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Foundational number sense: Summarising the development of an analytical framework

Judy Sayers and Paul Andrews

Stockholm University, Stockholm, Sweden, judy.sayers@mnd.su.se, paul.andrews@mnd.su.se

What number-related competences do grade one students need to ensure later success and avoid later failure? We address this question by summarising recent work on the development of an eight component framework, which we call foundational number sense (FoNS), in which those necessary learning outcomes are categorised. We then present summaries of three case studies undertaken to evaluate the robustness of the FoNS framework. Each case study, which focused on the teaching of a different mathematical topic, was undertaken in two different European grade one classrooms. Analyses confirm not only the sensitivity of the FoNS framework to both cultural and mathematical contexts but also its power as a tool for both cross cultural research and teacher education practices.

Keywords: Foundational Number Sense, grade one, teaching framework.

INTRODUCTION

The influence of a child's basic number understanding in later mathematical successes (Aubrey & Godfrey, 2003; Aunola et al., 2004) and failures (Geary, 2013; Gersten et al., 2012) is well known internationally. Consequently, it would seem sensible for researchers and teachers to be clear as to what such basic understandings entail. In this paper, focusing on the first year of schooling and from an international perspective, we do two things. Firstly, we summarise the competences research has shown will both avoid later difficulties and ensure later success in an eight component framework. Secondly, we summarise three recent case studies, each of which evaluates the framework's analytical efficacy by comparing specific topic-related practices of teachers from two different European countries.

DEVELOPING THE FRAMEWORK

In relation to the role of children's basic number understanding in their later mathematical development, the expression *number sense* dominates the literature. However, despite its ubiquity its definition has been elusive (Griffin, 2004). Indeed, despite its importance, "no two researchers have defined number sense in precisely the same fashion" (Gersten et al., 2005, p. 296). Our reading indicates that this has, in no small way, been due to psychologists and educators working with different conceptualisations (Berch, 2005), a problem exacerbated by psychologists employing different definitions according to whether they work in general cognition or learning disabilities. That said, our reading reveals three distinct but related perspectives on number sense, which we label *preverbal*, *applied* and *foundational*.

Preverbal number sense reflects those number insights innate to all humans and comprises an understanding of small quantities that allows for comparison (Ivrendi, 2011; Lipton & Spelke, 2005). For example, young babies can discern 1:2 but not 2:3 ratios (Feigenson et al., 2004). This, numerical discrimination is independent of formal instruction and develops as a consequence of human, and other species' evolution (Dehaene, 2001; Feigenson et al., 2004). *Applied number sense* concerns those number competences related to arithmetical flexibility that prepare learners for an adult world (McIntosh et al., 1992). *Foundational number sense* (FoNS) comprises those understandings that precede *applied*, typically arise during the first year of school and require instruction (Ivrendi, 2011; Jordan & Levine, 2009). Unlike preverbal number sense, it is a "construct that children acquire or attain, rather than simply possess" (Robinson et al. 2002, p. 85). Unlike applied number sense, it does facilitate a world beyond school but later arithmetical competence. FoNS is to the development

of mathematical competence what phonic awareness is to reading (Gersten & Chard, 1999).

Below we summarise the key components of FoNS. Our intention was not to offer an extensive list of learning outcomes, as found in Berch (2005) or Howell and Kemp (2006), but a concise conceptualisation that would support a range of activities, including developments in curriculum, teacher education or assessment, as well as cross-cultural classroom analyses. To achieve these objectives we exploited the constant comparison analysis advocated by grounded theorists, a process we describe in full in Andrews and Sayers (2015).

In brief, research papers typically addressing grade one students' acquisition of number-related competence were identified. These were read and FoNS-related categories identified. With each new category, previous articles were re-examined for evidence of the new. This approach, drawing on literature from psychology, mathematics education, learning difficulties and generic education, placed, for example, *rote counting to five* and *rote counting to ten*, two narrow categories discussed by Howell and Kemp (2005), within the same broad category of systematic counting.

Number recognition: Children recognise number symbols and know their vocabulary and meaning. They can identify a particular number symbol from a collection of number symbols and name a number when shown that symbol;

Systematic counting: Children count systematically and understand ordinality and cardinality. They count to twenty and back, or count upwards and backwards from arbitrary starting points, knowing that each number occupies a fixed position in the sequence of all numbers.

Awareness of the relationship between number and quantity: Children understand not only the one-to-one correspondence between a number's name and the quantity it represents but also that the last number in a count represents the total number of objects.

Quantity discrimination: Children understand magnitude and can compare different magnitudes. They use language like bigger than or smaller than. They know that eight represents a quantity that is bigger than six but smaller than ten.

An understanding of different representations of number: Children understand that numbers can be represented differently, including the number line, different partitions, various manipulatives and fingers.

Estimation: Children can estimate, whether it be the size of a set or an object. Estimation involves moving between representations of number; for example, placing a number on an empty number line.

Simple arithmetic competence: Children perform simple arithmetical operations, which Jordan and Levine (2009) describe as the transformation of small sets through addition and subtraction.

Awareness of number patterns: Children extend and are able to identify a missing number in simple number sequences.

In sum, our systematic review identified eight distinct and not unrelated FoNS components. The fact that they are not unrelated is important as number sense "relies on many links among mathematical relationships, mathematical principles..., and mathematical procedures" (Gersten et al., 2005, p. 297). In other words, without the encouragement of such links children may be able to count but not understand that four is less than six.

EVALUATING THE FRAMEWORK'S EFFICACY

Having derived an eight component FoNS-related entitlement for grade one students, our purpose was to evaluate the framework's efficacy for identifying FoNS-related opportunities in different cultural contexts. Such a process facilitates both instrument refinement and an evaluation of its sensitivity to cultural nuances. In the following we summarise three recently reported case studies in which we evaluated the efficacy of the FoNS framework. The first examined the teaching of sequences in England and Hungary (Back et al., 2014), the second the development of students' conceptual subitising in Hungary and Sweden (Sayers et al., 2014) and the third focused on teachers' use of the number line in Poland and Russia (Andrews et al., 2015).

Each examined lesson, typically drawn from video-based teacher professional development programmes independent of not only the research presented here but also each other, involved a teacher

construed against local criteria as effective. Thus, no lesson was captured with a FoNS-related analysis in mind. Two lessons were identified from those available for topic-based case studies. Such an approach, drawing on data intended for purposes other than a FoNS analysis, made these lessons ideal for evaluating the framework's capacity for identifying topic-related, but essentially incidental, FoNS-related occurrences. In all cases teachers had been video-recorded in ways that would optimise the capture of their actions and utterances. Each video, with transcripts, was repeatedly scrutinised for evidence of FoNS components by two researchers independently. These analyses were then compared and agreements reached with respect to which FoNS-related components were being encouraged at different times. Significantly, such an approach allowed lesson episodes to be multiply-coded according to which components were observed.

As data derived from different projects in five different countries, ethical procedures were managed according to local norms. In all countries permission from school principals and participating teachers was obtained by means of letters confirming the right of teachers to withdraw without notice or reason and anonymity. With respect to the Hungarian, Polish and Russian students, all parents, at the point at which their children entered their school, had signed to agree their child's participation in ethically conducted classroom based research. In England and Sweden, parental permission letters explained the projects and, alongside the promise of minimal classroom disruption, guaranteed the same protective principles as above.

RESULTS

In the following we do two things. Firstly, we summarise qualitatively the pilot studies introduced above. Space prevents detailed summaries but we believe that sufficient has been included to demonstrate the FoNS framework's sensitivity to both cultural and mathematical context. Secondly, acknowledging the limitations of case study, we present a simple, frequency-based, quantitative analysis to highlight not only how FoNS-related learning was managed but also interesting similarities and differences in the ways the various codes interact in the case study episodes. In so doing we show how the FoNS framework can facilitate the sorts of complex analyses discussed above in

relation to our earlier European study of mathematics teaching.

The qualitative analyses

In the first study (Back et al., 2014) episodes focused on number sequences were analysed. In addition to examining the functionality of the FoNS framework an aim was to examine how teaching, focused explicitly on one FoNS component, would yield other components. The analyses, based on three episodes from each lesson sequence, indicated that Klara in Hungary addressed six of the eight FoNS components while Sarah in England addressed four. Both encouraged, throughout their respective episodes, students' recognition of number symbols, vocabulary and meaning. Both encouraged the awareness of number patterns and the identification of missing numbers and both exploited simple arithmetical operations, typically to facilitate finding the next values in a sequence. In respect of differences Klara addressed three categories, the relationship between numbers and quantities, comparisons of magnitude and different representations of number that Sarah did not, while Sarah was seen to address systematic counting when Klara did not.

However, while both teachers encouraged various FoNS components, Klara's teaching was more didactically complex, with an average of four components per episode, than Sarah's, with an average of barely two. Moreover, Klara's practice resonated with earlier research highlighting the cognitively demanding but coherent learning outcomes of Hungarian classrooms, while Sarah's reflected the relatively unsophisticated promotion of modest and less coherent goals of her English colleagues.

In the second study (Sayers et al., 2014), analyses focused on conceptual subitising in grade one lesson sequences taught by Klara, again, in Hungary, and Kerstin, in Sweden. Conceptual subitising, the ways in which individuals identify large quantities through identifying smaller quantities that comprise the whole, has been promoted as a key component of early number learning. In both cases, an average of five FoNS components were identified in each of the teacher's three analysed episodes, indicating that claims for the efficacy of teaching focused on conceptual subitising are not without warrant.

It was interesting that in neither case was conceptual subitising an explicit intention, nor were teachers expecting to address FoNS categories of learning. It is also interesting to note that despite substantial differences in the management of their lessons - Klara spent all her lesson orchestrating whole class activity with only occasional expectations of students working individually, while Kerstin spent the great majority of her time managing and supporting students working in pairs - the FoNS components addressed in their respective excerpts were remarkably similar.

Finally, the third pilot study (Andrews et al., 2015) examined episodes drawn from lesson sequences focused on the introduction and exploitation of the number line taught by Olga, in Russia, and Maria, in Poland. Here the analyses, as in the first case study, showed that such a didactical emphasis on one FoNS component does not necessarily restrict opportunities for other FoNS outcomes. For example, Olga's episodes addressed an average of almost five components, while Maria's almost four. Not surprisingly, bearing in mind the number line emphasis, all analysed episodes addressed number recognition and systematic counting, while all but one showed evidence of children being asked to work with a different representation of number.

With respect to differences, whenever Olga asked her students to represent a number on the number line, she insisted on their pointing simultaneously to zero with their left hand and the desired number with their right. In this manner her teaching focused on the relationship between number and quantity. By way of contrast, Maria presented simultaneously three distinct number lines, each showing zero to eight but with different sized intervals. In so doing she highlighted the arbitrary size of the interval alongside the need for a consistent interval size. Both teachers also used the number line to facilitate simple arithmetical operations, including tasks involving several operations simultaneously. Finally, Maria used the number line in relation to number patterns, particularly even numbers and the identification of missing numbers. Interestingly, key differences were also found in Maria's frequent use of number line representations drawn from the real world, thermometers, measuring tapes, measuring jugs and so on, something that Olga did not do.

The quantitative analysis

It is important to remember that when teaching their respective classes, none of the case study teachers were focusing explicitly on FoNS-related learning opportunities. Moreover, despite the quality of instruction focused on it, neither Kerstin nor Klara were explicitly aware of conceptual subitising as a learning goal. In other words, all project teachers, in varying degrees and in varying ways, addressed a range of FoNS-related learning outcomes in incidental rather than planned ways. The extent to which these varying ways played out can be seen in Table 1. This shows a summary of the codes applied to each analysed episode. In addition, the mean number of codes calculated for each teacher's three episodes is included alongside the teacher's name. Finally, the table shows the total number of occurrences for each FoNS component.

At a very crude level, one could argue that the mean number of categories applied to a teacher's episodes could be construed as a measure of didactical complexity. For example, Sarah's number patterns-related practice, as reflected in a mean of 2.3 categories per episode, seemed considerably lower than that of her colleagues. In this respect, the next lowest mean, Maria's 3.7, was almost one and a half categories per episode more. Thus, Sara's practice seemed to lack the didactical complexity typically found in her colleagues' episodes. However, the more interesting differences, it could be argued, emerged at the level of the topic. Notwithstanding Sarah's low didactical complexity in relation to Klara when teaching number patterns, Olga's episodes, with respect to the number line, appeared more didactically complex than Maria's, particularly in the former's repeated opportunities for her students to explore the relationship between number and quantity. With respect to conceptual subitising such differences were minor, although it could be argued that Klara paid much more attention to systematic counting than did Kerstin. However, such conclusions remain tentative, although they allude to the sensitivity of the FoNS framework to culturally-located differences.

It is also interesting to note that the topics themselves seemed to invoke different levels of complexity. Admittedly, such distinctions are crude, but are supported by the fact that Klara was involved in both the highest and lowest topic means. As can be seen from the topic means in Table 1, episodes focused on

number patterns invoked, in general terms, relatively few FoNS categories, while those focused on conceptual subitising the most. Indeed, the data suggest that some topics have a greater propensity for teachers to address a range of FoNS-related learning possibilities. However, such conclusions, while tentative, tend to confirm the sensitivity of the FoNS framework to topical differences.

Finally, with respect to this particular analysis, Table 1 highlights the relative paucity of opportunities for students to engage in quantity discrimination and the complete absence of encouragement to estimate. While it could be argued that such FoNS categories may not be suited to the three topics examined here, the fact that Klara managed to invoke quantity discrimination in two of her three number sequence-related episodes may suggest otherwise. Also, the com-

plete lack of invitation to estimate may say something different about how teachers construe mathematics as a precise rather than imprecise discipline, not least because it is not difficult to imagine a teacher asking students to estimate, say, the twentieth member of a sequence or where a given number would be placed on an empty number line.

DISCUSSION

Our aim was to introduce and summarise recent work on the development of foundational number sense (FoNS). Our uncovering of three forms of number sense has gone a long way with respect to the problem of definition. Our atypical use of constant comparison has facilitated the development of an eight component FoNS framework that we have shown to be functional in different cultural contexts and with

			FoNS categories							
	Topic and mean	Episode	Number symbols, vocabulary and meaning	Systematic counting	Relationships between numbers and quantities	Quantity discrimination	Different representations of number	Estimation	Simple arithmetical operations	Number patterns and missing numbers
Sarah (2.3)	Number patterns (3.2)	1	X							X
		2	X						X	X
		3	X	X						
Klara (4.0)	Number patterns (3.2)	1	X						X	X
		2	X		X	X	X		X	X
		3	X				X	X		
Olga (4.7)	Number line (4.2)	1	X	X	X		X			
		2	X	X	X		X		X	
		3	X	X	X		X		X	
Maria (3.7)	Number line (4.2)	1	X	X			X			
		2	X	X			X			X
		3	X	X			X		X	
Kerstin (4.7)	Subitising (4.8)	1			X		X		X	X
		2	X	X	X		X		X	
		3	X		X		X		X	X
Klara (5.0)	Subitising (4.8)	1		X	X		X		X	X
		2		X	X		X		X	X
		3	X	X	X		X		X	
Totals			15	11	10	2	14	0	12	9

Table 1: Codes applied to each episode with summary statistics

different mathematical topics. Importantly, the case studies show that different teachers, while attending to similar core topic-related outcomes, also privilege different things. For example, Olga and Maria, in their number line-related teaching, encouraged several similar outcomes, although Olga focused attention on the relationship between number and quantity while Maria emphasised real world representations of the number line. In similar vein, when working on number patterns, Klara addressed almost twice as many FoNS-related outcomes as Sarah, confirming earlier research about the relative didactical complexity of Hungarian and English mathematics teaching. The analyses also show that the three topics failed to yield any episodes in which teachers encouraged their students to estimate, a key predictor of later arithmetical competence (Booth & Siegler, 2006), while only two, identified during Klara's teaching of number patterns, alluded to quantity discrimination, also a key predictor of later competence (Titeca et al., 2014).

In closing we speculate a little and suggest that the FoNS framework has the potential to inform the practices of teacher education for elementary teachers; its simple structure makes it a suitable starting point for students' professional learning, particularly from the perspective of practicum-related planning and teaching. It can also be used as a simple assessment tool for provoking post lesson discussion.

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