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# iPTF methods: How Green's identity and FEM solver can be used for acoustic inverse methods

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## Summary

Green's identity is a well-known mathematical tool usually used to solve acoustic problems. If two functions are twice continuously differentiable, an integral over a volume can be replaced by an integral over the surfaces of this volume. Mostly, one of these functions is the acoustic pressure but the other one is completely arbitrary. The possibilities given by this arbitrary choice are numerous. In the present paper, the powerful capabilities of the Green's identity will be illustrated on a 3D acoustic problem consisting in an oil pan radiating in a semi-infinite medium. The radiated field obtained by Infinite Elements (considered here as a reference) will be compared to two solutions provided by the application of the Green's identity on a finite virtual volume surrounding the vibrating surface. Indeed, thanks to Green's identity, the choice of the boundary conditions of this virtual volume is arbitrary. The cases of uniform (Neumann) and mixed (Neumann and Dirichlet) boundary conditions will be presented. Finally, it will be shown how Green's identity and FEM solver can be used as acoustic inverse method. The so-called "uniform iPTF" (inverse Patch Transfer Functions with uniform BC) and "mixed iPTF" (inverse Patch Transfer Functions with mixed BC) will be presented and experimentally applied on the case of the oil pan. Velocity, pressure and intensity fields reconstructed by the inverse methods will be compared to direct measurements.

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## 1. Introduction

Source identification methods like Near-field Acoustic Holography [1, 2] aim to locate a source of noise or to reconstruct its vibratory field. The diversity of applications in this field of research has caused to develop several different methods with their own domain of validity (acoustic environment, frequency range, shape of the source, etc.) and their own experimental setup (synchronous or sequential measurements, microphones antennas, etc.).

Despite the number of existing methods, there are still some issues for industrial applications. Measurements in non-anechoic conditions, reconstruction of fields on 3D sources, field separation are some of the difficulties that still slow down the industrial

applications.

The present paper deals with a way to develop new source identification methods based on the use of Green's identity. Green's identity is a well-known technique to solve problems in acoustics. Two different developments will be shown here to illustrate the possibilities of the Green's identity in the framework of inverse methods. These developments, called u-iPTF [3, 4] (inverse Patch Transfer Functions with uniform boundary conditions) and m-iPTF (inverse Patch Transfer Functions with mixed boundary conditions) have several advantages: (i) the properties of the Green's identity ensure the deconfinement of the source from its acoustic environment (ii) the judicious choice of the associate problem opens a lot of development possibilities (iii) the use of a numerical solver permits to treat irregularly shaped sources (iv) the source is fully characterized (pressure, normal velocity, normal intensity fields, acoustic power, radiation efficiency). These examples of developments using

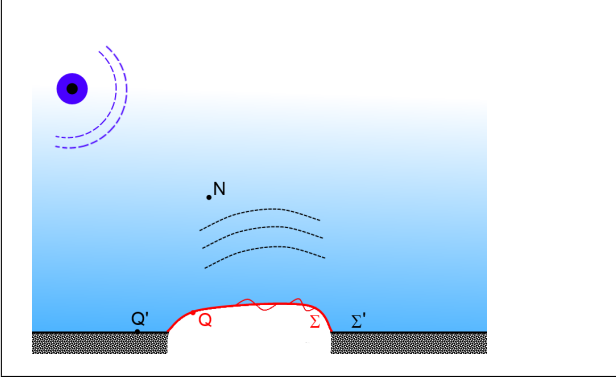


Figure 1. A baffled structure radiates in an acoustic environment.

Green's identity will be presented and illustrated by a numerical example and experimental applications.

## 2. Green's identity in acoustics

Let's take the classical case presented in Fig. 1 of a baffled structure radiating in any acoustic environment. In this acoustic environment, some other (stationary) sources may contribute to the noise measured at point N. To compute the pressure at point N, one can use the Green's identity. This well known method is extensively used in acoustics to compute the pressure anywhere in an acoustic volume knowing pressure and particle velocity fields at boundaries. In the case of Fig. 1 there is no strictly speaking acoustic volume, just an (semi-infinite) acoustic environment. In fact, in Green's identity, there is no need of a "real" acoustic volume, a "virtual" volume suits well. Let's define a "virtual" acoustic volume  $\Omega$  containing the point N and the vibrating surface  $\Sigma$  and excluding other sources as presented in Fig. 2. Obviously, the pressure and particle velocity fields on the "virtual" surface  $\Sigma''$  are due to the contribution of the radiating surface  $\Sigma$  and to the contribution of all other sources, reflections, absorption outside the volume  $\Omega$ . Let's consider that the pressure and the particle velocity are continuously known on surfaces  $\Sigma$  (vibrating surface),  $\Sigma'$  (physically rigid surface) and  $\Sigma''$  ("virtual" surface).

Then, the Green's identity writes

$$\begin{aligned} & \int_{\Omega} (\Delta p(N) + k^{*2} p(N)) \Phi(N) dN = \\ & \int_{\Omega} (\Delta \Phi(N) + k^{*2} \Phi(N)) p(N) dN \\ & + \int_{\Sigma} p(Q) \frac{\partial \Phi(Q)}{\partial n} - \Phi(Q) \frac{\partial p(Q)}{\partial n} dQ \\ & + \int_{\Sigma'} p(Q') \frac{\partial \Phi(Q')}{\partial n} - \Phi(Q') \frac{\partial p(Q')}{\partial n} dQ' \\ & + \int_{\Sigma''} p(Q'') \frac{\partial \Phi(Q'')}{\partial n} - \Phi(Q'') \frac{\partial p(Q'')}{\partial n} dQ'', \end{aligned} \quad (1)$$

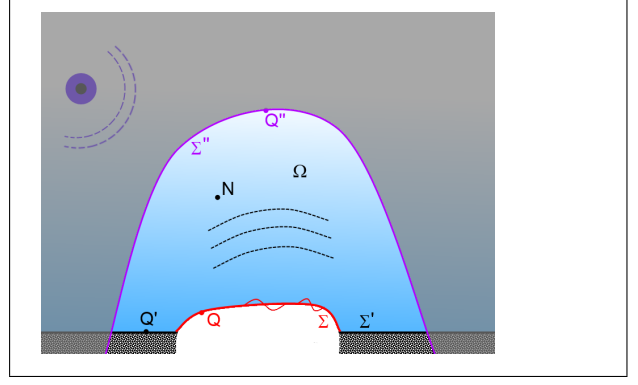


Figure 2. Definition of a "virtual" volume  $\Omega$  and its boundary surfaces  $\Sigma$ ,  $\Sigma'$  and  $\Sigma''$  for the application of Green's identity.

where  $k^* = k(1 - j\eta)$  is the complex wavenumber, with  $\eta$  is the damping loss factor and  $n$  is the outward normal. The boundary conditions are prescribed by the pressure and particle velocity on the surfaces  $\Sigma$ ,  $\Sigma'$  and  $\Sigma''$ . In addition, the Euler relation gives

$$\frac{\partial p(Q)}{\partial n} = -j\omega\rho_0 V_n(Q) \quad \forall Q \in \Sigma, \quad (2)$$

$$\frac{\partial p(Q')}{\partial n} = 0 \quad \forall Q' \in \Sigma', \quad (3)$$

$$\frac{\partial p(Q'')}{\partial n} = -j\omega\rho_0 V_n(Q'') \quad \forall Q'' \in \Sigma'', \quad (4)$$

where  $V_n(Q)$  is the normal velocity at point  $Q$ ,  $\omega$  is the angular frequency and  $\rho_0$  is density of air. In addition, if there is no source inside the volume  $\Omega$ , then the pressure  $p(N)$  is given by the Helmholtz equation

$$\Delta p(N) + k^{*2} p(N) = 0 \quad \forall N \in \Omega. \quad (5)$$

Considering Eqs. (1) to (5), one can write

$$\begin{aligned} & - \int_{\Omega} (\Delta \Phi(N) + k^{*2} \Phi(N)) p(N) dN = \\ & + \int_{\Sigma} p(Q) \frac{\partial \Phi(Q)}{\partial n} + j\omega\rho_0 V_n(Q) \Phi(Q) dQ \\ & + \int_{\Sigma'} p(Q') \frac{\partial \Phi(Q')}{\partial n} \\ & + \int_{\Sigma''} p(Q'') \frac{\partial \Phi(Q'')}{\partial n} dQ'' \\ & + \int_{\Sigma''} j\omega\rho_0 V_n(Q'') \Phi(Q'') dQ''. \end{aligned} \quad (6)$$

The main interest of Green's identity is that the function  $\Phi$  can be arbitrarily chosen provided that it is twice differentiable. Let's choose  $\Phi$  as a mode shape  $\phi_n(N)$  of the equivalent finite volume  $\Omega$  such as

$$\Delta \phi_n(N) + k_n^2 \phi_n(N) = 0 \quad \forall N \in \Omega, \quad (7)$$

where  $k_n$  is the natural wavenumber of mode  $n$  of the virtual acoustic volume  $\Omega$ . Mode shapes  $\phi_n(N)$

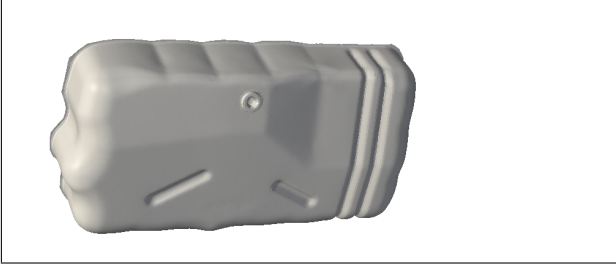


Figure 3. Oil pan.

constitute an orthonormal basis of functions on which the pressure at point  $N$  can be decomposed

$$p(N) = \sum_{n=0}^{\infty} a_n \phi_n(N) \quad (8)$$

Introducing Eqs. (7) and (8) in Eq. (6) and using the orthonormal property of mode shapes, one obtains

$$p(N) = \sum_{n=1}^{\infty} \frac{\phi_n(N)}{\Lambda_n(k^*2 - k_n^2)} C_n, \quad (9)$$

where

$$\begin{aligned} C_n = & \int_{\Sigma} p(Q) \frac{\partial \phi_n(Q)}{\partial n} dQ \\ & + j\omega \rho_0 \int_{\Sigma} V_n(Q) \phi_n(Q) dQ \\ & + \int_{\Sigma'} p(Q') \frac{\partial \phi_n(Q')}{\partial n} dQ' \\ & + \int_{\Sigma''} p(Q'') \frac{\partial \phi_n(Q'')}{\partial n} dQ'' \\ & + j\omega \rho_0 \int_{\Sigma''} V_n(Q'') \phi_n(Q'') dQ''. \end{aligned} \quad (10)$$

Eqs. (9) and (10) are valid whatever the boundary conditions of the "virtual" mode shapes  $\phi_n(N)$ . Depending on the chosen boundary conditions, some terms in Eq. (10) will be suppressed. Then, it has been here proven that the pressure at point  $N$  can be expressed using a "virtual" problem for which the boundary conditions can be arbitrarily chosen.

### 3. Acoustic radiation of an oil pan

A baffled oil pan, shown in Fig. 3 is excited by a point force and radiates in an semi-infinite acoustic environment.

#### 3.1. Reference computation

A classical computation based on the use of infinite elements has been done using ACTRAN software. An example of a pressure map is given in Fig. 4 at 2000Hz.

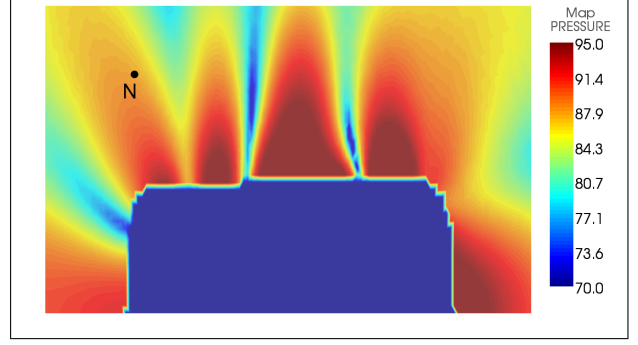


Figure 4. Map of the pressure field computed with infinite elements at 2000Hz.

#### 3.2. Green's function with Neumann boundary conditions

Let's consider the modes shapes  $\phi_n(N)$  with Neumann boundary conditions on surfaces  $\Sigma$ ,  $\Sigma'$  and  $\Sigma''$  as

$$\frac{\partial \phi_n(Q)}{\partial n} = 0 \quad \forall Q \in \Sigma \cup \Sigma' \cup \Sigma'' \quad (11)$$

In that case,  $C_n$  simplifies to

$$\begin{aligned} C_n = & j\omega \rho_0 \int_{\Sigma} V_n(Q) \phi_n(Q) dQ \\ & + j\omega \rho_0 \int_{\Sigma''} V_n(Q'') \phi_n(Q'') dQ'', \end{aligned} \quad (12)$$

and the pressure  $p(N)$  at any point of the domain  $\Omega$  is computed knowing the particle velocities on surfaces  $\Sigma$  and  $\Sigma''$  only. One can see in Eq. (12) that the choice of a Neumann condition on surface  $\Sigma'$  allows not to need to know information on this surface. This is actually due to the fact that the chosen boundary condition for the associate problem (the mode shapes) is the same as the real problem.

#### 3.3. Green's function with mixed boundary condition

A second possibility is to consider the boundary condition on the surface  $\Sigma''$  as a Dirichlet boundary condition. In that case,

$$\frac{\partial \phi_n(Q)}{\partial n} = 0 \quad \forall Q \in \Sigma \cup \Sigma' \quad (13)$$

and

$$\phi_n(Q'') = 0 \quad \forall Q'' \in \Sigma''. \quad (14)$$

Thus,  $C_n$  simplifies to

$$\begin{aligned} C_n = & j\omega \rho_0 \int_{\Sigma} V_n(Q) \phi_n(Q) dQ \\ & + \int_{\Sigma''} p(Q'') \frac{\partial \phi_n(Q'')}{\partial n} dQ'', \end{aligned} \quad (15)$$

and the pressure  $p(N)$  at any point in the domain  $\Omega$  is computed knowing the particle velocity on surface

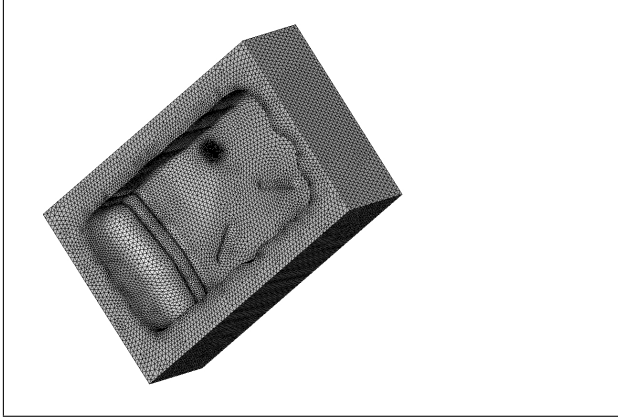


Figure 5. Virtual volume modeled by Finite Elements.

$\Sigma$  and pressure on surface  $\Sigma''$ . However, in this formulation, one has to know the spatial derivative of the mode shapes expressed in terms of pressure. These are proportional to the modes shapes expressed in terms of particle velocity as

$$\frac{\partial \phi_n(Q)}{\partial n} = -j\omega_n \rho_0 \chi_n(Q), \quad (16)$$

where  $\chi_n(Q)$  is the mode shapes of the virtual acoustic volume expressed in particle velocity. These mode shapes can be obtained with a numerical solver as ACTRAN.

### 3.4. Comparison of computations

The aim is here to verify if Eq. (12) (uniform conditions) and Eq. (15) (mixed conditions) permit to compute the pressure at point N by comparing the results to reference calculation presented in Fig. 4. For that purpose, the integrals are replaced by sums (discretisation of the surfaces) and the pressure and/or particle velocities on surfaces  $\Sigma$  and  $\Sigma''$  are taken from reference calculation. The modes shapes  $\phi_n(N)$  and its derivative  $\frac{\partial \phi_n}{\partial n}$  are computed using a numerical solver (ACTRAN) up to 5kHz on the virtual volume presented in Fig. 5. The pressures  $p(N)$  obtained with the three computations are compared in Fig. 6 which demonstrates that Green's identity can be used to compute the pressure in a domain whatever the chosen boundary conditions. The slight discrepancies between curves are due to the discretisation of the problem and to the convergence of the modal sums. It is important to underline that neither the Neumann nor the Dirichlet boundary conditions are representative of the real conditions. This does not prevent the calculation. However, the convergence of the calculation seems to be quicker with Dirichlet condition.

## 4. Source reconstruction using Green's identity

In both equations (12) and (15) the normal velocity of surface  $\Sigma$  appears. Let's consider now that this veloc-

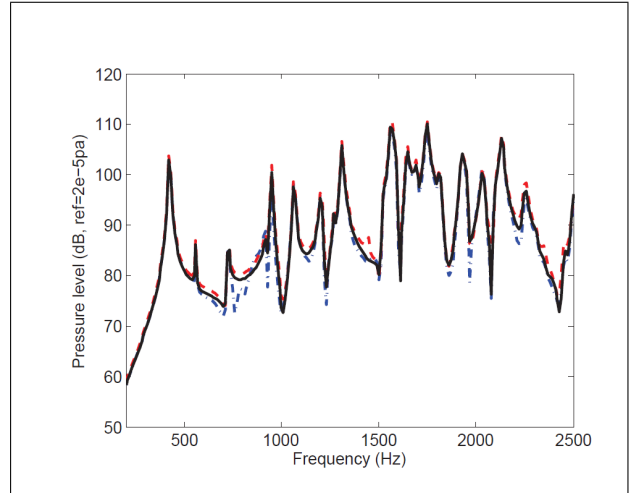


Figure 6. Pressure at point N as a function of frequency. Black solid line: reference computation using Infinite Elements; Blue dashed-dotted line: Green's function with Neumann BC; Red dotted line: Green's function with mixed BC.

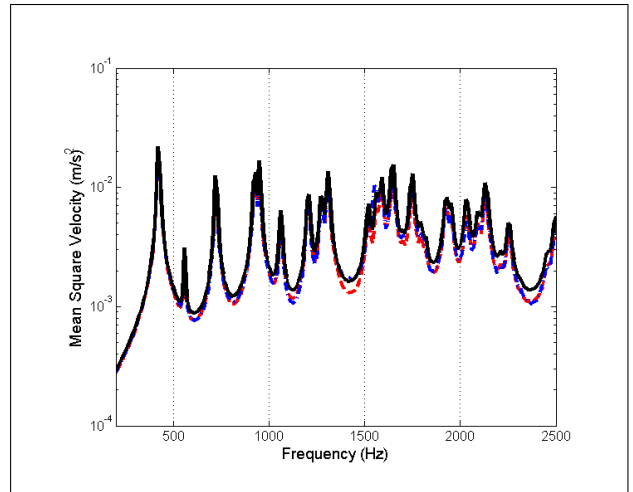


Figure 7. Mean Square Velocity as function of frequency. Black solid line: Reference; Blue dashed-dotted line: u-iPTF; Red dotted line: m-iPTF

ity field is unknown. If the pressure  $p(N)$  is measured at several points, these equations can be inverted and the velocity field can be identified. This is the principle of the method called inverse Patch Transfer Function [3, 4]. This method corresponds here to the inversion of the Green's identity with Neumann boundary conditions. In this paper, it will be called u-iPTF. The other possibility, also presented here, will be called m-iPTF.

### 4.1. Uniform BC: u-iPTF

If the vibrating surface  $\Sigma$  is discretized into  $N_s$  elementary surfaces called patches and the "virtual" sur-

face  $\Sigma''$  is discretized into  $N_m$  patches, then, Eqs. (9) and (12) lead to

$$\bar{p}_i = \sum_{j=1}^{N_m} Z_{ij} \bar{V}_j + \sum_{l=1}^{N_s} Z_{il} \bar{V}_l \quad (17)$$

where  $\bar{p}_i$  and  $\bar{V}_j$  are the space averaged pressure and particle velocity on patches  $i$  and  $j$  of surface  $\Sigma''$  and  $\bar{V}_l$  is velocity of patch  $l$  on surface  $\Sigma$ .  $Z_{ij}$  is the acoustic impedance between patches  $i$  and  $j$ . In a matrix form, one obtains

$$\bar{\mathbf{p}}_i = \mathbf{Z}_{ij} \bar{\mathbf{V}}_j + \mathbf{Z}_{il} \bar{\mathbf{V}}_l \quad (18)$$

The velocities space averaged on patches  $l$  on the vibrating surface  $\Sigma$  can then be deduced using

$$\bar{\mathbf{V}}_l = \mathbf{Z}_{il}^{-1} (\bar{\mathbf{p}}_i - \mathbf{Z}_{ij} \bar{\mathbf{V}}_j) \quad (19)$$

The velocity field on the surface of the source can be obtained measuring pressure and particle velocity on a virtual surface  $\Sigma''$ . These collocated measurements can be done using pU probe.

#### 4.2. Mixed BC: m-iPTF

Using Eqs. (9) and (15), the pressure at one point  $N$  in the domain can be expressed as

$$p(N) = \sum_{j=1}^{N_m} Y_{Nj}^* \bar{p}_j + \sum_{l=1}^{N_s} Z_{Nl}^* \bar{V}_l \quad (20)$$

If the pressure is measured at several points in the cavity, Eq. (20) can be written in a matrix form

$$\mathbf{p}_i = \mathbf{Y}_{ij}^* \bar{\mathbf{p}}_j + \mathbf{Z}_{il}^* \bar{\mathbf{V}}_l \quad (21)$$

Finally, the velocity field on the source is obtained by

$$\bar{\mathbf{V}}_l = \mathbf{Z}_{il}^{*-1} (\mathbf{p}_i - \mathbf{Y}_{ij}^* \bar{\mathbf{p}}_j) \quad (22)$$

Compared to Eq. (19), the velocity field  $\bar{\mathbf{V}}_l$  is obtained measuring the pressure on the virtual surface  $\Sigma''$  and at several points in the domain  $\Omega$ . In addition, impedance  $\mathbf{Z}_{il}^*$  and the admittance  $\mathbf{Y}_{ij}^*$  are computed with mode shapes with mixed BC while  $\mathbf{Z}_{il}$  and  $\mathbf{Z}_{ij}$  are obtained with Neumann BC.

#### 4.3. Application on the oil pan

In both cases, Eqs. (19) and (22), the problem is reduced to the classical form  $\mathbf{Ax} = \mathbf{b}$ . Both resolution are ill-posed and need regularization. In the following, the GCV method [5] will be used to find the best regularization parameter of the classical Tikhonov regularization technique.

Fig. 7 shows the mean square velocity of the vibrating surface  $\Sigma$  obtained with reference calculation and identification methods and Fig. 8 presents the comparison between the reference and the velocity fields identified by u-iPTF and m-iPTF at 700Hz. This application shows that these approaches, based on the use of the Green's identity coupled to FEM solver, can lead to powerful inverse methods. These methods are intrinsically independent of the acoustic environment and can handle sources with complex geometries.

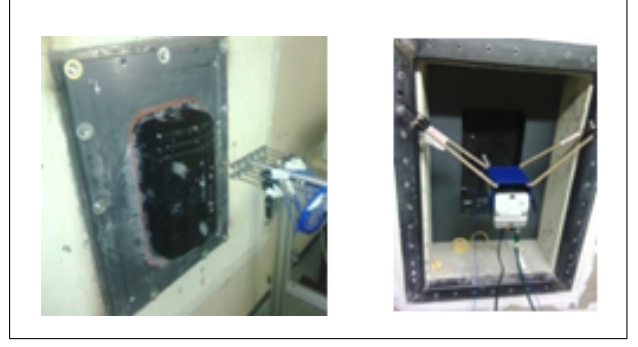


Figure 9. Experimental setup. Left: oil pan and line of microphones; right: electrodynamic shaker.

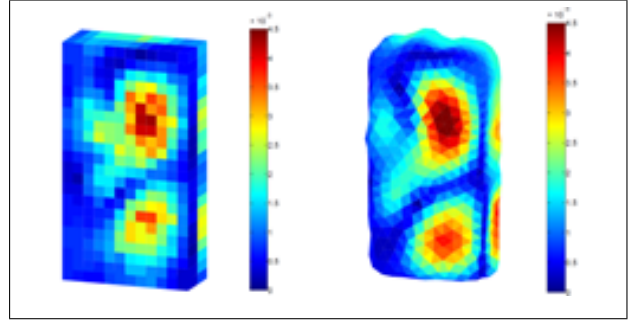


Figure 10. Amplitude of the velocity field at 976Hz. Left: measured with pU probe; Right: identification using m-iPTF.

#### 4.4. Experimental validation

An experimental validation of the described procedure has been performed with both approaches. However, as the results using u-iPTF have already been exposed in [4], only the results obtained using m-iPTF will be presented here.

The oil pan was fixed on a wall and excited by a shaker. It was radiating in a semi-anechoic chamber. Some reference measurements have been done using a pU probe near the oil pan (on a rectangular box close to the surface of the oil pan). An example of these measurements can be seen in Fig. 10 (left).

Then, to apply m-iPTF, the pressures have been measured on a virtual surface (a bigger rectangular box) and inside the virtual volume. The result obtained at 976Hz is shown in Fig. 10 (right). The comparison in Fig. 10 demonstrates that the source field reconstruction is possible in real conditions. In addition, contrary to reachable measurements, the identification is computed on the real surface of the structure.

Finally, the source is completely characterized as pressure and intensity fields can be deduced from identified velocity field and the direct formulation, Eq. (19) or (22). As a demonstration, the acoustic power of the source, measured with pU probe or identified, is plotted in Fig. 11. Both curves agree. As a consequence, the acoustic power of the source is obtained measuring pressures only thanks to m-iPTF.

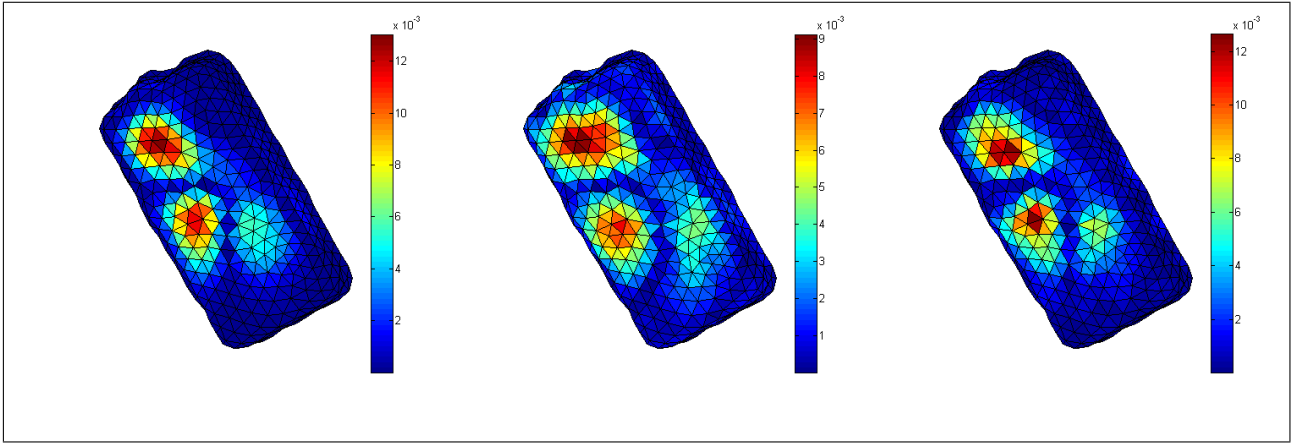


Figure 8. Velocity field of the vibrating surface  $\Sigma$  at 700Hz. Left: Reference; Center: u-iPTF; Right: m-iPTF

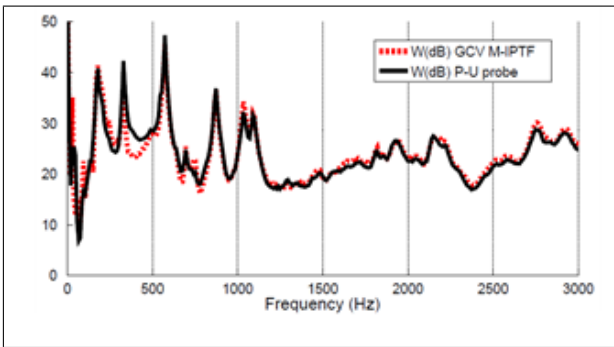


Figure 11. Acoustic power as a function of frequency. Black solid line: measured with pU probe; Red dotted line: identification using m-iPTF.

## 5. CONCLUSIONS

The Green's identity is a technique extensively used in acoustics to solve vibro-acoustic problems. In this paper, it is shown how the Green's identity and FEM solver can be used to develop source field reconstruction methods. Two developments are presented here. The first one, the u-iPTF, permits to reconstruct the velocity field of a source with complex geometry measuring colocated pressure and particle velocity on a virtual surface surrounding the source. The second one, the m-iPTF, only needs pressure measurements on a virtual surface and inside the virtual volume. Both of them need regularization as the problem is ill-conditioned. However, both of them allow source fields reconstructions in non-anechoic room on complex geometry. In addition, the source is then fully described by velocity, pressure, intensity fields and acoustic power and radiation efficiency.

## Acknowledgement

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