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GRAPHICAL REPRESENTATIONS ASSOCIATED WITH ALGEBRAIC TASKS IN APLUSIX

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Aplusix is a learning environment devoted to the practice of algebra in high school which has chosen a natural representation for algebraic expressions.

We will first present the didactical and technological choices for the development of the graphical representations. Then we will study the contributions of this didactical representation on various algebraic tasks such as factoring, solving equations. Finally, we will describe a first experiment.

INTRODUCTION

In Mathematics, notions can only be handled through semiotic representations such as the natural language register, numerical writing and graphical representations. As specified by (Duval, 1993), "semiotic representations are productions made of the use of signs belonging to a semiotic system of representation which has its own meaning and functioning constraints". In this approach, the conceptualisation of mathematical notions needs the handling of several registers for a same mathematical object thus enabling to separate it from its representations. Duval (ibid) distinguishes three types of activities: the construction of an identifiable representation responding to some given rules, the processing of a representation in its own register and the conversion of a representation from a register to another one.

Only the first two activities are generally considered by teaching. But the third one conversion plays an essential role in the conceptualisation. The understanding of a conceptual content relies on the coordination of at least two registers of representation, and this coordination manifests itself by the rapidity and the spontaneity of the cognitive activity of conversion Duval (ibid).

We rely on this didactical hypothesis to make the Aplusix software application evolve. Indeed, in the first version of the application, only one register was implemented: the usual register. This register enables the forming of algebraic expressions and their processing. As part of the European project ReMath, we made the application evolve by adding two other registers of representations: trees (Bouhineau, 2007) and graphs.

With each algebraic task, we associate a graphical representation to the usual representation which is the denotation of the mathematical object which is represented. The purpose is to develop the understanding that correct calculations lead to identical denotations.

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REGISTERS OF REPRESENTATION IN APLUSIX

The Aplusix application

The Aplusix environment (Nicaud et al., 2004) is an ILE (Interactive Learning Environment) to practice elementary algebra: transformations of algebraic expressions, solving of equations, inequations and systems, at a high school level.

The student's goal in Aplusix is to solve, as on paper, algebra problems by performing the calculation steps of his/her algebraic reasoning. The mathematical frame offered for this work is solving by equivalence. During training mode activities, feedback is given: the algebraic equivalence between steps is constantly calculated and displayed. There is also a test mode where this feedback is not available.

During the design of Aplusix, the authors made the screen representation of algebraic expressions very close to the usual representation, the one everybody uses on paper and blackboard. So, the first versions of Aplusix implement the usual algebraic register of mathematical expressions.

The graphical representation of the expressions

Mathematical objects available for a graphical representation in Aplusix are the algebraic expressions of one variable, equations and inequations with one unknown and linear systems with two unknowns. A « graphical representation » menu allows the user to get the graphical representation of the object contained in the current step. This representation is added to the graphical representations already displayed in a dedicated window. This is a graphical denotation of the reasoning step it comes from; it dynamically evolves according to the changes of the step. Changes can be performed on the graphical appearance of the curve: It is possible to choose the colour and the thickness for each curve.

More precisely, the graphical representations chosen for the different sorts of expressions are:

For an algebraic expression of one variable, Expr(x): the graph of the function $x \rightarrow$Expr(x) is drawn in the plane. See figure 1.

For an equation of one unknown, G(x) = R(x): the graphs of the functions $x \rightarrow$G(x) and $x \rightarrow$R(x) are drawn in the plane (this is an intermediary graphical representation and not a denotation of the equation) and the solution set of the equation is drawn on the real line (this is the denotation of the equation). See figure 2.

For a system with two unknowns, the lines representing the equations are drawn (intermediary representations which are not the denotation of the system) and the points which are common to all the straight lines (denotation of the system).

For an inequation of one variable, e.g., G(x) < R(x): the graphs of the functions $x \rightarrow$G(x) and $x \rightarrow$R(x) are drawn in the plane and the solution set is drawn on the real line. See figure 2.

The main challenge for designing interesting graphical representations is in the context of Aplusix (students work by equivalence) consisted of drawing the curves of objects having the same denotation, i.e. curves which coincide. We chose to use different thicknesses and colours. The new/last objects are drawn with the thinnest stroke above the others (their stroke thickness is increased if necessary), varying the colours. With the help of a palette, the user can change these settings to better observe each object.

EXPERIMENT WITH THE GRAPHICAL REGISTER

We experimented the use of this new representation with a pair of 10th grade students (15-16 years old). These students had already used Aplusix at school. Two sessions of 50 minutes were conducted by a researcher and the mathematics teacher of the class.

During the first phase, the researcher presented the graphical representation based on two examples: "expand $x(x+2)$" and "solving $2x+1= x-4$". The goal was to allow them to interpret algebraic calculations in the usual register with the help of the graphical register. Regarding the equation, we distinguished two levels of interpretation. The first level consisted of interpreting the equation $P(x) = Q(x)$ as an equality between two functions f and g defined by $f(x) = P(x)$ and $g(x) = Q(x)$. That is to say, we searched x belonging to the intersection of the definition domain of f and g, we noted it D_{fg}. Then, we interpreted the search of the equality between these two functions in the D_{fg} set as the search of the abscissa of the intersection points of the representative curves of f and g. The second level concerned the processing of the equation which had to be interpreted in the graphical register. Thus the student had to interpret the equivalence between two equations as the invariance of the intersection points of the curves associated with the two equations.

In order to accomplish this goal, we proposed to the pair to solve 4 exercises in the test mode (no display of the equivalence) with the possibility to display the graph. This way used the graph as a control tool for the algebraic solving by mobilising the two control levels developed before.

The pair made a relevant use of the graph as a control tool for the three first exercises: "Expand an simplify $(2x+3)(3-x)$", "Expand and simplify $(x-3)^2$" and "Factor $x(x+3)+(x+3)$". For example, in the first exercise, the pair made an error in step 3 (see figure 1), noticed the non superposition of the curves in the graph, then modified step 3 and obtained the superposition of the curves.

However, the fourth exercise "Solving $x(x-1)) -x^2+1$" the students had difficulties regarding the interpretation, because they tried to interpret the equivalence of the equations as a superposition of the curves. The situation required an intervention of the researcher. It was an opportunity to go back on the notion of equivalence between equations and equality between functions (see figure 2).

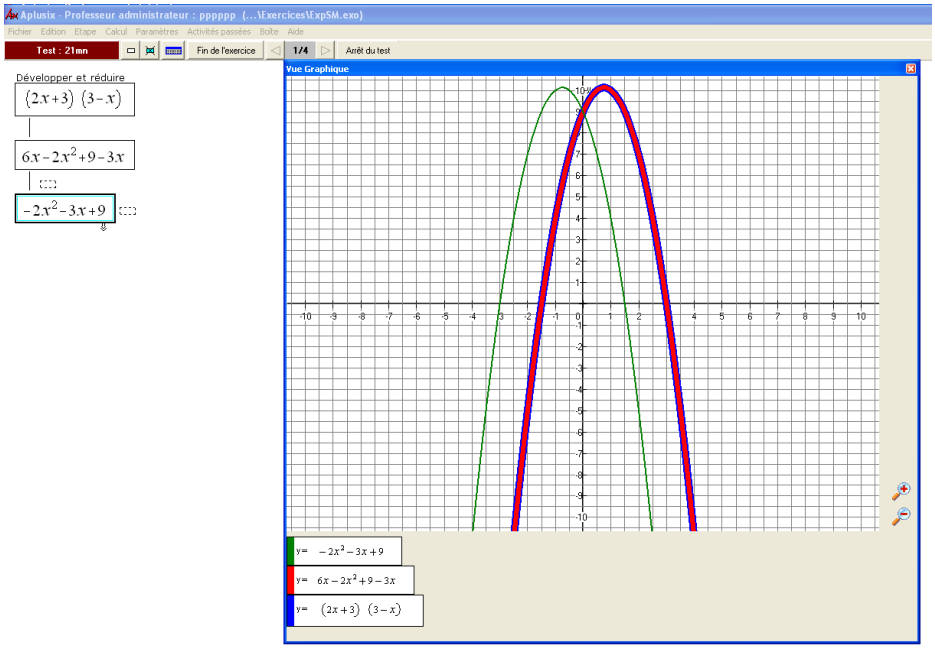

Figure 1. Solving an “expand and simplify” exercise. The non coincidence of the curves is a consequence of the non equality between the expressions of the two steps.

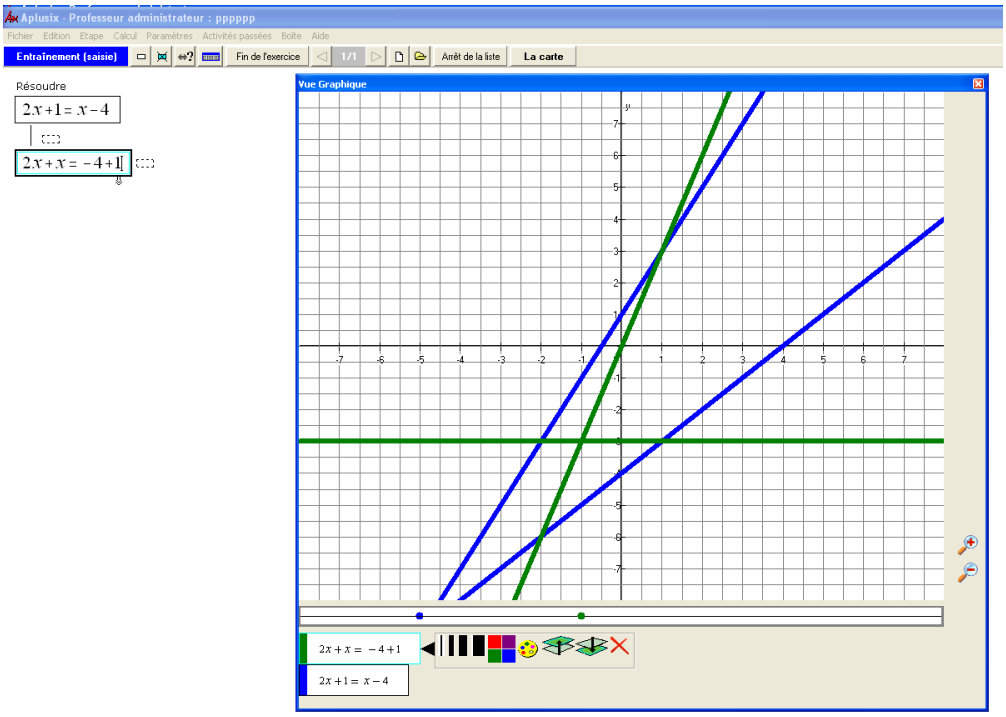

Figure 2. Solving an equation. The solutions, obtained from the abscissa of the intersection points of the curves are represented on the line under the plan. The non equivalence of the equations is represented by different set of solutions (associated to each equation).

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