil [19], the nature of the EPW is strongly modified and was observed as a Beam acoustic mode (BAM) in references [13,20] in presence of a ponderomotive driver (a situation usually met in laser-plasma interaction). However the BAM mode disappears when the driver is turned off indicating that the mode is indeed not self-sustained in spite of the presence of trapped electrons observed in phase space. Unlike EPWs, a nonlinear version of EAWs was introduced by several authors in [2,5,6] (usually referred as Trapped EAW) to indicate the persistence of such acoustic waves even in presence of strong Landau damping. However in view of (backward) Stimulated Raman Scattering (SRS) experiment, the central question of interest is how a long train of EAWs can be obtained when the ponderomotive driver is applied at the plasma frequency, which usually drives EPWs, i.e. waves well above the EAW frequency.

Another type of coherent kinetic structures were discovered and this is the case of the KEEN waves. KEEN waves<sup>2</sup> were numerically observed in [8,11], near the branch of the dispersion relation of EAWs but at frequency values smaller than  $1.3kv_{th}$ . However such waves are qualitatively different from EAWs, as they can contain multiple pronounced phase-locked harmonics. They can be considered as a non-stationary class of BGK solutions, with no linear and fluid counterpart. Like BGK modes which can be non linearly excited by instabilities at saturation (like the well-known electrostatic two-stream instability), it was recently proposed in [9,10], that KEEN waves can be generated by the negative mass instability (NMI) (see also [21] for the similar instability met in ac-

<sup>2</sup> It also appeared that it would be quite difficult to produce such waves at very low amplitudes, so they should not be confused with the result of any linear perturbation. Because these waves are essentially kinetic and electrostatic and because they involve the electrons and because they are, as it proves, essentially nonlinear, it was decided, at the suggestion of A. Bedros, that a distinctive acronym was required to emphasize this, the acronym of EAW being used to denote waves in the linear wave frequency gap between the electron and ion plasma frequencies. KEEN waves contains multiple robust phase-locked harmonics which differ strongly of EAWs. In a KEEN wave, the trapped electrons play the role that hot electrons play in a two-temperature EAWs, while the free electrons play the role of the cold population but with trapping eliminating the severe Landau damping. KEEN waves can be readily produced over 2:1 range of frequencies from  $0.8\omega_{EAW}$  till  $1.6\omega_{EAW}$ . The sensitivity of KEEN waves to the excitation process and the driver amplitude is still an open question. During the growth phase, high spatial harmonics of the plasma field become very noticeable as the KEEN’s net force nonsinusoidal waveform is set up and such a change seems essential to the transition to a free-running KEEN wave. It was recently proposed that such a sensitivity can depend on the negative mass instability, a collective property of trapped particles to bunch together with the formation of sideband oscillations of the plasma field.

celerator physics). Due to NMI – and collective effects – trapped electrons can form a “macroparticle” as the result of a condensation process. In particular mutually repelling electrons can undergo phase-bunching or a condensation mechanism as if they had negative masses. Indeed holes in phase space have the tendency to attraction (see [22]) in spite of the strictly repulsive character of the Coulomb force and such a counterintuitive behavior can be viewed as a negative mass instability, provided that the number of macroparticles is weak. Typically, two rotating macroparticles (phase space holes here) leads to NMI (see Ref. [9]). Thus the KEEN wave is expected to be formed as a result of the pairwise vortex merging at the saturation of NMI. This picture readily explains why the formation of KEEN waves is sensitive to the pairwise vortex merging scenario where only two vortices are implicated in the coalescence process while it is well known that BGK modes can be obtained after the coalescence of several tens of phase space holes.

As BGK modes, KEEN waves behave as quasi-particles and can also interact with electromagnetic (pump or idler) waves. However KEEN waves may be excited in a large band gap of frequencies located between linear EAW and linear EPW. The lack of low level narrow band resonance precludes the usual stimulated scattering scenario of the three-wave resonance amplification from a low-level seed as SRS scenario. Indeed KEEN waves are first created by the vortex merging process of two phase space holes (not necessary self-sustained structures, for instance BAM modes nonlinear driven at high frequencies) then, once formed, they can interact with the electromagnetic wave in a scattering process in a non resonant way. Such an interaction allows thus the possibility to detect their presence experimentally.

Let us now indicate briefly our numerical model used here. The plasma can be described by the usual Vlasov equation for the electron distribution function  $f(x, p_x, t)$ :

$$\frac{\partial f}{\partial t} + \frac{p_x}{m\gamma} \frac{\partial f}{\partial x} + e \left( E_x + \frac{P_y(x, t) B_z}{m\gamma} \right) \frac{\partial f}{\partial p_x} = 0, \quad (1)$$

with the Lorentz factor given by:

$$\gamma = \left( 1 + \frac{p_x^2}{m^2 c^2} + \frac{P_y^2(x, t)}{m^2 c^2} \right)^{\frac{1}{2}}. \quad (2)$$

Using the conservation of the canonical momentum in the transverse direction  $y$ ,  $P_{cy} = p_y + eA_y = \text{const}$ , we keep a “fluid” description in the transverse direction. Defining  $P_y$  as the fluid transverse momentum, we have then (exactly):

$$\frac{\partial P_y}{\partial t} = eE_y. \quad (3)$$

The field components ( $E_x, E_y, B_z$ ) are calculated using the standard Maxwell equations, making the model self-consistent since the source terms used in the Maxwell equations (charge and current densities) are determined using the data of  $f$ . Both periodic and non periodic (sometimes named “causal”) boundary conditions will be considered here.

### 3 KEEN waves observed in periodic simulation of SRS-B

While open (non periodic) Vlasov simulations are necessary to understand realistic situations, these simulations, as shown in the following section, are also difficult to analyze in detail. One of the difficulties is the occurring of several types of nonlinear processes (particle trapping, nonlinear frequency shift, secondary parametric instabilities of pump and/or probe beams, cascade, ...). In order to build a first physical picture of what happens in the SRS-B scenario (here B refers to the backward SRS case) in the kinetic regime of the instability (i.e. for  $k_B \lambda_D \geq 0.3$  and where  $k_B$  and  $\lambda_D$  are the wave vector of the EPW and the electron Debye length respectively), it is perhaps more fruitful to start with a periodic situation, which allows one to separate the different nonlinear processes.

In the standard three-wave parametric decay from the pump (here denoted by  $\omega_0$  and  $k_0$  in frequency and wave vector) to the backscattered Stokes mode (here denoted  $\omega_{sB}$  and  $k_{sB} < 0$ ) and to the electron plasma wave (or linear Langmuir wave denoted by  $\omega_B$  and  $k_B$ ), in a periodic case we have:

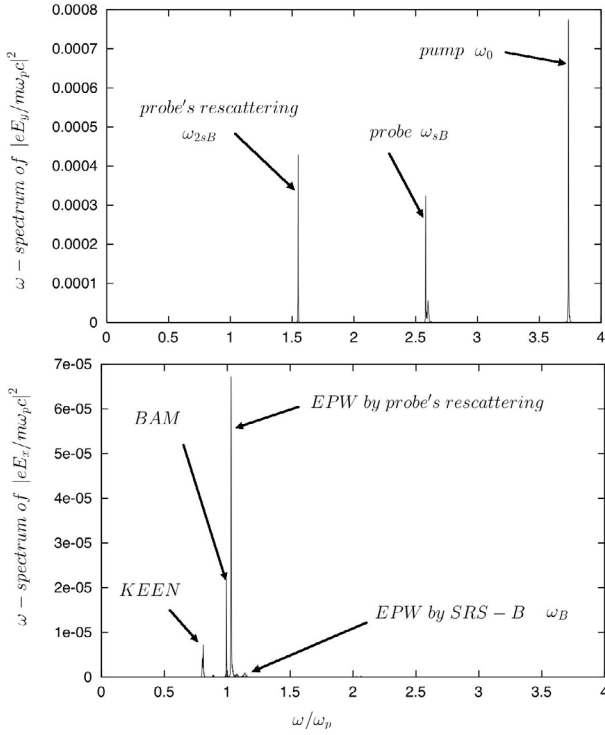
$$\omega_0(k_0) + \delta\omega = \omega_{sB}(k_{sB}) + \omega_B(k_B)$$

and

$$k_0 = k_{sB} + k_B. \quad (4)$$

Here we have assumed perfect  $k$ -matching (since the simulation box is periodic, the mode numbers are exactly matched) while a frequency mismatch  $\delta\omega$  is introduced in the first equation of (4).

Here the (linear) dispersion relations are given for EPW by the Bohm-Gross relation  $\omega_B^2 = \omega_p^2 + 3k_B^2 v_{th}^2$  and for electromagnetic waves by  $\omega_{0,sB}^2 = \omega_p^2 + k_{0,sB}^2 c^2$ . The matching conditions given in (4) can be satisfied only when  $n_0/n_c < 0.25$ , where  $n_c$  is the usual critical density above which electromagnetic waves will not propagate. Indeed, in a periodic simulation, the choice of  $k_0$  actually determines the plasma length  $L$  in terms of  $c\omega_p^{-1}$ . Here the velocities are normalized to the light velocity  $c$  and frequencies to the plasma frequency  $\omega_p$ , which is the conventional choice for periodic or uniform plasmas. Note that in the case of a parabolic plasma density,  $\omega_p$  is usually replaced by the value of the pump frequency  $\omega_0$  calculated at the maximum value of the density. For an electron temperature of  $T_e = 2$  keV, a good frequency matching was obtained by choosing  $k_0 = 3\Delta k = 3.6\omega_p/c$  (and where  $\Delta k = 2\pi/L$  is the fundamental mode with  $\Delta kc/\omega_p = 1.20$ ). Using the linear dispersion relation, for the corresponding pump frequency we so obtain  $\omega_0 \simeq 3.736\omega_p$ , that is a ratio of the electron mean density to the critical density of  $n_0/n_c = \omega_p^2/\omega_0^2 \simeq 0.0716$ . For the corresponding values in wave vectors for the Stokes mode and the corresponding EPW, we have  $k_{sB}c/\omega_p = -2.40$  (i.e.  $k_{sB} = -2\Delta k$ ) and  $k_Bc/\omega_p = 6$  (or equivalently  $k_B = 5\Delta k$ ). Using the linear dispersion relation the corresponding frequencies are then  $\omega_{sB} \simeq 2.60\omega_p$  and  $\omega_B \simeq 1.193\omega_p$ . Here we have a frequency mismatch given by  $\delta\omega = \omega_0 - \omega_{sB} - \omega_B \simeq 0.056\omega_p$ .



**Fig. 1.** Spectra in frequency obtained in the case of Vlasov-Maxwell simulation using periodic boundary conditions. On top panel, the electromagnetic spectrum showing the main actors: pump, probe and rescattered Stokes mode from the probe. On bottom panel the corresponding electrostatic spectrum in frequency. Note the occurring of unexpected signals associated with the growth of the BAM and low-frequency KEEN mode.

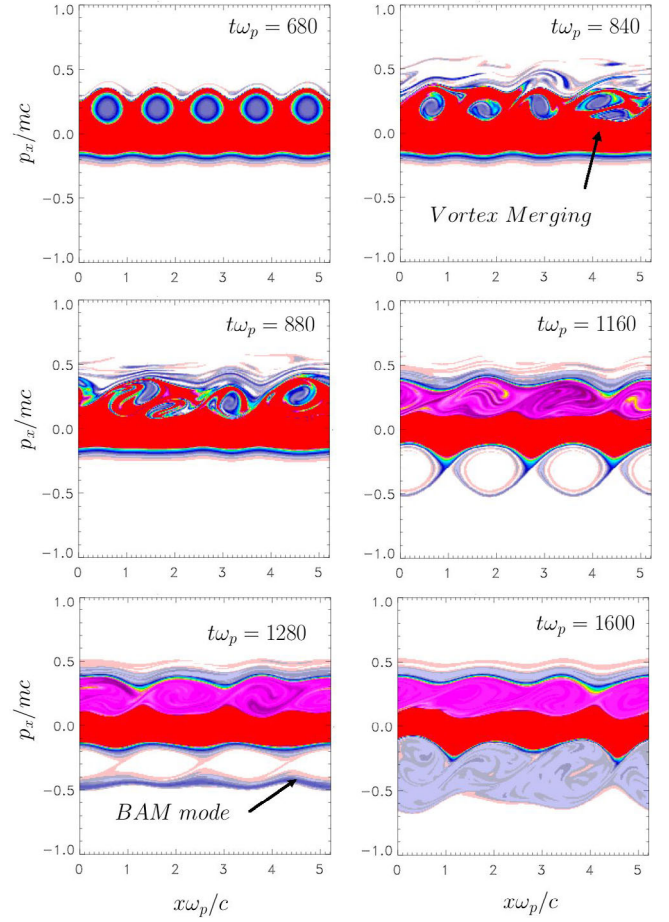
Numerical results are shown in Figures 1 and 2. In Figure 1 the top panel shows the electromagnetic spectrum in frequency, i.e. of the quantity  $|eE_y/m\omega_p c|^2$ . The figure shows that the expected values are obtained with high accuracy: the pump frequency is observed at  $\omega_0 \simeq 3.740\omega_p$  in good agreement with the expected value of  $\omega_0 = \sqrt{n_0/n_c} \simeq 3.736\omega_p$ . The backscattered mode frequency is found close to  $\omega_{sB} \simeq 2.580\omega_p$  in the spectrum (the exact value being  $\omega_{sB} \simeq 2.60\omega_p$ ). However another electromagnetic narrow peak can be also observed at  $\omega_{2sB} \simeq 1.56\omega_p$ . This peak is the signature of a cascade, that is the backward Raman rescattering of the probe beam. The probe light couples with the plasma to generate a backscattered mode, a counterpropagating electromagnetic wave ( $\omega_{2sB}, k_{2sB}$ ) at the mode  $k_{2sB} = \Delta k$ , which propagates in the opposite direction of the probe beam and which is resonant in frequency and in wavevector, according to the following matching conditions:

$$\omega_{sB}(k_{sB}) + \delta\omega = \omega_{2sB}(k_{2sB}) + \omega_{2B}(k_{2B})$$

and

$$k_{sB} = k_{2sB} + k_{2B}. \quad (5)$$

Here  $k_{sB} < 0$  since we have considered a SRS-B process. Therefore we expect  $k_{2sB} > 0$ . Accordingly, the simulation



**Fig. 2.** Corresponding phase space representation of the electron distribution function obtained for periodic simulation. We see clearly the formation of phase space electron holes induced by SRS-B (left top) followed by the pairwise vortex merging and finally the rescattering of the probe takes place.

shows that  $k_{2sB}c/\omega_p = 1.20$  (i.e. on the fundamental mode  $\Delta k$ ) and that  $k_{2B} = -3\Delta k$  is connected to the growth of a second EPW (with  $k_{2B}c/\omega_p = -3.60$ ). From the linear dispersion relations of electromagnetic waves and EPWs, we have  $\omega_{2sB} \simeq 1.562\omega_p$  and  $\omega_{2B} \simeq 1.073\omega_p$ , leading to a mismatch of  $\delta\omega = \omega_{sB} - \omega_{2sB} - \omega_{2B} \simeq -0.035\omega_p$ . More surprising is the electrostatic spectrum (here given by the quantity  $|eE_x/m\omega_p c|^2$ ), shown in the bottom panel in Figure 1. Due to a pairwise vortex merging scenario for the trapping vortices corresponding to the nonlinear version of the EPWs driven by SRS-B, the signal of the corresponding (linear) Langmuir wave  $\omega_B$  has disappeared while the EPW driven by the Raman rescattering of the probe beam appears now to clearly be the dominant mode with a frequency close to  $\omega_{2B} \simeq 1.04\omega_p$ , a value somewhat smaller than the expected value of  $\sqrt{\omega_p^2 + 3k_{2B}^2 v_{th}^2} \simeq 1.073\omega_p$ . However, when nonlinear effects become important, the electrostatic spectrum exhibits two further narrow peaks: the first one, due to the pairwise vortex merging scenario, is located at  $\omega_K \simeq 0.81\omega_p$ , a value well below the plasma frequency, and the second one is close to  $\omega_{BAM} \simeq 0.99\omega_p$ .

The latter corresponds to the generation of a beam acoustic mode (BAM), starting with the nonlinear shift induced by trapping effect of the Langmuir mode  $\omega_B$ , well before the vortex merging begins. In addition to the BAM mode as observed by Vu et al. [20], Yin et al. [23] and Strozzi et al. [13], another low-frequency KEEN mode is clearly visible at  $\omega_K \simeq 0.81\omega_p$  resulting from the pairwise vortex merging. In the kinetic regime of SRS-B, electron trapping can modify the velocity distribution, resulting in an effective beam-like low density population. The BAM mode, seen in the electrostatic spectrum, shows a damped reduction and a weak frequency downshift (compared to the EPW for a Maxwellian distribution).

To help the understanding of what is happening, we present in Figure 2 snapshots of the electron distribution function in phase-space ( $p_x/mc$  versus  $x\omega_p/c$ ), at six different times. The top left panel in Figure 2 clearly exhibits the trapping and formation of five holes induced by the EPW in the initial phase of the instability, with the dominant mode  $k_B = 5\Delta k$ . As a consequence of nonlinear effects, there is a change in the nature of this mode, initially born as an EPW: it is no longer a simple Langmuir wave associated to SRS-B but, as we hypothesize, a more complex structure characterized by trapping effects in the form of phase-space holes. Such structures eventually turn out to be unstable and not obviously self-sustained, and are apparently able to merge together in a pairwise vortex-merging mechanism, as can be seen at times  $t\omega_p = 840$  and  $t\omega_p = 880$ , leading to the occurrence of three larger holes in the phase-space. As a remarkable consequence the obtained trapping structures, driven by the vortex coalescence, can now be considered as BGK-type structures or more precisely as KEEN waves.

In the first step of the pairwise vortex merging in Figure 2, just before the coalescence takes place, just consider a pair of macroparticles (two holes) bouncing in the wave potential, i.e. rotating in phase space around a local equilibrium, as time  $t\omega_p = 880$  located at  $x\omega_p/c \sim 2$ . These holes have a tendency to behave as macroparticles [22] and their mutual attraction phenomenon belongs to a much larger class of behavior called NMI [9,10,22]. It becomes evident that collective effects and in particular the negative mass instability and its connection with the pairwise vortex merging instability must be correctly kept into account to fully understand the formation of KEEN waves at the saturation. Thus the KEEN wave is formed at the saturation of NMI when the two macroparticles meet, thus giving rise to the vortex coalescence. The process seems to be similar to that met in the formation of BGK waves in the (electrostatic) two-stream instability which may lead to the formation of holes in phase space, and to their subsequent merging, with generation of BGK equilibria in references [24,25].

As we will see in the next section, the pairwise vortex merging mechanism leads, in the case of a large size parabola-shaped plasma, to the formation of a self-sustained and low-frequency KEEN wavetrain. At time  $t\omega_p = 1160$  the coalescence ends and the rescattering of the probe takes place. Something similar to this vortex

merging was observed by Bertrand et al. [24,25], where in a periodic two-stream instability, the merging went as far as it could go, by proceeding until only one vortex was left in the simulation box. This is not what happens here. The wavevector is reduced by a factor  $3/5$  but the frequency  $\omega_K$  of the KEEN wave is found at  $\omega_K \simeq 0.80\omega_p$ , a somewhat larger value than  $\frac{3}{5}\omega_p = 0.71\omega_p$ . This pairwise vortex merging was also observed in [14] in the case of a more realistic parabolic-type profile, which proves to be pertinent to the result here presented, even though we deal here with a larger, open and inhomogeneous system.

In reference [26], we have studied the dynamics of trapped particles which becomes stochastic when repeated wave-particle interactions take place. For small values of the Kubo parameter defined by the ratio  $\tau_{auto}/\tau_b$  between the autocorrelation time to the bounce time of electrons, the turbulent regime results in the merging of phase space trapping vortices according to a weak turbulence scenario. There is also some evidence to suggest the way the chaotic behavior arises here to be linked to the occurrence of secondary instabilities, i.e. the rescattering of the probe signal by EPW. The second EPW with parameters  $(\omega_{2B}, k_{2B})$  exhibits a negative phase velocity  $v_{\varphi,2B} = \omega_{2B}/k_{2B}$  since  $k_{2B} < 0$ , and we now observe phase-space holes for negative values of the momentum at times  $t\omega_p = 1160$  and  $t\omega_p = 1280$  (the corresponding phase-velocity is then  $v_{\varphi,2B} \simeq -0.275c$ , or equivalently  $p_{\varphi,2B} \simeq -0.28mc$ ). At time  $t\omega_p = 1280$ , when nonlinear effects are dominant, the mode BAM occurs with a downshifted frequency and the mode is less Landau damped than the (linear) Maxwellian EPW  $(\omega_{2B}, k_{2B})$ . The plasma wave undergoes a frequency decrease, in agreement with the basic idea of Morales and O'Neil [19] regarding the effect of trapped electrons on the frequency of a Langmuir wave. However the downshift frequency remains weak. What then appears to happen is that, instead of the vortex-merging scenario met in the region of positive velocity phase-space, the EPW becomes now a BAM mode, dominated by trapping phenomena and the resulting downshift in frequency. At time  $t\omega_p = 1160$  the dominant mode is  $k_{2B} = 3\Delta k$ . At that time the system is not stochastic and is driven by a filamentary structure in phase space as particles are smoothly spiralling around the local O points. Particles interacting resonantly with high-amplitude electric field can be trapped (it is the case here at time  $t\omega_p = 1160$ ), and subsequently detrapped when the electric field decreases. This population of detrapped particles can then form an accelerated particle beam-type distribution: a structure which may be referred as a BAM mode with the same wave number  $3\Delta k$  but with a negative momentum to which no pairwise vortex merging is associated, can be seen at later time  $t\omega_p = 1280$ , close to a momentum  $p_{\varphi} \simeq -0.5mc$ .

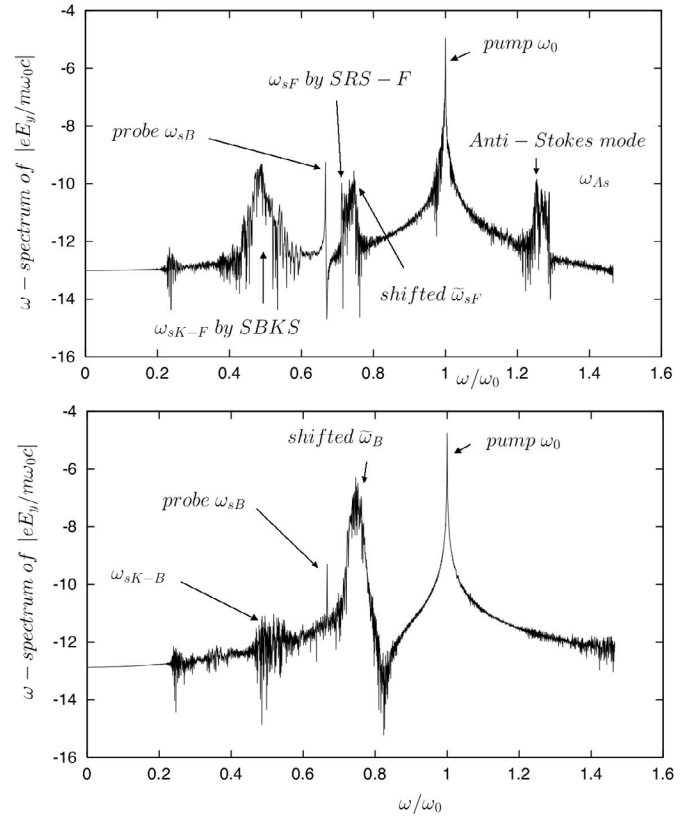
#### 4 Vlasov-Maxwell simulations using a parabolic profile

Aiming at applications to experimental plasmas, the uniform slab, while excellent for untangling the basic physics,

is however quite unrealistic. Such periodic simulations in underdense plasmas, are necessarily modest in size (i.e. thickness) and time because of the periodicity constraint). In larger scale (causal) simulations, which can be therefore run for longer times, closer to inertial confinement fusion operating conditions, a somewhat different picture emerges.

- (i) First the EPW undergoes a stronger frequency decrease, according again to the basic idea of Morales and O'Neil [19] regarding the effect of trapped electrons on the frequency of a Langmuir wave, while the wavevector slightly increases (or “chirp”) so to maintain the SRS-B matching conditions, through a mechanism of parametric resonance type, as first shown and discussed in reference [27] and then in reference [14]. Here instead both the frequency and the wave vector change downward in frequency for EPW (leading to a shifted-value  $\tilde{\omega}_B$ ) and upward in the wavevector (i.e giving  $\tilde{k}_B$ ). As a consequence the nature of this initially EPW is modified and dominated now by electron trapping. Such a structure corresponds to the concept of BAM mode observed in [20,23].
- (ii) In reference [14] we have shown that there is a SRS limiting process of the downward shift in frequency of EPWs, in the form of a symmetry-breaking instability of pairwise hole merging, which was also previously observed in the case of a periodic system to lead to the formation of KEEN waves. More interesting is now the possibility of direct interaction of the KEEN waves with the electromagnetic wave. When created at a self-sustained level, the KEEN waves can now interact with electromagnetic waves propagating in the plasma in a non-resonant way in a three-wave parametric-type process with the probe beam. Thus, such interaction with the electromagnetic component of the nonlinear wave, would allow a clear detection of such KEEN structures in laser-plasma experiments.

We begin with a simulation where just SRS-B is excited by injecting a seed (probe) light and a counterpropagating pump ( $\omega_0, k_0$ ) with relative intensities  $I_{probe} = 10^{-4} I_{pump}$ . The physical system consists in a plasma with a parabolic profile of width  $3000 c/\omega_0$  peaked at  $x = 2400 c/\omega_0$ , surrounded by a vacuum of length  $900 c/\omega_0$  at both sides. The total length of the system is then  $L = 4800 c/\omega_0$ , corresponding to  $731.7\lambda_0$  (or  $256 \mu\text{m}$ ) with an incident pump wavelength of  $\lambda_0 = 0.35 \mu\text{m}$ . The quiver momentum of the pump wave is  $a_0 = p_{osc}/mc = 0.025$  which corresponds to a pump intensity of  $I_{pump} \simeq 7 \times 10^{15} \text{ W cm}^{-2}$ . The maximum of the electron density is taken to be  $n_0 \simeq 0.0825 n_c$  for an electron temperature  $T_e$  of 2 keV. Here ions are taken into account with a temperature of  $T_i = 0.25 T_e$  and with a mass  $M = 1836 m$ . For matching conditions between waves in the SRS-B case we use again equations (4) taking now  $\delta\omega = 0$ . We have determined the plasma profile, the pump and probe frequencies so that the perfect matching conditions of SRS-B (with  $\delta\omega = 0$ ) occurs right on the top of the parabolic profile where both



**Fig. 3.** Spectra in frequency of the forward scattered Raman light (top panel) and of the backscattered Raman light on bottom panel (we have added probe plus pump signals on both plots). The simulation corresponds to an open parabolic-shape plasma profile in density. Both pump and probe beams are continuously injected respectively at the leftside and at the rightside of the system.

the pump and the probe beams met. The matching conditions are however chosen to be close to those used in Section 3 in the case of a periodic system, in order to make direct comparison with previous numerical results.

For the parabolic profile, we have normalized the frequencies to  $\omega_0$ , the time to the inverse pump frequency  $\omega_0^{-1}$  and the space variable to  $c/\omega_0$ , so the field components are normalized to  $m\omega_0 c/e$ . With such a normalization we obtain  $\omega_{sB} = 2/3\omega_0$  and  $k_0 c/\omega_0 = 0.9578$  for the pump wave and  $k_{sB} c/\omega_0 = -0.6016$  for the probe. For the EPW we have  $\omega_B = 1/3\omega_0$  and  $k_B c/\omega_0 = 1.5594$ , which corresponds to a value of  $k_B \lambda_D = 0.34$ , well into the kinetic regime of SRS-B. In this simulation we let the plasma evolve freely without electromagnetic field till  $t\omega_0 = 50000$  and then we inject the pump and probe pulses (see later, after Eq. (7)).

In Figure 3 on top panel, we have plotted the  $\omega$ -spectrum amplitude of the electromagnetic field  $eE_y/m\omega_0 c$  located in the vacuum region at  $x\omega_0/c = 4350$ , at right of the parabolic profile. The bottom panel in Figure 3 corresponds to measure of the spectrum in the vacuum region located now at left at  $x\omega_0/c = 450$ . Both spectra are calculated in the time interval

$60\,000 \leq \omega_0 \leq 75\,000$ . On top panel the two narrow peaks correspond to the pump at the frequency  $\omega_0$  and to the probe signal at  $\omega_{sB} = \frac{2}{3}\omega_0$ . We see also clearly the presence of the Forward stimulated Raman scattering (SRS-F) with a frequency of about  $\omega_{sF} \simeq 0.71\omega_0$  but also of an anti-Stokes mode at  $\omega_{as} = 1.289\omega_0$ . We can write the frequency and wavevector matching for SRS-F in the form  $\omega_0 = \omega_{sF} + \omega_F$  (i.e.  $1 = 0.7108 + 0.2892$  in  $\omega_0$  units) and  $k_0 = k_{sF} + k_F$  (i.e.  $0.9578 = 0.650 + 0.3078$  in normalized values). The corresponding matching conditions for the anti-Stokes process is  $\omega_0 = \omega_{as} + \omega_A$  (i.e.  $1.0 + 0.2891$  in normalized units) while for the wavenumbers we can write  $k_{as} = k_0 + k_A$  ( $1.2565 = 0.9578 + 0.2987$ ). On bottom panel, the probe signal is clearly visible at  $\frac{2}{3}\omega_0$ . Once again the pump is located at 1, in  $\omega_0$  units. Because we have plotted the electromagnetic field in the time interval  $60\,000 \leq t\omega_0 \leq 75\,000$ , i.e. well into the nonlinear regime, the electromagnetic spectra of Figure 3 are a combined result of the nonlinear shift due to particle trapping and of the parametric resonance which is rigorously maintained in time in the optical mixing scenario. Thus, starting from the matching conditions (4), the maintenance in time of the parametric resonance conditions writes:

$$\begin{aligned}\omega_0 &= \tilde{\omega}_{sB} + \tilde{\omega}_B = \omega_{sB}(k_0 - k_B - \delta k) + \omega_B - \delta\omega; \\ k_0 &= \tilde{k}_{sB} + k_B + \delta k,\end{aligned}\quad (6)$$

where we have assumed that  $\tilde{k}_B = k_B + \delta k > 0$  and  $\tilde{\omega}_B = \omega_B - \delta\omega$ . Here we have  $\delta k, \delta\omega > 0$  and  $\omega_{sB}(k)$  is the scattered wave frequency written as a function of its wavevector  $k$ . The variation in frequency is close to  $\delta\omega = \tilde{\omega}_{sB} - \omega_{sB} = 0.7650 - 2/3 \simeq 0.098\omega_0$ . Note that a similar behavior can be observed for the anti-Stokes and the SRS-F scattered signals on top panel. Here the shift induced by trapped electrons leads to a value of  $\tilde{\omega}_B = 0.235\omega_0$ . We have  $k_0 = \tilde{k}_{sB} + \tilde{k}_B$  (i.e.  $0.9578 = -0.710 + 1.6670$  in normalized units). This new wave with parameters  $(\tilde{\omega}_B, \tilde{k}_B)$  corresponds to a BAM mode with a frequency well below the plasma frequency  $\omega_p = \sqrt{n_0/n_c}\omega_0 \simeq 0.287\omega_0$ , which is first generated as a classic EPW associated with SRS-B process with a frequency  $\omega_B$  and then nonlinearly lowered by the Morales and O'Neil retuning. In addition to this behavior an unusually interesting signal can also be noticed in the top panel spectrum at the frequency  $\omega_{sK-F} \simeq 0.50\omega_0$ . Clearly, since the normalized frequency difference of this wave with the pump is 0.50, larger than  $\omega_p$ , some more involved explanation is required. Two candidates are suggested.

(i) One possible explanation involves a SRS cascade, concerning the backward rescattering of the nonlinearly shifted Stokes mode with a BAM mode in a three-wave resonant parametric-type instability, which can be written in the form  $\tilde{\omega}_{sB} = \omega_s + \omega_{BAM}$  (i.e.  $0.765 = 0.50 + 0.265$  in normalized units) with wave vectors  $\tilde{k}_{sB} = k_s + k_{BAM}$  (i.e.  $-0.71 = 0.40 - 1.110$  in normalized units). Thus, the strangely created leftward-going nonlinear Langmuir wave (a BAM-type mode) also

satisfies the three-wave condition for SRS-B from the originally scattered light  $(\tilde{\omega}_{sB}, \tilde{k}_{sB})$  (here acting as a pump), in what appears as a cascade process. However, when a classic Langmuir wave travels towards a lower density region, its phase velocity drops rapidly, the Landau damping increases and the EPW dies out in a welter of energy transferred to moderately fast electrons. In presence of a small trapped electron population, the EPW is transformed in a BAM mode which is also subject to strong Landau damping in particular when the density is not uniform.

(ii) Another explanation for the generation of the mode  $(\omega_{sK-F}, \tilde{k}_{sK-F})$  is the possibility of interaction of the probe light with KEEN waves. The KEEN waves resulting from the pairwise vortex merging, are very stable and can propagate toward the right side of the plasma by being reflected from the electrostatic ambipolar sheath potential at the boundaries. Such a behavior has been already observed in Vlasov simulations. In a result which differs remarkably from the standard linear treatment of the Vlasov's equation, Fijalkow and Nocera in [28] obtained, using a Vlasov-Poisson solver in the case of an open system, the periodic revolution of phase-space holes connected with oscillations in time of the mean electron momentum. When approaching the plasma/vacuum boundary, holes of positive velocities are first decelerated, then accelerated to acquire negative velocities in a reflection-type behavior induced by the ambipolar field. Here we hypothesize that, since the sheath has attained equilibrium before the injection of the pump and probe beams, we have a reflection of the KEEN waves at constant frequency and reversed wavevector and phase velocity. Such reflected KEEN waves can then interact with the presence of the propagating probe signal in a non resonant way.

The underlying process for the generation of the scattered light  $(\omega_{sK-F}, k_{sK-F})$  observed on top panel in Figure 3 is thus a stimulated backward KEEN scattering of the probe light, similar to a stimulated electron acoustic scattering and resembling a cascade process of the probe signal [29]. The new process involves the back-rescattering of the probe beam to a reflected KEEN wave of negative momentum, according to the matching conditions:

$$\omega_{sB} = \omega_{sK-F} + \omega_{KEEN}$$

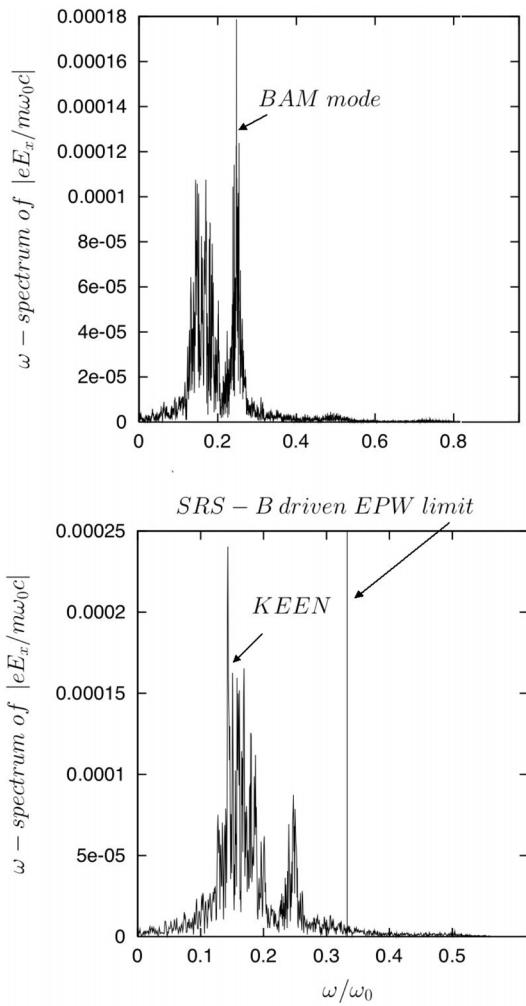
and

$$k_{sB} = k_{sK-F} + k_{KEEN}. \quad (7)$$

In normalized units, the matching condition in frequencies writes  $0.6666 = 0.506 + 0.1606$  and the corresponding equation in wave vectors is  $-0.6015 = 0.41 - 1.1015$ .

Further information on these physical processes in the parabolic profile is available in Figures 4 and 5, in the form of the  $\omega$ -spectrum of the electrostatic component  $eE_x/m\omega_0c$  obtained at point  $x\omega_0/c = 3000$ , and of the representation of a typical phase-space region of  $1700 \leq x\omega_0/c \leq 1800$ .

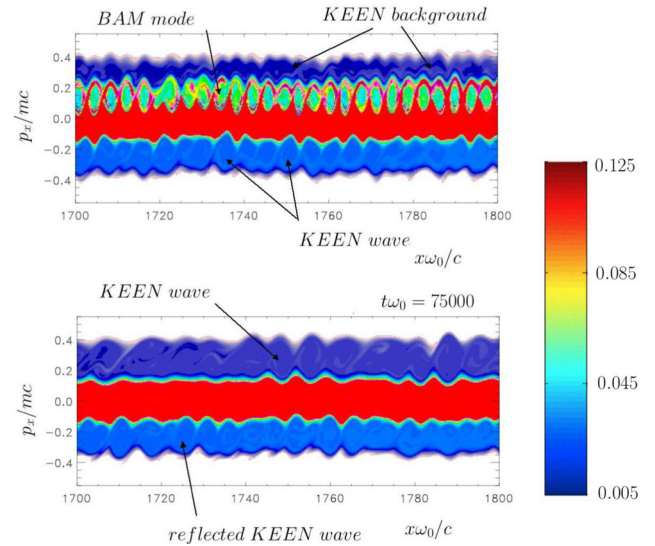




**Fig. 4.** Spectra in frequency of the electrostatic field component. On top panel both KEEN and BAM modes are excited. On bottom panel the BAM mode is strongly Landau damped when the pump plus probe laser beams are turned off. We have added on the bottom panel the frequency value  $\omega_B$  (by a straight vertical line) indicating the location in the spectrum of the initially SRS-B driven Langmuir wave.

The behavior is not the same as in the periodic case, previously shown in Section 3, the most striking difference being the possibility of a super-reflection of KEEN waves from the electrostatic ambipolar sheath potential, which may exist throughout the plasma because of the electron tendency to leave the plasma much faster than the ions. The strong reflection which may be induced by such amplified ambipolar field, is the reason we have chosen to let the plasma freely evolve till the time  $t\omega_0 = 50\,000$ , which corresponds to roughly one electron transit period, before starting the laser beams. To check the nature of the emitted plasma waves (BAM or KEEN) the self-sustained character of the wave is also analyzed by turning off the laser beams after the creation of the plasma structures.

Plots on top panels of Figures 4 and 5 correspond to the same simulation presented in Figure 3, where laser beams are injected at time  $t\omega_0 = 50\,000$ , after one electron



**Fig. 5.** Corresponding phase space plots of a selected part of the electron distribution function at the same time. On top panel, we see clearly a new SRS interaction cycle with the growth of trapping structures (BAM mode) on a enhanced density background constituted by fast particles. On bottom panel we show the same diagnosis when both pump plus probe lasers are turned off: only KEEN waves are now present in plasma while the BAM mode has disappeared.

transit time. To test the self-sustained character of the phase space vortices we have also plotted on bottom panels of Figures 4 and 5 the numerical results of a simulation, where, without modifying the other physical parameters, the laser beams (pump plus probe) are turned off at time  $t\omega_0 = 65\,000$  for a sufficiently long time after the generation of KEEN waves or BAM modes in plasma. On top panel in Figure 4 we have represented the electrostatic  $\omega$ - spectrum obtained by the simulation when the laser beams are injected from the initial time  $t\omega_0 = 50\,000$  till the final time  $t\omega_0 = 75\,000$ . Both modes are clearly excited and well separated: a KEEN wave with a broad frequency close to  $\omega_{KEEN} \sim 0.16 - 0.19 \omega_0$  plus the BAM mode with a frequency of order of  $\omega_{BAM} = \tilde{\omega}_B \sim 0.23 - 0.26 \omega_0$ ; both are below the plasma frequency  $\omega_p \simeq 0.287 \omega_0$ . The same diagnostics is now performed when both the pump and the probe lights are turned off. The result is shown in bottom panel of Figure 4. We clearly see that in this case the BAM mode is strongly Landau damped, while the KEEN mode seems to be somewhat amplified, by showing not to be affected by the disappearance of the electromagnetic waves.

The confirmation of the self-sustained character of the KEEN mode is confirmed by the plot of the distribution function in phase space in Figure 5. Again, the top panel corresponds to the case where the pump and the probe are continuously injected inside the plasma, whereas in the bottom panel the laser beams are turned off at time  $t\omega_0 = 65\,000$ . Both plots correspond to the same region in phase space and to the same time,  $t\omega_0 = 75\,000$ . The top panel shows the behavior of the electron distribution function when SRS-B is still present: we observe a

combined action of two phenomena: the presence of a fast electron population acting as a density background, and a situation predicted in the modelling of the BAM mode, when the Maxwellian distribution evolves in time leading to a symmetry-breaking in velocity of the distribution function. More precisely, such a background population corresponds to the initial process of creation of EPWs driven by SRS-B, followed by their pairwise vortex-merging leading to the growth of KEEN waves. More surprisingly, the SRS-B process is still capable of continuing and producing new BAM modes from the non-Maxwellian electron distribution. Indeed, the trapping structures observed for positive velocities are formed with the mode  $\tilde{k}_{BC}/\omega_0 = 2\pi/3.846 \simeq 1.633$  (i.e. roughly 26 vortices are formed in the region of length  $100c/\omega_0$ ), close to the expected value of the BAM mode  $k_{BAM}c/\omega_0 = 1.667$ . Finally, such trapping structures (BAM modes) have completely disappeared when the SRS is turned off (bottom panel). We can also observe, for negative velocities, the presence of reflected KEEN waves at the wavenumber  $k_{KEEN}c/\omega_0 = 2\pi/5.555 \simeq 1.13$  (corresponding to roughly 18 vortices in a phase space region of size  $100c/\omega_0$ ). The persistence of these KEEN waves far enough to be reflected is a good indication of the remarkable stability of these structures.

## 5 Conclusion

In summary the SRS-B instability was numerically investigated using a semi-lagrangian Vlasov code with periodic and open boundary conditions. In the last case a parabolic type profile was used. Our results confirm that kinetic effects cannot be ignored when studying the nonlinear behaviour of the EPW initially created by the parametric SRS-B instability. Secondly the results support the idea that the pairwise vortex merging of EPWs, downward shifted in frequency, leads to the formation of KEEN waves, whose self-sustained character allows them to persist over a long time. The pair vortex merging mechanism is interpreted as an example of negative mass instability, and fosters the interpretation of KEEN waves as inherently nonlinear electrostatic structures, generated from electrostatic modes as a consequence of a secondary instability which necessarily involves a pairwise coalescence of trapped electron populations. In the case of an open system, the reflection of these KEEN waves from the electrostatic ambipolar sheath potential, offers the possibility for them to interact with the injected probe light in a three-wave parametric-type decay in optical mixing, a process which allows the characterization of such kinetic structures in spectrum in frequency. Remarkably, such a behaviour opens for the detection of these structures in laser-plasma experiments.

To end we would like to make just one comment on terminology, namely on the use of terms EAWs and KEEN waves. It would be probably more appropriate to call about BGK structures; however BGK states are stationary and ideal and smooth (without velocity filamentation)

states in the asymptotic limit, not necessarily accessible by the plasma. The crucial role is here played by the excitation level and the corresponding response of the plasma. Usually in Vlasov-Poisson numerical experiments EAWs were introduced when the external driver is applied in a low-frequency regime and at weak intensities allowing the excitation and the growth of EAWs even in the linear regime. However such simulations have shown that the birth of phase space holes is complex and depends not only on the level of excitation but also on the driver imposition time. Alternatively what happens when we choose to apply now an external high-frequency driver at high intensity, a situation met in laser-plasma experiments, which now precludes the initial linear excitation of EAWs. The possibility of excitation of self-sustained phase space holes by a secondary pairwise vortex merging of high frequency nonlinear waves (as a result of the parametric Raman instability for instance) makes clear that the fundamental nature of the phase space hole (in the asymptotic limit) must be modified and cannot be considered as (trapped) EAWs and the term of KEEN waves term was introduced for these reasons. The most important result is the following: the excitation of KEEN waves now requires an external physical phase-mixing-type mechanism allowing a stable solution. Several processes have been already observed in the literature. For instance in reference [24,25] in the study of nonlinear Landau damping, the occurring of phase space holes is always accompanied by the cleaning of the small-scale filaments in velocity space, a process usually intensified by the numerical diffusion, although weak in the Vlasov codes. On the other hand the negative mass instability, introduced in the bifurcation-type of EAWs towards KEEN waves was recently introduced. Here a similar role is played by a secondary instability, having a very strong similarity, the pairwise vortex merging, acting as a phase space mixing mechanism. This gives to the KEEN waves a non resonant character that was clearly absent in the nature of the EAWs.

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