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Cascading dust inflation in Born-Infeld gravity

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Abstract. In the framework of Born-Infeld inspired gravity theories, which deviates from General Relativity (GR) in the high curvature regime, we discuss the viability of Cosmic Inflation without scalar fields. For energy densities higher than the new mass scale of the theory, a gravitating dust component is shown to generically induce an accelerated expansion of the Universe. Within such a simple scenario, inflation gracefully exits when the GR regime is recovered, but the Universe would remain matter dominated. In order to implement a reheating era after inflation, we then consider inflation to be driven by a mixture of unstable dust species decaying into radiation. Because the speed of sound gravitates within the Born-Infeld model under consideration, our scenario ends up being predictive on various open questions of the inflationary paradigm. The total number of e-folds of acceleration is given by the lifetime of the unstable dust components and is related to the duration of reheating. As a result, inflation does not last much longer than the number of e-folds of deceleration allowing a small spatial curvature and large scale deviations to isotropy to be observable today. Energy densities are self-regulated as inflation can only start for a total energy density less than a threshold value, again related to the species' lifetime. Above this threshold, the Universe may bounce thereby avoiding a singularity. Another distinctive feature is that the accelerated expansion is of the superinflationary kind, namely the first Hubble flow function is negative. We show however that the tensor modes are never excited and the tensor-to-scalar ratio is always vanishing, independently of the energy scale of inflation.

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1 Introduction

General Relativity (GR) explains gravitational phenomena in a wide range of scales, from sub-millimeter to Solar System scales [1] and (if we accept the existence of dark matter and dark energy) even on cosmological scales from Big Bang Nucleosynthesis (BBN) time until today [2]. Despite its observational success, attempts to modify GR have been proposed in the aim of alleviating some observational and theoretical problems.

On the one hand, the dark energy problem motivates the search for modifications of the infrared (IR) regime of gravity in form of scalar-tensor [3–9], vector-tensor [10–18], tensor-tensor [19–24] and non-local [25–27] theories among others, so that the cosmic acceleration would be caused by a modification of GR on cosmological scales rather than induced by the cosmological constant or some new components [28, 29]. Also at a phenomenological level, some models have been put forward to account for the dark matter [30–33]. From a more theoretical motivation, IR modifications of gravity have been considered as potential mechanisms to tackle the old cosmological constant problem [34, 35], like theories exhibiting degravitating solutions [36–39], where the cosmological constant is effectively decoupled from gravity on large scales, or unimodular gravity, where the additional Weyl symmetry is claimed to prevent quantum corrections in the form of a cosmological constant [40–42] (see however [43, 44] for claims in the opposite direction).

On the other hand, the non-renormalizability of GR also calls for modifications of gravity, in the ultraviolet (UV) regime this time, which should lead to a gravitational framework compatible with quantum physics as it is currently understood. These quantum effects are usually expected to regularise classical singularities present in Einstein equations. However, modifications of the high curvature regime have also been explored as possible mechanisms able to resolve geometric singularities without invoking quantum effects. From a more phenomenological point of view, very much like IR modifications modify the late-time cosmology

and, therefore, can be used to explain the current acceleration of the Universe, UV modifications are expected to be at work in the early Universe. Therefore, one may wonder whether they could provide a new mechanism for Cosmic Inflation [45–51]. In the standard picture, inflation is sourced by a self-gravitating scalar field in its potential dominated regime. Among the currently favoured single field models [52], let us notice that some of them are modified gravity theories, such as the Starobinsky (or Higgs inflation) model [46, 53] which belongs to $f(R)$ theories [54].

In this work we explore the possibility of realizing an inflationary phase through a Born-Infeld modification of gravity in the high curvature regime. The specific theory that we will consider here corresponds to the class of modifications introduced in Ref. [55].

The original Born-Infeld theory of electrodynamics was introduced as a way of regularising the self-energy of point-like charged particles in electromagnetism [56]. This was achieved by introducing a square root structure that gives rise to an upper bound for the allowed electromagnetic fields. Deser and Gibbons suggested to use the same idea to resolve the singularities encountered in GR [57]. However, their proposal suffered from an ambiguity, that originated from the necessity to remove a ghost present in the metric formulation of the theory. In Ref. [58], the theory was considered within the Palatini formalism, putting forward that, in that approach, the ghost is naturally avoided.

The phenomenological consequences and viability of Born-Infeld gravity theories have been extensively explored in cosmology [59–69], astrophysics [70–74], the problem of cosmic singularities [75, 76], black holes [77, 78], wormhole physics [79–82] and various extensions of the original formulation have also been considered [83–97]. Another interesting property of these theories is that they give specific realizations of Cardassian-like models [98, 99]. Recently, a natural extension of the theory was introduced in Ref. [55] where it was shown that the high curvature regime is free of singularity and may support a quasi-de Sitter expansion when the Universe is dominated by a perfect fluid having a vanishing equation of state (see also Ref. [91]). UV modifications of GR with additional degrees of freedom have been shown to provide potential candidates for dark matter [16, 100–105] so that finding an UV modification that may also support accelerated expansion opens the possibility of unifying dark matter and cosmic inflation.

A notable topic of discussion concerning Palatini theories is the potential existence of anomalies and surface singularities around sharp variations of the energy and pressure densities [106–111]. As argued in Ref. [112], gravitational theories containing auxiliary fields necessarily introduce new couplings to matter that involve derivatives of the energy-momentum tensor as source of the "Einstein equations". Thus, systems in which there is a sharp change in the density profile may lead to strong tidal forces, although Ref. [108] has argued that backreaction effects might cure such a pathology. We expect these effects to appear for variations on the density profile of the order of the new mass scale of the theory, since the corrections with respect to GR are determined by such a scale. However, in Ref. [107] it is argued that for a specific class of polytropic equations of state, and in a particular realization of Born-Infeld like theories of gravity, the appearance of the surface singularity is independent of the scale that suppresses the corrections with respect to GR. This could also be the case for the theory that we consider here and addressing this issue would require a detailed analysis that is beyond the scope of the present work. We would like to point out, however, that this pathology is very specific for one theory and one class of equation of state, so such a problem may be avoided by assuming a high enough scale suppressing the new corrections. Additionally, when tidal forces start growing, additional terms in the fluid description may

become relevant.

In the following, we consider the Born-Infeld theory introduced in Ref. [55] and show that gravitating dust in the high curvature regime may be used to support Cosmic Inflation. The inflationary dynamics in the high curvature regime strongly depends on the speed of sound. In particular, getting enough e-folds of inflation to solve the usual problems of the Friedmann-Lemaître model, together with having a graceful exit and a reheating era is a non-trivial problem. We show that the minimal model satisfying all these conditions involves a cascade of at least two decaying dust components ultimately decaying into a radiation fluid. Moreover, using Big-Bang Nucleosynthesis (BBN) constraints, such a setup gives new constraints on the energy scale at which the gravity modifications may take place, which ends up being complementary to the ones coming from astrophysical processes. The inflationary phase itself exhibits unique properties as for instance a bounded total number of e-folds, a maximal energy density and a Hubble parameter which is slowly increasing during inflation. Although in GR with scalar fields, such a feature would produce a blue spectral index for the gravitational wave spectrum, we emphasize here that tensor modes remain unamplified and inobservable, independently of the energy scale of inflation.

The paper is organized as follows. In section 2 we give a brief summary of the Born-Infeld inspired modification of gravity that we will use as well as some of its main properties. Then, in section 3 we study isotropic and homogeneous cosmological solutions taking into account the speed of sound and show in section 4 how accelerated expansion can be obtained in the presence of dust. In sections 4.2 to 4.3, we show that a viable model incorporating a graceful exit together with a reheating before BBN requires the presence of at least two unstable dust components decaying one into another and ultimately into radiation. Finally, we discuss various attractive aspects of the model as well as some properties of the tensor perturbations in the conclusion.

2 Minimal Born-Infeld theory

The minimal extension of the Born-Infeld inspired gravity considered in the following is described by the action[1] [55]

$$\mathcal{S} = m_\lambda^2 M_{\rm Pl}^2 \int \mathrm{d}^4x\sqrt{-g}\,\mathrm{Tr}\left(\sqrt{\mathbb{1} + m_\lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)} - \mathbb{1}\right), \tag{2.1}$$

where $M_{\rm Pl}$ is the reduced Planck mass, m_λ is some energy scale (in principle unrelated to $M_{\rm Pl}$), $\hat{g}^{-1}$ $(= g^{\mu\nu})$ denotes the inverse of the metric tensor, $\hat{R}$ $(= R_{\mu\nu})$ is the Ricci tensor matrix, $\mathbb{1}$ $(= \delta^\mu{}_\nu)$ is the 4×4 identity matrix and Tr( ) stands for the trace operator. This action is treated within the Palatini formalism, i.e., the Ricci tensor is constructed out of an independent connection field $\Gamma^\alpha_{\mu\nu}$. The factor in front of the action is chosen so that we recover the Einstein-Hilbert action at small curvatures and we subtract the identity inside the square brackets to guarantee the existence of Minkowski as vacuum solution. This can be seen by expanding the action at curvatures much smaller than m_λ^2:

$$\mathcal{S} = \frac{1}{2}M_{\rm Pl}^2 \int \mathrm{d}^4x\sqrt{-g}g^{\mu\nu}R_{\mu\nu}(\Gamma)\left[1 + \mathcal{O}\left(\frac{\hat{g}^{-1}\hat{R}}{m_\lambda^2}\right)\right], \tag{2.2}$$

[1]Throughout this work, a hat will be used to denote matrix representation of the corresponding tensor.

where we see that the leading order term is nothing but Einstein-Hilbert action in the Palatini formalism without cosmological constant.

Varying the action (2.1) with respect to the metric tensor yields the metric field equations

$$(M^{-1})^{\alpha}{}_{(\mu}R_{\nu)\alpha} - \mathrm{Tr}(\hat{M} - \mathbb{1})m_\lambda^2 g_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}, \tag{2.3}$$

where we have defined

$$M^{\mu}{}_{\nu} \equiv \left(\sqrt{\mathbb{1} + m_\lambda^{-2}\hat{g}^{-1}\hat{R}}\right)^{\mu}{}_{\nu}. \tag{2.4}$$

Here, we demand that $\mathbb{1} + m_\lambda^{-2}\hat{g}^{-1}\hat{R}$ is a positive definite matrix on physically admissible solutions and we define $\hat{M}$ as the only positive definite matrix such that $\hat{M}^2 = \mathbb{1} + m_\lambda^{-2}\hat{g}^{-1}\hat{R}$.

The metric field equations can be written in an alternative form by using the above definition of the fundamental matrix in order to express the Ricci tensor as $\hat{R} = m_\lambda^2\,\hat{g}(\hat{M}^2 - \mathbb{1})$. One then obtains

$$\frac{1}{2}\left[\hat{g}(\hat{M} - \hat{M}^{-1}) + (\hat{M} - \hat{M}^{-1})^T\hat{g}\right] - \mathrm{Tr}(\hat{M} - \mathbb{1})\hat{g} = \frac{1}{m_\lambda^2 M_{\rm Pl}^2}\hat{T}, \tag{2.5}$$

where we have used matrix notation and the superscript "T" stands for the transposition operator. This equation allows, in principle, to express the matrix $\hat{M}$ in terms of the metric tensor and the matter content by solving an algebraic set of equations. These equations are, in general, non-linear and several branches may arise in the theory. Nonetheless, not all of them will be physical since one must require the matrix $\hat{M}$ to be positive definite. In addition, there is only one branch of solutions that will be continuously connected with GR at low curvatures (or densities). This is indeed the branch that we will choose in this work, although other branches can also have interesting phenomenologies [55].

Variations of the action (2.1) with respect to the connection Γ gives the remaining field equations in the Palatini formalism:

$$\nabla_\lambda\left(\sqrt{-g}W^{\beta\nu}\right) - \delta_\lambda^\beta\nabla_\rho\left(\sqrt{-g}W^{\rho\nu}\right) - 2\sqrt{-g}\left(\mathcal{T}^{\kappa}_{\kappa\lambda}W^{\beta\nu} - \delta_\lambda^\beta\mathcal{T}^{\kappa}_{\kappa\rho}W^{\rho\nu} + \mathcal{T}^{\beta}_{\lambda\rho}W^{\rho\nu}\right) = 0. \tag{2.6}$$

in which we have introduced the torsion tensor $\mathcal{T}^{\mu}_{\alpha\beta} \equiv \Gamma^{\mu}_{[\alpha\beta]}$ and

$$W^{\mu\nu} \equiv (\hat{M}^{-1})^{\mu}{}_{\alpha}g^{\alpha\nu}. \tag{2.7}$$

Assuming all fields to be minimally coupled to the metric, the connection equations are not sourced by the matter sector, i.e., the right hand side (RHS) of the connection equations identically vanishes[2]. In the following, we will be interested in torsion-free solutions and therefore set $\mathcal{T}^{\alpha}_{\mu\nu} = 0$ from now on. Obviously one needs to check the consistency of this condition and we show below that it is the case for the cosmological solutions we are interested in.

For vanishing torsion, taking the trace of (2.6) gives $\nabla_\lambda\left(\sqrt{-g}\,W^{\lambda\nu}\right) = 0$ such that the connection equations reduce to

$$\nabla_\lambda\left(\sqrt{-g}\,W^{\beta\nu}\right) = 0, \tag{2.8}$$

[2]Some subtleties might arise when considering fermions, but we will not consider that case here.

for all indices. As shown in Ref. [55], if the matrix $\hat{W}$ is symmetric, an elegant way to solve these equations is to introduce the auxiliary metric $\tilde{g}$ defined by

$$\tilde{g}^{\mu\nu} \equiv \sqrt{\det \hat{M}} g^{\alpha\mu} (\hat{M}^{-1})^{\nu}{}_{\alpha}. \tag{2.9}$$

Plugging equations (2.7) and (2.9) into (2.8) gives

$$\nabla_\lambda \left(\sqrt{-\tilde{g}} \tilde{g}^{\beta\nu} \right) = 0. \tag{2.10}$$

These equations require Γ to be the Levi-Civita connection associated with the auxiliary metric $\tilde{g}$. One can check a posteriori that having a vanishing torsion is consistent with the solutions where $W^{\mu\nu}$ is actually symmetric. For a more detailed discussion on these points see Ref. [113].

3 Cosmological solutions

Homogeneous and isotropic solutions associated with the action (2.1) have been discussed in [55] for barotropic fluids and we generalize this approach to any perfect fluid below.

3.1 Hubble parameter

We assume the Friedmann-Lemaître-Robertson-Walker metric

$$\mathrm{d}s^2 = -n^2(t)\mathrm{d}t^2 + a(t)\mathrm{d}\vec{x}^2, \tag{3.1}$$

where $n(t)$ and $a(t)$ are the lapse and scale factor functions respectively. The matter sector is modeled by a homogeneous perfect fluid so that the stress tensor and the fundamental matrix are assumed to be of the form

$$T_\mu{}^\nu = \mathrm{diag}[-\rho(t), P(t), P(t), P(t)], \qquad M^\mu{}_\nu = \mathrm{diag}[_0(t), M_1(t), M_1(t), M_1(t)], \tag{3.2}$$

i.e., compatible with the spacetime symmetries. Since the matrix $\hat{M}$ must be positive definite, one has $M_0 > 0$ and $M_1 > 0$. The metric field equations (2.5) for the assumed background become

$$\begin{aligned} \frac{1}{M_0} + 3M_1 &= 4 + \bar{\rho}, \\ M_0 + 2M_1 + \frac{1}{M_1} &= 4 - \bar{P}, \end{aligned} \tag{3.3}$$

where we have defined the dimensionless quantities

$$\bar{\rho} \equiv \frac{\rho}{m_\lambda^2 M_{\mathrm{Pl}}^2}, \qquad \bar{P} \equiv \frac{P}{m_\lambda^2 M_{\mathrm{Pl}}^2}. \tag{3.4}$$

These equations allow to obtain $M_0(\rho, P)$ and $M_1(\rho, P)$ algebraically so that we can compute the auxiliary metric that generates the connection in terms of the matter content. According to (2.9), the auxiliary metric also takes a FLRW form

$$\mathrm{d}\tilde{s}^2 = -\tilde{n}^2(t)\mathrm{d}t^2 + \tilde{a}^2(t)\mathrm{d}\vec{x}^2, \tag{3.5}$$

where the auxiliary lapse and scale factor functions are given by

$$\tilde{n}^2(t) = n^2(t)\sqrt{M_0 M_1^{-3}}, \qquad \tilde{a}^2(t) = \frac{a^2(t)}{\sqrt{M_0 M_1}}. \tag{3.6}$$

Since the connection $\Gamma^{\alpha}_{\mu\nu}$ is associated to the auxiliary metric $\tilde{g}_{\mu\nu}$, the corresponding time-time component of its Einstein tensor is given by the usual expression in terms of the auxiliary Hubble parameter $\tilde{H} \equiv \dot{\tilde{a}}/\tilde{a}$, namely

$$G_{00}(\tilde{g}) = 3\tilde{H}^2 = 3\left[H - \frac{1}{4}\frac{\mathrm{d}\ln(M_0 M_1)}{\mathrm{d}t}\right]^2, \tag{3.7}$$

with $H = \dot{a}/a$ the Hubble parameter associated to the spacetime metric $g_{\mu\nu}$. On the other hand, we can use the definition of $\hat{M}$ in (2.4) to express the Einstein tensor in terms of $\hat{M}$ from its definition as follows:

$$\begin{aligned}\hat{G}(\tilde{g}) &\equiv \hat{R}(\tilde{g}) - \frac{1}{2}\,\hat{\tilde{g}}\,\mathrm{Tr}(\hat{\tilde{g}}^{-1}\hat{R}) \\ &= m_\lambda^2\,\hat{g}\left[\hat{M}^2 - \mathbb{1} - \frac{1}{2}\hat{M}\,\mathrm{Tr}\left(\hat{M} - \hat{M}^{-1}\right)\right].\end{aligned} \tag{3.8}$$

Equations (3.7) and (3.8) will lead to the equivalent of the usual Einstein equations with a modified source term, since the matrix $\hat{M}$ is an algebraic function of the matter content. For the FLRW solutions, the right hand side of equation (3.8) reads

$$G_{00}(\tilde{g}) = -\frac{m_\lambda^2 n^2(t)}{2}\left(M_0^2 - 1 - 3M_0M_1 + 3\frac{M_0}{M_1}\right) = m_\lambda^2 n^2(t)\left[M_0^2 + \frac{3}{2}\left(\bar{\rho} + \bar{P}\right)M_0 - 1\right], \tag{3.9}$$

where we have used the equation (3.3) to express everything in terms of M_0 only. Similarly, the auxiliary Hubble parameter $\tilde{H}$ can be expressed in terms of M_0 by noticing that equation (3.3) implies

$$M_0 M_1 = \frac{(4+\bar{\rho})M_0 - 1}{3}. \tag{3.10}$$

Then, from equation (3.7) and using (3.3), one gets

$$\tilde{H} = H\left\{1 - \frac{M_0}{4\left[(4+\bar{\rho})M_0 - 1\right]}\left[\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N} + (4+\bar{\rho})\frac{\mathrm{d}\ln M_0}{\mathrm{d}N}\right]\right\}, \tag{3.11}$$

where $N \equiv \ln[a(t)]$ is the "e-fold" time variable and $M_0[\bar{\rho}(N), \bar{P}(N)]$ stands for the physical solution of equations (3.3). Expanding now the total derivative of M_0 as

$$\frac{\mathrm{d}\ln M_0}{\mathrm{d}N} = \frac{\partial \ln M_0}{\partial\bar{\rho}}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N} + \frac{\partial \ln M_0}{\partial\bar{P}}\frac{\mathrm{d}\bar{P}}{\mathrm{d}N} = \frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N}\left(\frac{\partial \ln M_0}{\partial\bar{\rho}} + c_{\mathrm{s}}^2\frac{\partial \ln M_0}{\partial\bar{P}}\right), \tag{3.12}$$

where $c_{\mathrm{s}}^2 \equiv \dot{\bar{P}}/\dot{\bar{\rho}}$ is the sound speed, one obtains

$$\tilde{H} = H\left\{1 - \frac{M_0}{4\left[(4+\bar{\rho})M_0 - 1\right]}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N}\left[1 + (4+\bar{\rho})\left(\frac{\partial \ln M_0}{\partial\bar{\rho}} + c_{\mathrm{s}}^2\frac{\partial \ln M_0}{\partial\bar{P}}\right)\right]\right\}. \tag{3.13}$$

From equations (3.9) and (3.13) one finally obtains the modified Friedmann-Lemaître equation

$$\bar{H}^2 = \frac{1}{3}\frac{M_0^2 + \frac{3}{2}(\bar{P}+\bar{\rho})M_0 - 1}{\left\{1 - \frac{M_0}{4\left[(4+\bar{\rho})M_0 - 1\right]}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N}\left[1 + (4+\bar{\rho})\left(\frac{\partial \ln M_0}{\partial\bar{\rho}} + c_{\mathrm{s}}^2\frac{\partial \ln M_0}{\partial\bar{P}}\right)\right]\right\}^2}, \tag{3.14}$$

where $\bar{H} \equiv H/m_\lambda$ is the dimensionless Hubble parameter in units of the new scale m_λ. As previously advertised, our Born-Infeld inspired gravity theory within the Palatini formalism has led us to a version of the Friedmann-Lemaître equation where the matter source is modified. Let us stress again that $M_0 = M_0(\bar{\rho}, \bar{P})$ so that the RHS of the above equation is also a function of $\bar{\rho}$ and $\bar{P}$. A remarkable feature that should not be unnoticed is the appearance of derivatives of $\bar{\rho}$ and $\bar{P}$, which will play a crucial role in the dynamics of the system, as we show below. Moreover, this introduces the novel effect that the background cosmological evolution is not only determined by the equation of state parameter $w \equiv \bar{P}/\bar{\rho}$ of the fluid as in the standard case, but also by the sound speed of the fluid $c_{\rm s}$.

To end this section we stress that the matter sector is assumed to be minimally coupled to the metric tensor $g_{\mu\nu}$. In that situation, conservation of the stress-tensor yields the usual equation

$$\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N} = -3(\bar{P} + \bar{\rho}). \tag{3.15}$$

3.2 Physical branch

The metric field equations (3.3) generically lead to several branches for $M_0(\bar{\rho}, \bar{P})$ and $M_1(\bar{\rho}, \bar{P})$ due to their non-linearity. These have been thoroughly discussed in Ref. [55] and we simply summarize the results here. Solving for M_0 gives the cubic equation

$$(4+\bar{\rho})M_0^3 + \left[\bar{P}(4+\bar{\rho}) + \frac{2}{3}(1+\bar{\rho})^2 - 4\right] M_0^2 - \left[\bar{P} + \frac{4}{3}(1+\bar{\rho})\right] M_0 + \frac{2}{3} = 0. \tag{3.16}$$

There are three different branches of solution, but only two of them are admissible on physical grounds, i.e., obtained by imposing the positivity of the fundamental matrix $\hat{M}$. Out of those two physical solutions, only one branch matches GR at low energy densities. Although the expression is not particularly illuminating, it is explicit:

$$M_0 = \frac{1}{18(\bar{\rho}+4)} \left[2^{2/3} \sqrt[3]{27\sqrt{3}(4+\bar{\rho})\sqrt{-B} - A} + \frac{2^{1/3} C}{\sqrt[3]{27\sqrt{3}(4+\bar{\rho})\sqrt{-B} - A}} - D \right], \tag{3.17}$$

where

$$\begin{aligned}
A &= 3456\bar{P}^3 + \left(108\bar{P}^2 + 576\bar{P} + 168\right)\bar{\rho}^4 - 4752\bar{P}^2 + \left(54\bar{P}^3 + 1080\bar{P}^2 + 1206\bar{P} + 680\right)\bar{\rho}^3 \\
&\quad + \left(648\bar{P}^3 + 3159\bar{P}^2 + 720\bar{P} + 2670\right)\bar{\rho}^2 + \left(2592\bar{P}^3 + 1080\bar{P}^2 + 4302\bar{P} + 2616\right)\bar{\rho} \\
&\quad + (72\bar{P} + 96)\bar{\rho}^5 + 9144\bar{P} + 16\bar{\rho}^6 + 1456, \\
B &= 144\bar{P}^4 - 1248\bar{P}^3 + \left(4\bar{P}^2 - 32\bar{P} + 32\right)\bar{\rho}^4 + 2404\bar{P}^2 + \left(9\bar{P}^4 - 468\bar{P}^2 + 384\bar{P} + 52\right)\bar{\rho}^2 \\
&\quad + \left(12\bar{P}^3 - 80\bar{P}^2 - 24\bar{P} + 96\right)\bar{\rho}^3 + \left(72\bar{P}^4 - 504\bar{P}^3 + 256\bar{P}^2 - 920\bar{P} + 1024\right)\bar{\rho} \\
&\quad - 73456\bar{P} + 1600, \\
C &= \left(18\bar{P}^2 + 144\bar{P} + 24\right)\bar{\rho}^2 + \left(144\bar{P}^2 + 126\bar{P} + 200\right)\bar{\rho} + 288\bar{P}^2 + (24\bar{P} + 32)\bar{\rho}^3 + 8\bar{\rho}^4 \\
&\quad - 264\bar{P} + 488, \\
D &= (6\bar{P} + 8)\bar{\rho} + 24\bar{P} + 4\bar{\rho}^2 - 20.
\end{aligned} \tag{3.18}$$

Both M_0 and M_1 are well-defined, i.e., real and positive, only on a given domain in the plane $(\bar{\rho}, \bar{P})$ which has been represented in Figure 1. We have also represented in this figure various barotropic equations of state $P = w\rho$, with constant w, and one can single out three typical behaviours:

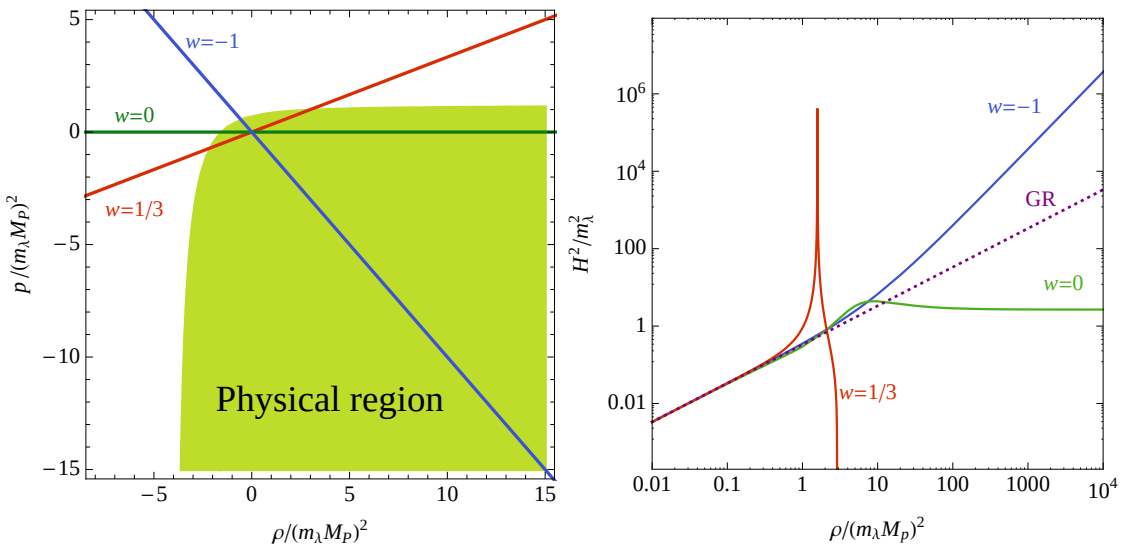

Figure 1. Left panel: Region in the plane $(\bar{\rho}, \bar{P})$ obtained by imposing that the fundamental matrix $\hat{M}$ is positive definite, i.e., that both M_0 and M_1 are real and positive valued. We see that such a region is basically defined by the simultaneous conditions $\rho \gtrsim -4m_\lambda^2 M_{\rm Pl}^2$ and $\bar{P} \lesssim m_\lambda^2 M_{\rm Pl}^2$. For an idealized barotropic fluid $P = w\rho$, with a constant equation of state parameter w, the pressure and/or the energy densities may end up being bounded. Right panel: We show the Hubble function as a function of the energy density for different equation of state parameters illustrating the 3 behaviours discussed in the main text. We can see that at low energy densities they all follow the usual GR behavior (dotted line), whereas at high energy densities ($\rho \gg m_\lambda^2 M_{\rm Pl}^2$) the differences appear. In particular, for a dust component with $w = 0$ we see that H^2 goes to a constant value, i.e., it gives a de Sitter phase.

- For fluids with positive pressure $0 < w < 1$ we find that there is a maximum value for ρ which is given by $\rho \lesssim m_\lambda^2 M_p^2$. This is the desired property of Born-Infeld inspired theories, i.e., we find an upper bound for the allowed energy densities.

- If $0 \leq w < -2/3$, there is no upper bound on ρ. As discussed below, depending on the behaviour of $c_{\rm s}^2$, the Hubble function can take a constant value at high energy densities. Even though $\bar{\rho}$ can grow indefinitely, the curvature, here parametrically given by H, remains bounded.

- Finally, for $-2/3 < w \leq -1$ we find that the Hubble function grows as $H^2 \propto \rho^2$. In this case there is no realization of the Born-Infeld mechanism. Interestingly enough, such a behaviour is also typical of theories with extra-dimensions [114].

As mentioned above, these three ideal cases are only illustrative and one should keep in mind that in the cosmological context $c_{\rm s}^2$ is expected to be a function of time, and thus of the Hubble parameter. As a result, the trajectory followed by any realistic gravitating fluid in the plane $(\bar{\rho}, \bar{P})$ of Figure 1 is in general a curve becoming strongly non-linear as soon as $\bar{\rho}$ or $\bar{P}$ becomes of order unity.

On the contrary, in the low energy and pressure limit, plugging the expression for M_0 back into the Hubble parameter (3.14) and expanding everything for small $\bar{\rho}$ and $\bar{P}$, one gets

$$\bar{H}^2 = \frac{1}{3}\bar{\rho} + \frac{2c_{\rm s}^2 - 1}{4}\bar{P}\bar{\rho} + \frac{4c_{\rm s}^2 - 1}{8}\bar{\rho}^2 - \frac{1}{8}\bar{P}^2 + \mathcal{O}\big(\bar{\rho}^3, \bar{P}^3\big)\,. \tag{3.19}$$

As expected, this expression matches the usual Friedmann-Lemaître equation for $\rho \ll m_\lambda^2 M_{\rm Pl}^2$ and $P \ll m_\lambda^2 M_{\rm Pl}^2$. As a result, the energy scale at which one should expect deviations from the standard GR case is determined by the geometrical mean of m_λ and $M_{\rm Pl}$.

4 Inflationary scenario

In the previous section we have reviewed the cosmological evolution for the Born-Infeld inspired gravity theory under consideration. We have shown that for a perfect fluid with equation of state parameter satisfying $-2/3 < w \leq 0$, the energy density may not be bounded from above, but the curvature is. In particular, as we show in more detail in the next section, for a dust gravitating fluid having $w = 0$, the Hubble parameter becomes nearly constant at high energy densities thereby allowing a quasi-de Sitter expansion typical of an inflationary epoch [115]. Since the theory matches GR at low energies, the inflationary graceful exit is always naturally realised within our Born-Infeld inspired gravity when such a regime is reached. However, with only dust, the Universe would end up being matter dominated after inflation and, in order to produce a radiation dominated Universe, one needs to implement some mechanism allowing the dust to decay into radiation. This is analogous to the reheating period within the standard inflationary picture where the inflaton decays at the end of inflation giving rise to radiation made out of relativistic degrees of freedom.

Thus, in section 4.2, instead of stable dust, we will consider an unstable dust fluid decaying into radiation. However, in this scenario the sound speed is no longer vanishing, see equation (3.14), and this has a crucial effect since now the existence of a quasi-de Sitter era becomes possible only within a finite range of energy densities. Although inflation can be made to last long enough by adequately fixing the decay rate of the dust component, we show that the duration of the reheating era ends up being necessarily longer than the inflationary period. On the other hand, the decay rate will also determine the beginning of the radiation era, so the duration of inflation and the reheating period are linked. This in fact prevents the model from solving the flatness problem of the standard FLRW cosmology.

In section 4.3, this issue is solved by considering a cascade of decaying dust fluids which ultimately end into radiation. In such a case, inflation can be realized with a graceful exit onto a reheating era. The solution comes about because while the duration of inflation depends on the whole cascade of dust fluids, the reheating duration depends only on the decay rate of the last component thereby making the model viable.

4.1 Stable dust

Let us start by considering the gravitating fluid to be pure dust and conserved characterized by

$$w = \frac{\bar{P}}{\bar{\rho}} = 0, \qquad c_{\rm s}^2 = 0, \qquad \frac{{\rm d}\bar{\rho}}{{\rm d}N} = -3\bar{\rho}. \tag{4.1}$$

Plugging these conditions into equations (3.14) and (3.17) gives the Hubble parameter as an algebraic function of $\bar{\rho}$ only. If we take the high energy density limit $\bar{\rho} \gg 1$ (i.e., in the

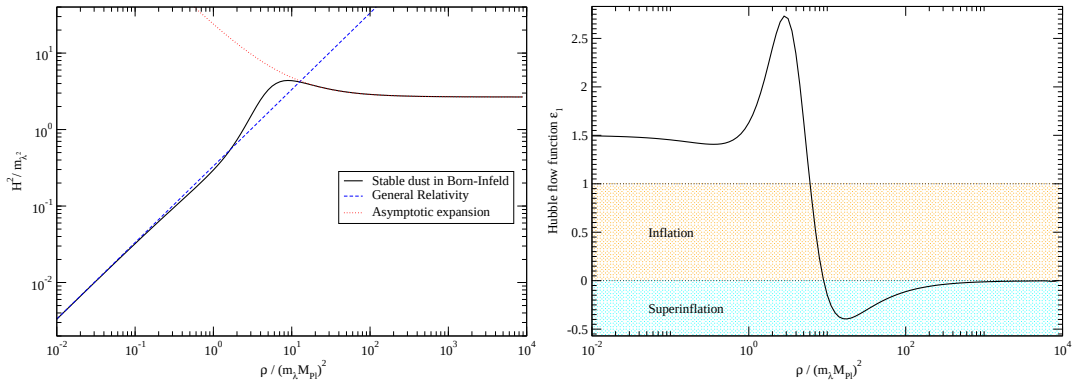

Figure 2. Hubble parameter square $\bar{H}^2$ as a function of the stable dust energy density $\bar{\rho}$ (left panel). In the Born-Infeld regime, $\bar{\rho} \gg 1$, the Hubble parameter is nearly constant and slowly grows $\dot{H} > 0$ to generate a superinflationary era. The red dotted curve corresponds to the asymptotic expansion of equation (4.2). In the right panel, the first Hubble flow function ϵ_1 is represented as a function of $\bar{\rho}$. In the superinflationary regime $\epsilon_1 < 0$.

Born-Infeld regime), we find

$$\bar{H} = \sqrt{\frac{8}{3}} + \frac{8\sqrt{3} - 3\sqrt{6}}{\bar{\rho}} + \mathcal{O}\left(\frac{1}{\bar{\rho}^2}\right), \tag{4.2}$$

so that the Hubble parameter becomes nearly constant and the expansion of the Universe is accelerated. The exact dependence of $\bar{H}$ with respect to $\bar{\rho}$ as well as the above expansion are represented in Figure 2. Strictly speaking, acceleration of the scale factor, i.e. inflation, occurs as long as the first Hubble flow function $\epsilon_1 \equiv -\mathrm{d}\ln(H)/\mathrm{d}N$ is less than unity. From equation (4.2) we obtain

$$\epsilon_1 = -\frac{9(4\sqrt{2} - 3)}{2\bar{\rho}} + \mathcal{O}\left(\frac{1}{\bar{\rho}^2}\right), \tag{4.3}$$

such that for $\bar{\rho} \gg 1$ one has $\epsilon_1 \lesssim 0$. Negative values of ϵ_1 cannot be obtained within General Relativity and correspond to superinflation [116]. Given that the gravitational sector is not described by the usual Einstein-Hilbert term, let us emphasize that it is not possible to deduce from $\epsilon_1 < 0$ that the primordial perturbations will have a nearly scale invariant power spectrum with a blue spectral index. The cosmological perturbations are discussed in more detail in the conclusion where we show that tensor perturbations are in fact not generated.

Finally, as can be seen in Figure 2, as soon as $\bar{\rho} \lesssim 1$, the Hubble parameter evolution becomes nearly identical to GR (up to some relaxation oscillations) and inflation naturally ends. However, in the subsequent decelerated expansion, the Universe is and will remain matter dominated, which is in contradiction with both BBN and the existence of the Cosmic Microwave Background (CMB). The most natural extension to produce a radiation dominated Universe after inflation is to assume that the dust component is actually decaying into radiation. We explore this possibility in the next section.

4.2 Decaying dust and radiation

In order to generate a radiation dominated universe after inflation, let us now consider an unstable dust component that decays into radiation at a constant rate Γ. Thus, the matter

sector consists of two fluids, decaying dust and radiation, in interaction, which are described by the following coupled equations:

$$\frac{\mathrm{d}\bar{\rho}_1}{\mathrm{d}t} + 3H\bar{\rho}_1 = -\Gamma\bar{\rho}_1, \qquad \frac{\mathrm{d}\bar{\rho}_\mathrm{r}}{\mathrm{d}t} + 4H\bar{\rho}_\mathrm{r} = \Gamma\bar{\rho}_1, \tag{4.4}$$

where $\bar{\rho}_1$ and $\bar{\rho}_\mathrm{r}$ are the energy densities of dust and radiation respectively and H is given by equation (3.14). Again, since the matter sector is minimally coupled, we have conservation of the total energy density $\bar{\rho} = \bar{\rho}_\mathrm{r} + \bar{\rho}_1$. However, the total pressure does no longer vanish and reads $\bar{P} = \bar{\rho}_\mathrm{r}/3$. In addition, we are also in the presence of a time-dependent sound speed c_s^2.

We can rewrite the equations of the interacting dust and radiation system in a more convenient manner by introducing the radiation fraction

$$X \equiv \frac{\bar{\rho}_\mathrm{r}}{\bar{\rho}} = \frac{\bar{\rho}_\mathrm{r}}{\bar{\rho}_\mathrm{r} + \bar{\rho}_1}\,, \tag{4.5}$$

so that equation (4.4) can be recast into equations of evolution for $\bar{\rho}$ and X

$$\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N} = -(3+X)\bar{\rho}, \qquad \frac{\mathrm{d}X}{\mathrm{d}N} = (X-1)X + (1-X)\frac{\bar{\Gamma}}{\bar{H}}\,, \tag{4.6}$$

where $\bar{\Gamma} \equiv \Gamma/m_\lambda$ and $\bar{P} = X\bar{\rho}/3$. From the definition of $c_\mathrm{s}^2 \equiv \dot{\bar{P}}/\dot{\bar{\rho}}$, one gets

$$c_\mathrm{s}^2 = \frac{4X}{3(3+X)} - \frac{1-X}{3(3+X)}\frac{\bar{\Gamma}}{\bar{H}}\,. \tag{4.7}$$

Because the above expression explicitly involves $\bar{H}$, and equation (3.14) depends on c_s^2, the resulting evolution of the system is highly non-linear and crucially depends on the functional form of c_s^2. As discussed before, because the speed of sound "gravitates" within Palatini theories, this is expected and emphasizes the importance of considering more than the equation of state parameter in the fluid description in order to obtain the background cosmological evolution.

Plugging equation (4.7) into equation (3.14) yields an algebraic equation for $\bar{H}$ that can be analytically solved in terms of $\bar{P}$ and $\bar{\rho}$ as[3]

$$\bar{H} = \frac{\sqrt{\frac{M_0^2}{3} + \frac{1}{2}(\bar{P}+\bar{\rho})M_0 - \frac{1}{3}} + \frac{3(\bar{P}+\bar{\rho})}{4}\frac{(4+\bar{\rho})\,{c_\mathrm{s}}_1^2\,\bar{\Gamma}}{(4+\bar{\rho})M_0 - 1}\frac{\partial \ln M_0}{\partial \bar{P}}}{1 + \frac{3(\bar{P}+\bar{\rho})}{4}\frac{M_0}{(4+\bar{\rho})M_0-1}\left[1 + (4+\bar{\rho})\left(\frac{\partial \ln M_0}{\partial \bar{\rho}} + {c_\mathrm{s}}_0^2\frac{\partial \ln M_0}{\partial \bar{P}}\right)\right]}\,, \tag{4.8}$$

where we have defined

$${c_\mathrm{s}}_0^2 \equiv \frac{4X}{3(3+X)}\,, \qquad {c_\mathrm{s}}_1^2 \equiv \frac{1-X}{3(3+X)}\,. \tag{4.9}$$

This expression matches equation (3.14) if one sets both the decay rate $\bar{\Gamma}$ and the radiation fraction X to zero, as one may expect. However, for non-vanishing $\bar{\Gamma}$, the limit $X \to 0$ does not give back the pure dust behaviour as there will always be a term proportional to $\bar{\Gamma}$ in the expression of $\bar{H}$. This is important for the initial conditions of the inflationary scenario

[3] When solving this equation we obtain two branches. We choose the one corresponding to an expanding rather than contracting universe.

since it will set a maximum value for the total energy density. More precisely, expanding equation (4.8) at large $\bar{\rho}$ and small X we obtain for the initial value of the Hubble parameter

$$\begin{aligned}\bar{H}_{\rm ini} = &\sqrt{\frac{8}{3}} - \frac{21+\sqrt{2}}{24}\,\bar{\Gamma} - \frac{\bar{\Gamma}}{6\sqrt{2}}\,\bar{\rho} + \left(\frac{4}{3\sqrt{3}} - \frac{4+149\sqrt{2}}{288}\,\bar{\Gamma}\right) X\bar{\rho} - \frac{4+\sqrt{2}}{72}\,\bar{\Gamma}\,X\bar{\rho}^2 \\ &+ \left(\frac{-124+189\sqrt{2}}{12\sqrt{3}} + \frac{-8548+4217\sqrt{2}}{1152}\,\bar{\Gamma}\right) X + \left(8\sqrt{3} - 3\sqrt{6} + \frac{248-249\sqrt{2}}{64}\,\bar{\Gamma}\right)\frac{1}{\bar{\rho}} \\ &+ \left(\frac{829}{3\sqrt{3}} - 59\sqrt{6} + \frac{454560-292333\sqrt{2}}{4608}\,\bar{\Gamma}\right)\frac{X}{\bar{\rho}} + \mathcal{O}\left(\frac{1}{\bar{\rho}^2}, X^2\right).\end{aligned} \tag{4.10}$$

As opposed to pure stable dust, there is now a maximal value of $\bar{\rho}$ at which the Hubble parameter vanishes. Such a behaviour happens for any value of X, even for $X = 0$, and therefore is not due to non-vanishing pressure but induced by a non-vanishing speed of sound. Let us stress that this is only relevant for the initial conditions since, if we have a non-vanishing value of Γ, the evolution of the system will immediately generate a certain amount of radiation. Under these considerations and at leading order in small X and $\bar{\Gamma}$, one gets $\bar{H}(\bar{\rho}_{\rm max}) = 0$ for

$$\bar{\rho}_{\rm max} = \sqrt{\frac{8}{3}}\left(\frac{\bar{\Gamma}}{6\sqrt{2}} - \frac{4}{3\sqrt{3}}X\right) \tag{4.11}$$

where we see that for $X = 0$, we obtain the non-vanishing maximum value

$$\bar{\rho}_{\rm max} = \frac{24}{\sqrt{3}\bar{\Gamma}}\,. \tag{4.12}$$

This condition guarantees that the universe starts in an expanding phase, so the inflationary period will be achieved. We should emphasize that this bound appears if we impose a vanishing radiation component initially. If we have some radiation initially, then (4.11) should be used instead. Interestingly, initial energy densities higher than $\bar{\rho}_{\rm max}$ continuously map into negative values of the initial Hubble parameter. This suggests that some of our inflationary solutions could be extended back in time with a bounce [117–119].

In order to discuss the inflationary phase only, we will assume in the following that the Universe starts its evolution with $\bar{\rho}(0) = \bar{\rho}_{\rm ini} \lesssim \bar{\rho}_{\rm max}$, in the regime in which $\bar{H}$ is independent of $\bar{\rho}$, and with only decaying dust, i.e. having $X(0) = 0$. Equations (4.6) can be approximated by

$$\frac{{\rm d}X}{{\rm d}N} \simeq -X + \frac{\bar{\Gamma}}{\bar{H}}\,, \qquad \frac{{\rm d}\bar{\rho}}{{\rm d}N} \simeq -3\bar{\rho}, \tag{4.13}$$

the solutions of which are

$$X(N) \simeq \frac{\bar{\Gamma}}{\bar{H}}\,, \qquad \bar{\rho}(N) = \bar{\rho}_{\rm ini}\, e^{-3N}. \tag{4.14}$$

Therefore, while the Universe is inflating, the radiation fraction remains constant and given by $\bar{\Gamma}/\bar{H}$. This in turn allows to obtain the condition guaranteeing that the system remains inside the physical region of the theory as depicted in Figure 1. The radiation component gives rise to a positive pressure given by $\bar{P} \simeq \frac{1}{3}X\bar{\rho}$ so that, according to the above solution for X, we have $\bar{P} \simeq \bar{H}\bar{\rho}/(3\bar{\Gamma})$ so the condition to be inside the physical region $\bar{P} \lesssim 1$ leads to

a bound for $\bar{\rho}$ given by $\bar{\rho} \lesssim \sqrt{8}/(3\sqrt{3}\bar{\Gamma})$. It is of the same order of magnitude as the bound given by (4.12), although obtained from a different condition.

On the other hand, the total energy density is driven by the dust component and decreases exponentially fast. Inflation ends for $\epsilon_1(\bar{\rho}) = 1$, i.e. when we exit the Born-Infeld regime and the evolution equations become close to those in GR. In order to obtain the duration of the inflationary era, we only need an order of magnitude estimate for $\bar{\rho}(N_{\rm end})$ which can be obtained by solving $\bar{H}(\bar{\rho}) = \sqrt{\bar{\rho}/3}$ by using the asymptotic expansion (4.10). At leading order, one gets $\bar{\rho}(N_{\rm end}) \simeq 8$ and, therefore, the maximum number of e-folds of inflation is

$$\Delta N_{\rm inf} = \frac{1}{3}\ln\left(\frac{\bar{\rho}_{\rm max}}{8}\right) \simeq \frac{1}{3}\ln\left(\frac{\sqrt{3}}{\bar{\Gamma}}\right). \tag{4.15}$$

In order to have more than 60 e-folds of inflation (which is the typically required minimum duration of inflation), the decay rate of the dust component should be very small, namely $\Gamma < 10^{-78} m_\lambda$, which compromises the appearance of the radiation era after inflation. For m_λ of the order of the Planck mass, this implies that the dust component should have a lifetime around 10^{34} s, i.e. much longer than the (GR) age of the Universe $\simeq 10^{17}$ s.

Considering a mass scale m_λ much larger than the Planck mass does not help either. Indeed, the reheating proceeds only when the cosmological evolution matches GR, and the radiation era starts when the reheating is completed at $X(N_{\rm reh}) \simeq 1$. Assuming $X \ll 1$ during reheating, we can approximate the evolution equations by

$$\frac{{\rm d}X}{{\rm d}N} \simeq -X + \frac{\sqrt{3}\,\bar{\Gamma}}{\bar{\rho}^{1/2}}, \qquad \bar{\rho}(N > N_{\rm end}) \simeq \bar{\rho}(N_{\rm end})\, e^{-3\Delta N} \simeq 8 e^{-3\Delta N}, \tag{4.16}$$

where $\Delta N \equiv N - N_{\rm end}$. Therefore, with $X(N_{\rm end}) \simeq 0$, one gets

$$X(N > N_{\rm end}) \simeq e^{-\Delta N} \int_{N_{\rm end}}^{N} e^{\Delta n} \frac{\sqrt{3}\bar{\Gamma}}{\rho^{1/2}(n)}\, {\rm d}n \simeq \frac{\sqrt{3}\,\bar{\Gamma}}{5\sqrt{2}} e^{3\Delta N/2}, \tag{4.17}$$

and solving for $X \simeq 1$ gives

$$\Delta N_{\rm reh} \simeq \frac{2}{3}\ln\left(\frac{5\sqrt{2}}{\sqrt{3}\,\bar{\Gamma}}\right) = 2\Delta N_{\rm inf} + \frac{1}{3}\ln\left(\frac{50}{9}\right). \tag{4.18}$$

One concludes that, independently of $\bar{\Gamma}$ and m_λ, the reheating era typically lasts twice the number of e-folds of inflation. Because reheating is a decelerating era, this catastrophically prevents the inflationary era to solve the flatness and horizon problem. The problem is that the decay rate determines both the duration of inflation and the reheating period so that both are intimately linked. In the next section we avoid this difficulty by introducing more than one decaying dust component.

4.3 Cascading dust and radiation

In order to achieve a viable inflationary scenario leading to a radiation dominated universe, one is led to consider a cascade of unstable dust components decaying one into another and ultimately producing radiation. Intuitively, the number of e-folds required to reheat the Universe will be set by the lifetime of the most stable specie whereas the speed of sound evolution, and thus inflation, is expected to be sensitive to all species or, equivalently, all the

unstable components will contribute to generating a radiation fraction and, thus, to the total pressure. For this scenario, one finds that the relation between $\Delta N_{\rm reh}$ and $\Delta N_{\rm inf}$ is indeed relaxed. The conservation equations for each species are given by

$$\begin{aligned} &\frac{\mathrm{d}\bar{\rho}_i}{\mathrm{d}t} + 3H\bar{\rho}_i = \Gamma_{i-1}\bar{\rho}_{i-1} - \Gamma_i\bar{\rho}_i \qquad i = 1, ..., n \\ &\frac{\mathrm{d}\bar{\rho}_{\rm r}}{\mathrm{d}t} + 4H\bar{\rho}_{\rm r} = \Gamma_n\bar{\rho}_n, \end{aligned} \tag{4.19}$$

with $\Gamma_0 = \bar{\rho}_0 = 0$ and where $\bar{\rho}_i$ are the energy densities of the dust components, Γ_i are the decay rates of the corresponding particles and $\bar{\rho}_{\rm r}$ is the energy density of the radiation component as before. Introducing the relative fractions

$$X_i \equiv \frac{\bar{\rho}_i}{\sum_{i=1}^n \bar{\rho}_i + \bar{\rho}_{\rm r}}\,, \qquad X = \frac{\bar{\rho}_{\rm r}}{\sum_{i=1}^n \bar{\rho}_i + \bar{\rho}_{\rm r}}\,, \tag{4.20}$$

one has now to solve the coupled system of equations

$$\frac{\mathrm{d}X_i}{\mathrm{d}N} = \left(X - \frac{\bar{\Gamma}_i}{\bar{H}}\right) X_i + \frac{\bar{\Gamma}_{i-1}}{\bar{H}} X_{i-1}\,, \qquad \frac{\mathrm{d}X}{\mathrm{d}N} = (X-1)X + \frac{\bar{\Gamma}_n}{\bar{H}} X_n, \tag{4.21}$$

with $\bar{H}$ given in equation (3.14). Again, the total energy density $\bar{\rho}$ is conserved so that it satisfies (4.6). The total pressure is still driven by the radiation component and reads $P = X\bar{\rho}/3$ whereas the sound speed becomes

$$c_{\rm s}^2 = \frac{4X}{3(3+X)} - \frac{X_n}{3(3+X)}\frac{\bar{\Gamma}_n}{\bar{H}}\,. \tag{4.22}$$

4.3.1 Inflationary regime

Comparing this expression to equation (4.7), one deduces that the Hubble parameter for cascading dust and radiation is given by equation (4.8) with

$$c_{{\rm s}_0}^2 \equiv \frac{4X}{3(3+X)}\,, \qquad c_{{\rm s}_1}^2 \equiv \frac{X_n}{3(3+X)}\,. \tag{4.23}$$

As a result, the Hubble parameter has an explicit dependence only on the total energy density $\bar{\rho}$, the radiation fraction X, the fraction X_n of the last specie in the decaying cascade as well as its associated decay rate $\bar{\Gamma}_n$ (into radiation). Expanding the Hubble parameter at large value of $\bar{\rho}$ and small values of X and X_n, one gets

$$\begin{aligned} \bar{H} = &\sqrt{\frac{8}{3}} - \frac{21+\sqrt{2}}{24}\bar{\Gamma}_n X_n - \frac{\bar{\Gamma}_n X_n}{6\sqrt{2}}\bar{\rho} + \left(\frac{4}{3\sqrt{3}} - \frac{4+173\sqrt{2}}{288}\bar{\Gamma}_n X_n\right) X\bar{\rho} - \frac{4+\sqrt{2}}{72}\bar{\Gamma}_n X_n X\bar{\rho}^2 \\ &+ \left(\frac{-124+189\sqrt{2}}{12\sqrt{3}} + \frac{-9556+4169\sqrt{2}}{1152}\bar{\Gamma}_n X_n\right) X + \left(8\sqrt{3} - 3\sqrt{6} + \frac{248-249\sqrt{2}}{64}\bar{\Gamma}_n X_n\right)\frac{1}{\bar{\rho}} \\ &+ \left[\frac{\sqrt{3}\left(856+493\sqrt{2}\right)}{90+63\sqrt{2}} + \frac{7\left(67488-44323\sqrt{2}\right)}{4608}\bar{\Gamma}_n X_n\right]\frac{X}{\bar{\rho}} + \mathcal{O}\left(\frac{1}{\bar{\rho}^2}, X^2, X_n^2\right). \end{aligned} \tag{4.24}$$

Up to some numerical coefficients, this expression is formally identical to equation (4.8) with the replacement $\bar{\Gamma} \to \bar{\Gamma}_n X_n$. Therefore, starting within the superinflationary regime at vanishing radiation now requires $\bar{\rho} < \bar{\rho}_{\rm max}$ with

$$\bar{\rho}_{\rm max} \simeq \frac{24}{\sqrt{3}\bar{\Gamma}_n X_n}\,, \tag{4.25}$$

and the duration of inflation can, a priori, be made longer by having small values of X_n instead of $\bar{\Gamma}_n$. However, X_n is not arbitrary but given by solving the set of equations (4.21). The crucial point to realize here is that, in the previous case with only one dust component, having small X required having small Γ as well, i.e., the two conditions were related, whereas in the cascading case the condition will depend on the whole set of decay rates, as we show in the following.

Assuming $\bar{H}$ to be almost constant with $\bar{\rho}(0) = \bar{\rho}_{\rm ini} \lesssim \bar{\rho}_{\rm max}$ together with $X_1(0) = 1$, $X_{i\neq 1}(0) = 0$ and $X(0) = 0$, one can find an approximate solution for all the species. One gets

$$\frac{{\rm d}X_1}{{\rm d}N} \simeq -\frac{\bar{\Gamma}_1}{\bar{H}} X_1, \qquad \frac{{\rm d}X_i}{{\rm d}N} \simeq -\frac{\bar{\Gamma}_i}{\bar{H}} X_i + \frac{\bar{\Gamma}_{i-1}}{\bar{H}} X_{i-1}. \tag{4.26}$$

The first dust component can be immediately integrated as

$$X_1(N) = \exp\left(-\frac{\bar{\Gamma}_1}{\bar{H}} N\right) \simeq 1, \tag{4.27}$$

where the last approximation requires $N \ll \bar{H}/\bar{\Gamma}_1$. For instance, if we require the duration of inflation to be $\mathcal{O}\big(10^2\big)$, we need $\bar{\Gamma}_1 \lesssim 10^2 \bar{H}$. Obviously this is nothing but the condition that the dust component should remain stable throughout the whole inflationary phase. The remaining components of the cascade X_i's are given by the quadrature

$$X_i(N) \simeq \frac{\bar{\Gamma}_{i-1}}{\bar{H}} \int_0^N e^{(\bar{\Gamma}_i/\bar{H})(y-N)} X_{i-1}(y){\rm d}y. \tag{4.28}$$

Plugging $X_1 \simeq 1$ into this equation for $i = 2$ gives

$$X_2(N) \simeq \frac{\bar{\Gamma}_1}{\bar{\Gamma}_2}\left[1 - e^{-(\bar{\Gamma}_2/\bar{H})N}\right] \simeq \frac{\bar{\Gamma}_1}{\bar{H}} N\,, \tag{4.29}$$

where the last approximation is valid for $N \ll \bar{H}/\max(\bar{\Gamma}_1, \bar{\Gamma}_2)$ (which is again related to the stability condition of the cascade during inflation). From this expression for $X_2(N)$ one can compute $X_3(N)$ and so on such that the full hierarchy is given by

$$X_n(N) \simeq \frac{N^{n-1}}{(n-1)!} \prod_{j=1}^{n-1} \frac{\bar{\Gamma}_j}{\bar{H}}\,, \tag{4.30}$$

again for $N \ll \bar{H}/\max(\{\bar{\Gamma}_i\}_{i=1,n-1})$. As a result, concerning the inflationary phase, the cascade of dust components behave as one unstable dust fluid with an effective decay rate given by

$$\bar{\Gamma} = \bar{\Gamma}_n X_n \simeq \frac{\prod_{i=1}^n \bar{\Gamma}_i}{\bar{H}^{n-1}} \simeq \frac{1}{(n-1)!}\left(\frac{3N^2}{8}\right)^{\frac{n-1}{2}} \prod_{i=1}^n \bar{\Gamma}_i\,. \tag{4.31}$$

Although $\bar{\Gamma}$ is no longer constant, its dependence on the e-fold number N is only a power law such that $\bar{\rho}_{\rm max}$ acquires the same dependence on N. Since the total energy density $\bar{\rho}(N)$ decreases much faster, as $\exp(-3N)$, it is only necessary to require $\bar{\rho} < \bar{\rho}_{\rm max}$ at the beginning of inflation. In order to show this more precisely, we need to impose $\bar{\rho}/\bar{\rho}_{\rm max} \lesssim 1$ throughout the whole inflationary period. Then, we only need to compute the number of e-fold at which $\bar{\rho}/\bar{\rho}_{\rm max} \lesssim$ takes its maximum value[4]. We find that this happens for $N = (n-1)/3$. Since we expect inflation to last for about 60 e-folds, the obtained value will only make sense if it is smaller than ~ 60. This is indeed the case if $n - 1 < 180$, so we will assume this in the following. Thus, taking $N = (n-1)/3$ in the previous expression gives the maximal number of inflationary e-folding

$$\Delta N_{\rm inf} \simeq \frac{1}{3}\ln\left(\frac{\sqrt{3}}{\prod_{i=1}^{n}\bar{\Gamma}_i}\right) + \frac{n-1}{6}\ln\left[\frac{24}{(n-1)^2}\right] + \frac{1}{3}\ln[(n-1)!]. \tag{4.32}$$

For instance, taking all the $\bar{\Gamma}_i$ equals, getting 60 e-folds of inflation requires $\Gamma_i < 10^{-26} m_\lambda$ for a mixture of $n = 3$ dust components. Taking $m_\lambda = \mathcal{O}(M_{\rm Pl})$, the minimum lifetime of each specie is required to be greater than 10^{-19} s.

4.3.2 Reheating

The reheating can be dealt as in the previous section. Provided the radiation component remains subdominant, $X \ll 1$, one has $\bar{H} \simeq \sqrt{\bar{\rho}/3}$ with $\bar{\rho}(N) \simeq \bar{\rho}(N_{\rm end})\exp(-3\Delta N)$. Plugging these expressions into equation (4.26) gives the evolution of the density fraction of all the species during reheating, up to the end at which the radiation fraction becomes significant. For X_1, one gets

$$X_1(N > N_{\rm end}) = X_1(N_{\rm end})\exp\left[-\frac{2}{3}\frac{\bar{\Gamma}_1}{\bar{H}_{\rm end}}\left(e^{3\Delta N/2} - 1\right)\right], \tag{4.33}$$

where $\bar{H}_{\rm end} = \sqrt{\bar{\rho}(N_{\rm end})/3} \simeq \sqrt{8/3}$. Moreover, under the reasonable assumption that X_1 does not evolve much during inflation, i.e. $\Delta N_{\rm inf} \ll \bar{\Gamma}_1^{-1}\sqrt{8/3}$, one has $X_1(N_{\rm end}) \simeq 1$. During reheating, $X_1(N)$ remains almost constant as long as $\Delta N \ll \Delta N_1$ where

$$\Delta N_i \equiv N_i - N_{\rm end} = \frac{2}{3}\ln\left(\frac{3}{2}\frac{\bar{H}_{\rm end}}{\bar{\Gamma}_i}\right) \simeq \frac{2}{3}\ln\left(\frac{\sqrt{6}}{\bar{\Gamma}_i}\right). \tag{4.34}$$

For the other species, one obtains for $i > 1$

$$\begin{aligned} X_i(N) \simeq\ & X_i(N_{\rm end})\exp\left[-\frac{2}{3}\frac{\bar{\Gamma}_i}{\bar{H}_{\rm end}}\left(e^{3\Delta N/2} - 1\right)\right] \\ & + \frac{\bar{\Gamma}_{i-1}}{\bar{H}_{\rm end}}\int_{N_{\rm end}}^{N}\exp\left[\frac{2}{3}\frac{\bar{\Gamma}_i}{\bar{H}_{\rm end}}\left(e^{3\Delta y/2} - e^{3\Delta N/2}\right) + \frac{3}{2}\Delta y\right]X_{i-1}(y)\,{\rm d}y. \end{aligned} \tag{4.35}$$

From the inflationary solution, $X_i(N_{\rm end})$ is given by equation (4.30). The first line in the previous expression encodes the evolution in the absence of sources, such that for $\Delta N < \Delta N_i$, X_i keeps its value acquired during inflation. For $\Delta N > \Delta N_i$, the source-free evolution is decaying exponentially of exponentially fast such that X_i is completely driven by the source

[4] As in the previous cases, this is quantitatively similar to imposing the total pressure to be smaller than $m_\lambda^2 M_{\rm Pl}^2$, i.e., $\bar{P} = 1/3X\bar{\rho} = 1/3\bar{\Gamma}_n X_n\bar{\rho} \lesssim 1$.

given in the second line of equation (4.35). As a result, either X_i takes its value at the end of inflation, or the source term drives its dynamics. For this reason, we only consider the source term from now on.

For X_2, plugging equation (4.33) into (4.35), one has

$$X_2(N) \simeq \frac{\bar{\Gamma}_1}{\bar{H}_{\rm end}} e^{-\frac{2}{3}\frac{\bar{\Gamma}_2}{\bar{H}_{\rm end}} e^{3\Delta N/2}} \int_{N_{\rm end}}^{N} \exp\left(\frac{2}{3}\frac{\bar{\Gamma}_2 - \bar{\Gamma}_1}{\bar{H}_{\rm end}} e^{3\Delta y/2} + \frac{3}{2}\Delta y\right) {\rm d}y. \tag{4.36}$$

The argument under the integral emphasizes the typical time scale ΔN_{12} involved in the evolution of X_2 sourced by X_1. It is defined by the transcendental equation

$$\frac{3}{2}\Delta N_{12} = \frac{2}{3}\frac{\left|\bar{\Gamma}_2 - \bar{\Gamma}_1\right|}{\bar{H}_{\rm end}} e^{3\Delta N_{12}/2}, \tag{4.37}$$

whose solution can be expressed in terms of the Lambert function

$$\Delta N_{12} = -\frac{2}{3}{\rm W}_{-1}\left(-\frac{2}{3}\frac{\left|\bar{\Gamma}_2 - \bar{\Gamma}_1\right|}{\bar{H}_{\rm end}}\right) \simeq \frac{2}{3}\ln\left(\frac{3}{2}\frac{\bar{H}_{\rm end}}{\left|\bar{\Gamma}_2 - \bar{\Gamma}_1\right|}\right) \simeq \frac{2}{3}\ln\left(\frac{\sqrt{6}}{\left|\bar{\Gamma}_2 - \bar{\Gamma}_1\right|}\right), \tag{4.38}$$

the asymptotic limit holding for $\left|\bar{\Gamma}_2 - \bar{\Gamma}_1\right| \ll \bar{H}_{\rm end}$. For $1 < \Delta N < \Delta N_{12}$, we obtain the approximate solution

$$X_2(1 < \Delta N < \Delta N_{12}) \simeq \frac{2}{3}\frac{\bar{\Gamma}_1}{\bar{H}_{\rm end}} \exp\left(-\frac{2}{3}\frac{\bar{\Gamma}_2}{\bar{H}_{\rm end}} e^{3\Delta N/2} + \frac{3}{2}\Delta N\right), \tag{4.39}$$

showing that X_2 first exponentially grows for $\Delta N < \Delta N_2$ and then disappears as a double decaying exponential for $\Delta N > \Delta N_2$. However, such an evolution may be interrupted before completion if $\Delta N_{12} < \Delta N_2$.

For $\Delta N > \Delta N_{12}$, one has to distinguish whether $\bar{\Gamma}_2 < \bar{\Gamma}_1$ or $\bar{\Gamma}_2 > \bar{\Gamma}_1$. In the former situation, the argument of the exponential under the integral of equation (4.36) becomes strongly negative and the integral saturates for $\Delta y \simeq \Delta N_{12}$. One has

$$X_2(\Delta N > \Delta N_{12}) \simeq \frac{2}{3}\frac{\bar{\Gamma}_1}{\bar{H}_{\rm end}} \exp\left(-\frac{2}{3}\frac{\bar{\Gamma}_2}{\bar{H}_{\rm end}} e^{3\Delta N/2} + \frac{3}{2}\Delta N_{12}\right), \qquad \bar{\Gamma}_2 < \bar{\Gamma}_1. \tag{4.40}$$

This expression shows that for the "interrupted" case, $\Delta N_{12} < \Delta N_2$, the exponential growth of X_2 is interrupted at ΔN_{12} and X_2 remains constant up to ΔN_2 at which it finally disappears. From equation (4.38), the stationary (and maximal) value of X_2 is

$$X_2(\Delta N_{12} < \Delta N < \Delta N_2) \simeq \frac{\bar{\Gamma}_1}{\bar{\Gamma}_1 - \bar{\Gamma}_2}. \tag{4.41}$$

Under the same hypothesis, $\Delta N > \Delta N_{12}$, let us discuss the case $\bar{\Gamma}_2 > \bar{\Gamma}_1$. The argument of the exponential in equation (4.36) becomes positive and the integral blows up as a double exponential such that one can neglect the term in $3\Delta y/2$. The integral is now given by an exponential integral function such that

$$\int_{N_{\rm end}}^{N} \exp\left(\frac{2}{3}\frac{\bar{\Gamma}_2 - \bar{\Gamma}_1}{\bar{H}_{\rm end}} e^{3\Delta y/2}\right) {\rm d}y \simeq \frac{2}{3}\,{\rm Ei}\left(\frac{2}{3}\frac{\bar{\Gamma}_2 - \bar{\Gamma}_1}{\bar{H}_{\rm end}} e^{3\Delta N/2}\right). \tag{4.42}$$

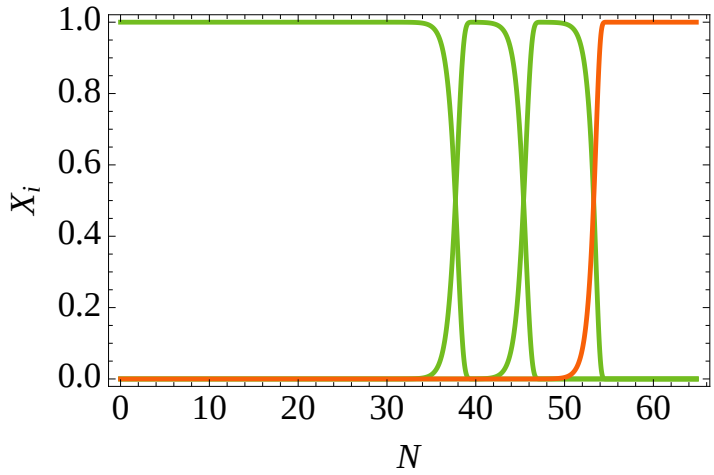

Figure 3. Evolution of the relative dust fraction X_i (green) and radiation X (red) for three dust components during reheating. One may notice the transient stationary behaviour described by equation (4.41). Reheating ends at the time the last dust component of the cascade decays.

Taking the large argument limit in the previous expression one finally gets

$$X_2(\Delta N > \Delta N_{12}) \simeq \frac{\bar{\Gamma}_1}{\bar{\Gamma}_2 - \bar{\Gamma}_1} \exp\left(-\frac{2}{3}\frac{\bar{\Gamma}_1}{\bar{H}_{\rm end}} e^{3\Delta N/2}\right), \qquad \bar{\Gamma}_2 > \bar{\Gamma}_1. \tag{4.43}$$

As one may expect, for $\bar{\Gamma}_2 > \bar{\Gamma}_1$, the late time evolution of X_2 becomes completely driven by the evolution of X_1 and thus X_2 disappears when $\Delta N \simeq \Delta N_1$.

Equations (4.39), (4.40) and (4.43) give a good approximation of the evolution of X_2 at all stages. Plugging back these expressions into equation (4.35) would give in the same way the evolution of all X_i by recurrence. In fact, we do not need to perform any additional calculations. Ultimately, either $\bar{\Gamma}_2 < \bar{\Gamma}_1$ and X_2 decays as in equation (4.40), i.e. completely disappears for $\Delta N > \Delta N_2$; or $\bar{\Gamma}_2 > \bar{\Gamma}_1$ and X_2 decays as in equation (4.43), i.e. for $\Delta N > \Delta N_1$. Both equations (4.40) and (4.43) are of the same functional form as equation (4.33) such that the ultimate behaviour of all the X_i will be of the same functional form. As a result, after some transient evolution, the component X_i is expected to disappear when $\Delta N \simeq \sup\{\Delta N_j, j < i\}$.

In figure 3, we have numerically solved the evolution equations for $n = 3$ decaying dust components in addition to radiation having $\Gamma_{i+1} < \Gamma_i$. As can be seen in this plot, the expected behaviours of X_i are recovered. While X_1 decays, $X_{i>1}$ exponentially grows to reach a stationary regime and finally disappears. Reheating ends when the Universe contains a significant amount of radiation, namely for $X(N_{\rm reh}) \simeq 1$. Because

$$X = 1 - \sum_{i=1}^{n} X_i, \tag{4.44}$$

the end of reheating corresponds to the time at which the last dust component disappears, i.e.

$$\Delta N_{\rm reh} = \max\left(\Delta N_i\right) \simeq \frac{2}{3} \ln\left[\frac{\sqrt{6}}{\min(\bar{\Gamma}_i)}\right]. \tag{4.45}$$

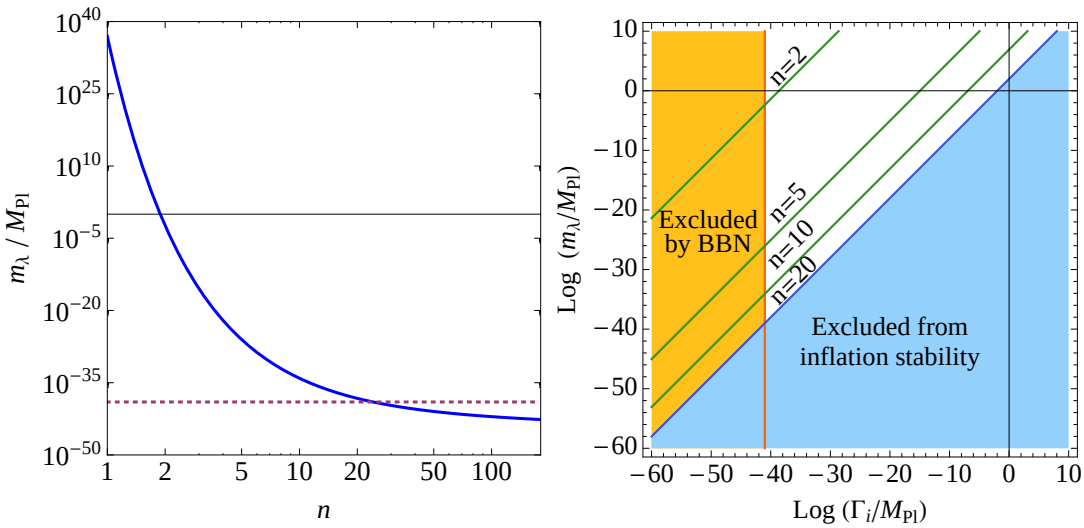

Figure 4. In the left panel we show the lower bound on m_λ from the constraint given in (4.49) as a function of the number of dust components n when they share the same decay rate Γ_i. The allowed region is above the curve so we see that only for $n = 1$ the constraint can not be made below the Planck scale. The lower bound (4.50) is represented by the dotted line and we see that is only satisfied for $n \lesssim 20$. In the right panel we show the different bounds in the plane (Γ_i, m_λ). The orange region is the bound on Γ_i for reheating to end before BBN. The blue region is excluded from the fact that the dust components need to be stable during inflation as expressed in (4.50). Finally, the green curves correspond to bounds on m_λ in order to have, at least, $\Delta N_{\rm inf} \simeq 60$ e-folds of inflation.

Notice that only the most stable component contributes to the duration of the reheating period, whereas, as can be seen in equation (4.32), the duration of inflation depends on the whole cascade. This is the crucial feature that allows to make the model viable by introducing the cascade. One can readily understand that with only one dust component, both durations are linked and this was precisely the obstruction to make the model viable with only one dust component.

4.3.3 BBN constraint

Comparing equations (4.32) and (4.45), one can obtain a long enough inflationary era to solve the usual Big-Bang problem without having a too long reheating era. Reheating should however end before the onset of Big-Bang Nucleosynthesis, i.e. for $\rho_{\rm reh} > \rho_{\rm nuc}$. According to the previous section,

$$\bar{\rho}_{\rm reh} = \bar{\rho}(N_{\rm end}) e^{-3\Delta N_{\rm reh}} \simeq \frac{4}{3} \left[\min(\bar{\Gamma}_i)\right]^2 , \tag{4.46}$$

such that the BBN bound for $\rho_{\rm nuc}^{1/4} \simeq 10\,{\rm MeV}$ reads

$$\min(\Gamma_i) \geq \frac{\sqrt{3\rho_{\rm nuc}}}{2M_{\rm Pl}} \simeq 10^{-41}\, M_{\rm Pl}. \tag{4.47}$$

For three fluids having the same decay rate, and $m_\lambda = \mathcal{O}(M_{\rm Pl})$, getting 60 e-folds of inflation were requiring only $\Gamma_i < 10^{-26}\, m_\lambda$ so that the BBN constraint is trivially satisfied for that case.

In the case where all the Γ_i are equal, one can use the BBN bound to actually constrain the acceptable values of m_λ in order to have a viable inflationary model. From equation (4.32), one has

$$3\Delta N_{\rm inf} \simeq -n \ln\left(\frac{\Gamma_i}{m_\lambda}\right) + \frac{n-1}{2}\ln\left[\frac{24}{(n-1)^2}\right] + \ln[(n-1)!] + \frac{\ln 3}{2}\,. \tag{4.48}$$

If we saturate the minimal value for Γ_i we can obtain the following lower bound for m_λ by requiring that inflation should last longer than ~ 60 e-folds:

$$m_\lambda > \frac{n-1}{\sqrt{24}}\left[\frac{\sqrt{8}}{(n-1)(n-1)!}\right]^{1/n}\frac{\sqrt{3\rho_{\rm nuc}}}{2M_{\rm Pl}}e^{\frac{3\Delta N_{\rm inf}}{n}} \simeq \frac{n-1}{\sqrt{24}}\left[\frac{\sqrt{8}}{(n-1)(n-1)!}\right]^{1/n} e^{\frac{-94+3\Delta N_{\rm inf}}{n}}. \tag{4.49}$$

This bound implies that for one single dust component $n = 1$ one would need $m_\lambda \gtrsim 10^{36} M_{\rm Pl}$, well above the Planck mass. In addition to be unnatural, we have shown earlier that the duration of reheating would be twice longer than inflation in this case. Already for $n = 2$ we however obtain the bound $m_\lambda \gtrsim 10^{-3} M_{\rm Pl}$, which is below the Planck scale. Finally, in the limit of many intermediate species (but smaller than 180 as we required before) the smallest possible bound is $m_\lambda \gtrsim 10^{-42} M_{\rm Pl}$. There is an additional constraint that we have on m_λ coming from the stability of the cascade during inflation, i.e., the decay rates should be smaller than the expansion rate during inflation for the dust components to remain throughout the inflationary phase. This was already stated as the condition $\Delta N_{\rm inf} \ll \bar{H}/\bar{\Gamma}_i$ in order to have $X_1 \simeq 1$. Thus, if we again saturate the value of Γ_i, we obtain the additional bound

$$m_\lambda \gtrsim \sqrt{\frac{8}{3}}\,\Gamma_i\,\Delta N_{\rm inf} \simeq 10^{-39} M_{\rm Pl}, \tag{4.50}$$

where we have used $\Delta N_{\rm inf} \simeq 60$. We see that this bound is more stringent than the one obtained in the large n limit. This means, that there will be a maximum value for the allowed number of species such that (4.50) is satisfied. Such a maximum value is $n \lesssim 20$ as can be clearly seen in Figure 4. In that figure we have also reported all the bounds in the plane (Γ_i, m_λ).

5 Conclusion

In this paper, we have discussed a scenario in which inflation is a natural outcome of a modification of gravity in the high curvature regime.

By considering minimal Born-Infeld gravity in the metric-affine formalism, we have shown that accelerated expansion of the Universe can be induced by gravitating dust at energy densities higher than $m_\lambda^2 M_{\rm Pl}^2$, m_λ being the new mass scale of the theory. At lower curvatures, or energy-densities, GR is recovered such that these theories automatically implement a graceful exit of the accelerated phase without the need of scalar fields (and/or negative pressure fluids). Graceful exit is however not enough to ensure a viable inflationary scenario as the Universe should reheat to become radiation dominated after inflation, and such a reheating era should end before BBN.

Reheating within our scenario can be implemented by considering that inflation in the Born-Infeld regime is supported by a chain of unstable dust components that ultimately decay into radiation when the GR regime is recovered. Because in the Palatini formalism "the speed of sound gravitates" or, in other words, the background evolution not only depends

on the equation of state parameter, but also on the sound speed, we have shown that such a scenario is severely constrained thereby making the model relatively predictive. For instance, the total number of e-folds cannot be made arbitrarily large, see equation (4.32), and is completely fixed by the decay rate of the dust species. As a result, observing a tiny but non-vanishing spatial curvature today would be a natural outcome of such a scenario, unlike in most inflationary scenarios where the typical duration is usually much larger than $\mathcal{O}(10^2)$ e-folds and, therefore, any initial spatial curvature is heavily suppressed. Another requirement of a successful reheating phase is that at least two decaying dust components should coexist during inflation, cascading one into another. Indeed, when only one unstable dust specie decays into radiation, both the duration of reheating and inflation are completely fixed by the dust lifetime. In that situation, one obtains that reheating lasts twice the duration of inflation, which is in tension with cosmological observations. For more than two dust components, the model works provided the new mass scale m_λ is not too low (see figure 4), although the constraints are fairly mild and having $m_\lambda \gtrsim 10^{-39} M_{\rm Pl}$ would suffice.

Interestingly, due to the existence of a non-vanishing speed of sound $c_{\rm s}^2$ during inflation, there is a maximum energy density $\bar{\rho}_{\rm max}$, given by equation (4.12), above which the inflationary phase cannot exist anymore and could be superseded by a bounce. In addition, the presence of a positive pressure given by the radiation component also sets an independent upper bound on the energy density in our Born-Infeld inspired modification of gravity. As a result, the energy densities of the (unstable) dust components are self-regulated and cannot take arbitrarily large values in the past. On the one hand, cascading dust inflation provides a framework in which, at the background level, various criticisms of the standard inflationary scenario, and criticisms of the alternative models, seem to be addressed. On the other hand, there are still various points to be clarified. For instance, the nature of the dust components has not being specified and could range from super-heavy or super-cold dark matter particles to black holes. Being more specific on the origin of the gravitating dust should put additional constraints on the acceptable decaying rates, and thus on the duration of inflation and reheating. It will be also interesting to discuss if these particles can play the role of dark matter. Most importantly, we have not derived the primordial power spectra that would source the cosmological perturbations. This is indeed a non-trivial problem within the Born-Infeld class of theories as we illustrate now.

A distinctive feature of the model is that it exhibits superinflation, namely a negative first Hubble flow function $\epsilon_1 < 0$. Within GR, one would immediately conclude that the spectral index of the gravitational waves is blue, and from equation (4.3), that the second Hubble flow function $\epsilon_2 \simeq 3$ such that the spectral index of the scalar modes would be super-red. However, this is not correct as in the Born-Infeld regime the perturbations evolve in a completely different manner than in GR. The problem for the tensor modes $h_{ij} \equiv a^{-2}\delta g_{ij}$ has been addressed in Ref. [120] for a large class of Palatini theories and it was shown that, in the absence of anisotropic stresses (as expected in our inflationary scenario), they verify

$$h_{ij} = \tilde{h}_{ij}, \qquad \tilde{\mu}_{ij}'' + \left(-\nabla^2 + \frac{\tilde{a}''}{\tilde{a}}\right)\tilde{\mu}_{ij} = 0, \tag{5.1}$$

where $\tilde{h}_{ij} = \tilde{a}^{-2}\delta\tilde{g}_{ij}$ stands for the tensor perturbations of the auxiliary metric and $\tilde{\mu}_{ij} \equiv \tilde{a}\tilde{h}_{ij}$ is the usual quantized mode function. In the above equation a "prime" denotes derivative with respect to the auxiliary conformal time $\tilde{\eta}$ (defined by $\tilde{n} = \tilde{a}$). The mode evolution for $\tilde{\mu}_{ij}$ is identical to the one of GR, but with respect to the auxiliary metric and coordinates. For a quasi-de Sitter expansion $\bar{H} \simeq \sqrt{8/3}$ in the metric g, one gets $\tilde{a}(\tilde{\eta}) \propto \tilde{\eta}^{1/2}$, i.e.

kination in the auxiliary metric. As a result, the effective mass in (5.1) reads $-\tilde{a}''/\tilde{a} = +1/(4\eta^2) > 0$ and no amplification of the tensor modes occurs. As a result, and in spite of the superinflationary nature of the cascading dust model, no primordial gravitational waves are generated, independently of the energy scales involved. This is indeed a very distinctive feature of this inflationary model. In particular, any detection of B-modes in the CMB generated by primordial gravitational waves would rule out the model.

The situation is more complex for the scalar modes and, at the time of this writing, no conclusion can be drawn on their viability. Their equations of motion will now involve the scalar sources, namely perturbations in the dust components and one would need to quantize the fluid degrees of freedom directly. This could be done either by resorting to an effective theory for fluids [121–125] or by computing the spectrum of thermal fluctuations. We leave however the explicit computation for a future work.

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