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BENCHMARK SOLUTION OF 3D NATURAL CONVECTION FLOWS WITH SURFACE RADIATION IN AIR-FILLED CAVITY

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ABSTRACT

A benchmark problem of 3D natural convection/surface radiation coupling in an air-filled square cavity is investigated. The benchmark problem and reference solutions are provided for Ra=10^4,10^5 and 10^6. The relationship between the two spatial resolutions (one for the Navier-Stokes and another for the surface radiation) is also discussed.

Key Words: *Natural Convection, Surface Radiation, Benchmark Problem*

1. INTRODUCTION

Natural convection in an air-filled square cavity became a reference problem for validating the CFD codes after the pineering work of De Vahl Davis [1]. There was still no benchmark problem in an air-filled cavity for the coupling of natural convection with surface radiation in 3D cases, and it is difficult to validate the parallel 3D code developed [2]. Colomer *et al.* [3] have provided the 3D simulation results of convection and radiation in a differentially heated cavity, but according to the dimensionless parameters that he supplied, it is hard to determine the values of physical quantities such as averaged temperature T_0, temperature difference ΔT, cavity height H and so on. The aim of this work is to provide the reference solutions on the purpose of validating numerical procedures developed for investigating the interaction of natural convection with surface radiation.

2. NUMERICAL METHODS AND BENCHMARK PROBLEMS

Spectral methods and MPI (Message Passing Interface) are used for both natural convection and surface radiation. Parallel DNS is performed for the coupling laminar natural convection with surface radiation in air-filled cavities. Parallel multi-domain has been implemented by using influence matrix technique for constructing Schur complement [2].

The Benchmark problem concerns a 3D air-filled cubic cavity of height H, width W and depth D, which is an extension of the 2D benchmark problem [2]. Air in the cavity is considered as a transparent medium. The two vertical walls at x=0 and W are maintained at constant but different temperatures (T_h, T_c and $\Delta T= T_h-T_c$) and other walls are adiabatic with the convection-radiation balance. No slip condition is applied on the cavity walls. Buoyancy-driven air flow in the cavity is governed by the Navier-Stokes equations under Boussinesq assumption. The representative parameters are the geometrical aspect ratios A_x=H/W and A_y=D/H(A_x= A_y=1), Prandtl number, Pr=ν/α=0.71, and Rayleigh number, Ra=$(g\beta\Delta TH^3)/(\nu\alpha)$. The six cavity walls (grey, diffuse and opaque) have the same emissivity, ε=0.1. $T_0(=(T_h+T_c)/2)$ is set to 293.5K, then λ at T is 0.025W/(m.K). $g\beta/\nu k$=1.08E8.Table 1 displays the values of H and ΔT corresponding to each Rayleigh number investigated.

Ra	10^4	10^5	10^6
H(m)	0.020	0.045	0.097
ΔT(K)	11.574074	10.1610527	10.1452100

TABLE 1. Ra investigated and the corresponding H and ΔT

3. RESULTS

At first, the simulation results are compared with that in [3](TABLE 2).Although Colomer *et al.* [3] did not give the clear physical parameters, we can estimate the value of ΔT according to the values of Ra, Pl, and Tc/ΔT. The estimated value of ΔT is 17.954K in case of T_0=278.29K, accordingly, H=0.03369m. TABLE 2 shows that our simulation results agree well with that of Colomer *et al.* [3]. For w_{max}(z=0.5), the sum of two y values approaches to 1 because the fluid field is symmetrical along y direction.

	U_{max}(x=0.5)	y	z	W_{max}(z=0.5)	x	y	Hot Nu_c	Hot Nu_r
Colomer *et al.* [3]	0.1869	0.285	0.869	0.2865	0.069	0.115	3.983	3.385
simulated	0.1874	0.2815	0.8746	0.2890	0.0744	0.8813	3.9761	3.6471

TABLE 2. Local extreme values for velocity and average Nusselt numbers in hot wall(Ra=10^5, Pl = 0.043, Tc/ΔT=15 , ε=1 and Pr =0.71)

TABLE 3 shows average convection Nusselt numbers, maximum and minimum velocity and their positions when y=0.5 at Ra=10^4. Numerical results are compatible with that in 2D cases [2].It indicates that $(Ni*Nj*Nk)_r$ has almost no influence and they can be considered as converged solutions. Convection Nusselt numbers are more sensible to $(Ni*Nj*Nk)_{NS}$. TABLE 4 lists average Nusselt numbers in planes, maximum velocity in the cavity and their positions at Ra=10^4. Numerical results also indicate that coarser $(Ni*Nj*Nk)_r$ can be used as is shown in 2D cases [2]. TABLE 5 and 6 lists the numerical solutions in planes and in the cavity at Ra =10^5 and Ra=10^6.

$(Ni*Nj*Nk)_{NS}$	32*32*8			48*48*12		
$(Ni*Nj*Nk)_r$	16*16*4	32*32*8	48*48*12	16*16*4	24*24*6	48*48*12
Hot Nu_c	2.2493E+00	2.2492E+00	2.2492E+00	2.2486E+00	2.2486E+00	2.2486E+00
W/2 Nu_c	2.3151E+00	2.3152E+00	2.3152E+00	2.3152E+00	2.3152E+00	2.3152E+00
Cold Nu_c	2.2549E+00	2.2549E+00	2.2549E+00	2.2543E+00	2.2543E+00	2.2543E+00
W/2 U_{max}	1.6967E-01	1.6967E-01	1.6967E-01	1.6967E-01	1.6967E-01	1.6967E-01
z	8.2796E-01	8.2796E-01	8.2796E-01	8.2796E-01	8.2796E-01	8.2796E-01
W/2 U_{min}	-1.6979E-01	-1.6979E-01	-1.6979E-01	-1.6979E-01	-1.6979E-01	-1.6979E-01
z	1.7186E-01	1.7186E-01	1.7186E-01	1.7185E-01	1.7185E-01	1.7185E-01
H/2 W_{max}	1.8774E-01	1.8774E-01	1.8774E-01	1.8774E-01	1.8774E-01	1.8774E-01
x	1.1651E-01	1.1651E-01	1.1651E-01	1.1653E-01	1.1653E-01	1.1653E-01
H/2 W_{min}	-1.8796E-01	-1.8796E-01	-1.8796E-01	-1.8796E-01	-1.8796E-01	-1.8796E-01
x	8.8353E-01	8.8353E-01	8.8353E-01	8.8355E-01	8.8355E-01	8.8355E-01

TABLE 3. Benchmark solutions at Ra=10^4 in a cubic cavity (y=0.5)

$(Ni*Nj*Nk)_{NS}$	32*32*8			48*48*12		
$(Ni*Nj*Nk)_r$	16*16*4	32*32*8	48*48*12	16*16*4	24*24*6	48*48*12
Hot Nu_c	2.0914E+00	2.0913E+00	2.0913E+00	2.0907E+00	2.0907E+00	2.0906E+00
W/2 Nu_c	1.9541E+00	1.9541E+00	1.9541E+00	1.9541E+00	1.9541E+00	1.9541E+00
Cold Nu_c	2.0973E+00	2.0972E+00	2.0972E+00	2.0966E+00	2.0966E+00	2.0966E+00
Hot Nu_r	2.2754E-01	2.2747E-01	2.2746E-01	2.2754E-01	2.2749E-01	2.2746E-01
Cold Nu_r	2.2160E-01	2.2154E-01	2.2153E-01	2.2160E-01	2.2156E-01	2.2153E-01
Front Nu_r	-1.7197E-03	-1.7195E-03	-1.7195E-03	-1.7197E-03	-1.7196E-03	-1.7195E-03
Rear Nu_r	1.7197E-03	1.7195E-03	1.7195E-03	1.7197E-03	1.7196E-03	1.7195E-03
Bottom Nu_r	-8.2632E-02	-8.2632E-02	-8.2632E-02	-8.2633E-02	-8.2633E-02	-8.2633E-02
Top Nu_r	-8.0132E-02	-8.0132E-02	-8.0132E-02	-8.0132E-02	-8.0133E-02	-8.0133E-02
U_{max}	1.6987E-01	1.6987E-01	1.6987E-01	1.6987E-01	1.6987E-01	1.6987E-01
x	5.1791E-01	5.1791E-01	5.1791E-01	5.1791E-01	5.1791E-01	5.1791E-01
y	5.0000E-01	5.0000E-01	5.0000E-01	5.0000E-01	5.0000E-01	5.0000E-01
z	8.2670E-01	8.2670E-01	8.2670E-01	8.2672E-01	8.2672E-01	8.2672E-01
V_{max}	2.1908E-02	2.1908E-02	2.1908E-02	2.1909E-02	2.1909E-02	2.1909E-02
x	1.1926E-01	1.1926E-01	1.1926E-01	1.1926E-01	1.1927E-01	1.1926E-01
y	7.8080E-01	7.8080E-01	7.8080E-01	7.8081E-01	7.8081E-01	7.8082E-01
z	1.5452E-01	1.5452E-01	1.5452E-01	1.5448E-01	1.5447E-01	1.5447E-01
W_{max}	1.9105E-01	1.9105E-01	1.9105E-01	1.9105E-01	1.9105E-01	1.9105E-01
x	1.1774E-01	1.1774E-01	1.1774E-01	1.1773E-01	1.1773E-01	1.1773E-01
y	7.2637E-01	7.2637E-01	7.2637E-01	2.7362E-01	2.7362E-01	2.7361E-01
z	4.8360E-01	4.8358E-01	4.8358E-01	4.8358E-01	4.8358E-01	4.8358E-01

TABLE 4. Benchmark solutions at Ra=10^4 in a cubic cavity

(Ni*Nj*Nk)r	24*24*6	48*48*12	24*24*6	48*48*12
D/2 U_{max}	1.4646E-01	1.4646E-01	1.4493E-01	1.4493E-01
x	3.2116E-01	3.2116E-01	1.9983E-01	1.9983E-01
z	8.8920E-01	8.8920E-01	9.4030E-01	9.4030E-01
D/2 W_{max}	2.0862E-01	2.0862E-01	2.1892E-01	2.1892E-01
x	6.4433E-02	6.4433E-02	3.7136E-02	3.7136E-02
z	5.0388E-01	5.0388E-01	4.8592E-01	4.8592E-01
Hot Nu_{cmax}	8.9824E+00	8.9753E+00	1.8685E+01	1.8666E+01
y	4.9999E-01	4.9999E-01	7.8530E-01	7.8541E-01
z	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
W/2 Nu_{cmax}	1.1247E+01	1.1247E+01	2.5672E+01	2.5672E+01
y	6.3992E-01	6.3995E-01	7.7637E-01	7.7637E-01
z	8.7567E-01	8.7567E-01	1.1460E-01	1.1462E-01
Cold Nu_{cmax}	8.9710E+00	8.9642E+00	1.8684E+01	1.8666E+01
y	4.9999E-01	4.9999E-01	2.5770E-01	7.4233E-01
z	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
Hot Nu_{rmax}	5.3527E-01	5.3530E-01	1.1566E+00	1.1566E+00
y	5.0004E-01	5.0000E-01	5.0004E-01	5.0000E-01
z	2.8060E-01	2.8050E-01	2.3773E-01	2.3753E-01

TABLE 5. Benchmark solutions at Ra=10^5 and 10^6 in a cubic cavity ($(Ni*Nj*Nk)_{NS}$=48*48*12)

(Ni*Nj*Nk)r	24*24*6	48*48*12	24*24*6	48*48*12
Hot Nu_c	4.2726E+00	4.2726E+00	8.3238E+00	8.3237E+00
W/2 Nu_c	4.0964E+00	4.0964E+00	8.1167E+00	8.1168E+00
Cold Nu_c	4.2848E+00	4.2848E+00	8.3500E+00	8.3499E+00
Hot Nu_r	5.1519E-01	5.1514E-01	1.1172E+00	1.1171E+00
Cold Nu_r	5.0300E-01	5.0295E-01	1.0904E+00	1.0903E+00
Front Nu_r	-3.8124E-03	-3.8123E-03	-8.5169E-03	-8.5167E-03
Rear Nu_r	3.8124E-03	3.8123E-03	8.5169E-03	8.5167E-03
Bottom Nu_r	-2.3525E-01	-2.3524E-01	-5.4921E-01	-5.4920E-01
Top Nu_r	-2.3066E-01	-2.3066E-01	-5.3942E-01	-5.3941E-01
U_{max}	1.4914E-01	1.4914E-01	1.4784E-01	1.4784E-01
x	3.2282E-01	3.2282E-01	2.0535E-01	2.0535E-01
y	2.7687E-01	2.7687E-01	7.9359E-01	7.9359E-01
z	8.9243E-01	8.9243E-01	9.4072E-01	9.4072E-01
V_{max}	3.3471E-02	3.3471E-02	3.2452E-02	3.2452E-02
x	8.5880E-02	8.5880E-02	5.5735E-02	5.5746E-02
y	8.3668E-01	8.3668E-01	8.9530E-01	8.9530E-01
z	1.1297E-01	1.1297E-01	6.6302E-02	6.6288E-02
W_{max}	2.2834E-01	2.2834E-01	2.4122E-01	2.4122E-01
x	7.0803E-02	7.0803E-02	4.0173E-02	4.0173E-02
y	8.7281E-01	8.7281E-01	7.0221E-02	7.0232E-02
z	5.0324E-01	5.0324E-01	4.7692E-01	4.7692E-01

TABLE 6. Benchmark solutions at Ra=10^5 and 10^6 in a cubic cavity ($(Ni*Nj*Nk)_{NS}$=48*48*12)

4. CONCLUSIONS

Benchmark solutions are provided for Ra=10^4,10^5 and 10^6 by code coupling between Navier-stokes and surface radiation parallel codes. The simulation results indicate that $(Ni*Nj*Nk)_r$ can be coarser than $(Ni*Nj*Nk)_{NS}$ in 3D cases.

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