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Modeling of Magnetic Field Perturbations on the Balance-Spring of a Mechanical Watch

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A magnetic field is a major enemy of a mechanical watch. Such field modifies the structure of the balance-spring that gives rhythm to the watch and thus changes its resonance frequency. Consequently, the watch loses its accuracy. The aim of this work is to quantify the impact of the magnetic field on the structure and the resonance of the balance-spring using a finite element approach. This coupled magneto-mechanical problem implies magnetic force computation, including both magnetostatic forces and magnetostriction effect, while accounting for a large rotation of the spring. An algorithm including a dynamic mechanical time-stepping scheme is proposed to quantify the change of the resonance frequency.

Index Terms— Mechanical watch, balance-spring, resonance frequency, magneto-mechanical problem.

I. INTRODUCTION

A MECHANICAL WATCH is a masterpiece of human creativity in mechanical engineering and arts. This watch uses only a mechanical system to measure the passage of time, as opposed to quartz watch which works electronically. Several parts of the mechanical watch are made of ferromagnetic materials that are sensitive to the magnetic field perturbations.

Watch manufactures are then currently studying the origin of these perturbations and the way to limit them. In this context, this work deals with the study of the impact of magnetic field on the mechanical watch, especially on its balance-spring. Spring deformations due to magnetic perturbations -magnetostriction and magnetostatic stress- are evaluated by using a finite element approach. Moreover, the modification of both amplitude and frequency of first resonance resulting from this deformation is studied to evaluate the corresponding time delay.

II. MECHANICAL WATCH MECHANISM

A mechanical watch contains four principal components: main spring, gear wheels, escapement, and balance-spring [1]. Fig. 1 shows the watch components.

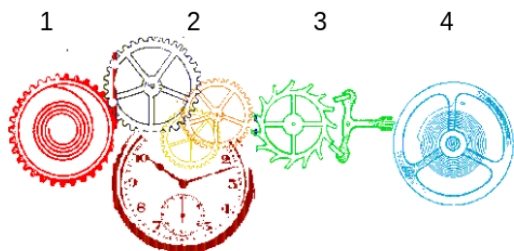


Fig. 1. Mechanical watch components

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Main spring (1) is a component to store the mechanical energy. The energy is then transferred to the escapement via the gear wheels (2) connected to the time indication. At the end of the system, a balance-spring (4) has to be installed to control the movement with a regular frequency.

The balance-spring is the heart of a mechanical watch and is suspected to be the part being the most sensitive to magnetic perturbations. It regulates the passage of time thanks to its natural oscillation and is responsible of the watch accuracy [2]. The balance-spring studied in this work is shown in fig. 2.

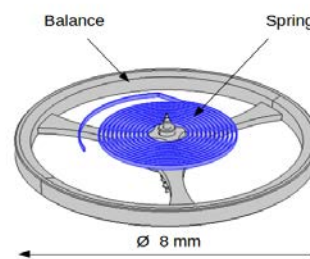


Fig. 2. Balance-spring of a mechanical watch

The frequency of the oscillation depends on the stiffness of the spring K and the moment of inertia of the balance J . For a simple geometry, the resonance frequency is given by:

$$f = \frac{1}{2\pi} \sqrt{K/J} \quad (1)$$

The stiffness and the moment of inertia depend on the geometry and the material properties of the spring. As a consequence, a small modification in the structure of the spring changes the resonance frequency and generates an error on the time indication. The norm NIHS 91-10 requires that this error should not exceed 30 seconds per day [3].

Generally, the balance is made of Glucydur (beryllium bronze). This is a non-magnetic material. The spring in the other side is made of iron-nickel alloys (Invar, Elinvar, Nivarox ...), notable for their low coefficient of thermal expansion [4]. These are ferromagnetic materials, and the structure can thus be impacted by magnetic fields. It has been observed that the presence of a magnetic field can imply an error from few seconds to few minutes per day [3].

III. MAGNETO-MECHANICAL PROBLEM

The problem requires coupled magneto-mechanical analysis. The magnetic problem deals with the computation of the magnetic field in material, and the mechanical problem treats the deformation resulting from the corresponding forces. To connect these two systems, one has to determine the forces related to magnetic origins. Fig. 3 shows a general scheme of the considered weak magneto-mechanical coupling modeling.

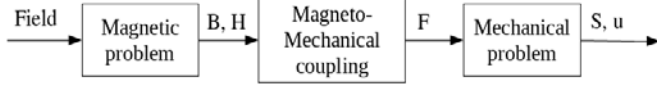


Fig. 3. Weak magneto-mechanical coupling problem

The magnetic equilibrium equation is represented by Maxwell's equations (neglecting the displacements currents):

$$\begin{aligned}\nabla \times \mathbf{H} &= \mathbf{J} \\ \nabla \cdot \mathbf{B} &= 0\end{aligned}\quad (2)$$

where \mathbf{H} is the magnetic field, \mathbf{B} the magnetic induction and \mathbf{J} the current density (here equal to zero). A magnetostatic problem can be solved using scalar or vector potential formulations. The scalar potential formulation that needs less unknown number is adopted in this work.

The mechanical equilibrium equation is given by Newton's second law:

$$\nabla \cdot \mathbb{T} + \mathbf{F} = \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2}\quad (3)$$

where \mathbb{T} is the stress tensor, \mathbf{F} is the driving force, \mathbf{u} the displacement and ρ_m is the mass density.

Magnetic forces link the magnetic and the mechanical systems. For the considered application, these forces can be divided into two types: magnetostatic forces and magnetostriction effect [5].

Considering homogeneous isotropic linear properties for the ferromagnetic material, magnetic forces are related to the structure of the spring. They can be obtained from Maxwell's stress tensor [6]:

$$\mathbf{F}^{mag} = \frac{1}{2} \left(\left(\frac{1}{\mu_0} - \frac{1}{\mu} \right) \mathbf{B}_n^2 - (\mu_0 - \mu) \mathbf{H}_t^2 \right) \mathbf{n}\quad (4)$$

where \mathbf{B}_n is the magnetic induction in normal direction, \mathbf{H}_t is the magnetic field in tangent direction, \mathbf{n} is the vector normal to the surface, μ_0 and μ are respectively the permeability of the air and the material.

Magnetostriction effect is a spontaneous strain in ferromagnetic material during the process of the magnetization [7]. This strain is a non-linear phenomenon that can be approached by [8]:

$$S_{kl}^\mu = \frac{\lambda_s}{M_s^2} \left(3M_k M_l - \delta_{kl} \|\mathbf{M}\|^2 \right)\quad (5)$$

where λ_s is magnetostriction constant in saturation, \mathbf{M} is the magnetization, M_s is the magnetization in saturation, and δ is

Kronecker symbol.

The magnetostriction effect can be represented by an equivalent magnetostrictive force. This force is said to be equivalent in the sense that it should produce the same elastic strain than the strain associated to magnetostriction. It can be determined using:

$$\mathbf{F}^m = -\nabla \cdot (\mathbf{C} : \mathbb{S}^\mu)\quad (6)$$

where \mathbf{C} is the elasticity tensor and \mathbb{S}^μ is the strain tensor.

Equations (2) - (6) combined with the constitutive relations in materials are then converted into weak formulations to obtain linear equations to be solved by a 3D finite element approach integrated in Comsol software:

$$\begin{aligned}[\mathbf{K}_{\phi\phi}] \phi &= \mathbf{g} \\ [\mathbf{K}_{uu}] \mathbf{u} + [\mathbf{C}] \dot{\mathbf{u}} + [\mathbf{M}] \ddot{\mathbf{u}} &= \mathbf{F}^{mag} + \mathbf{F}^m\end{aligned}\quad (7)$$

where $\mathbf{K}_{\phi\phi}$ is magnetic stiffness, \mathbf{K}_{uu} is mechanical stiffness, \mathbf{C} is damping coefficient, \mathbf{M} is mass and \mathbf{g} is used to impose the external magnetic field.

IV. SIMULATION

A. Static Modeling

The first simulation considers the balance-spring placed in a uniform magnetic field and does not consider its oscillations. The aim is to determine the relative impact of the direction of magnetic field and the contribution of magnetostriction and magnetostatic forces on its deformation.

Fig. 4 shows magnetic flux density and forces for a 4800 A/m magnetic fields from two directions (horizontal and vertical) and fig. 5 shows the total displacements of the spring due to these forces. It can be stated that the impact of the field perturbation from the horizontal direction is more important than the vertical one: displacements are 20 times greater. This information can be considered to design a shielding to the

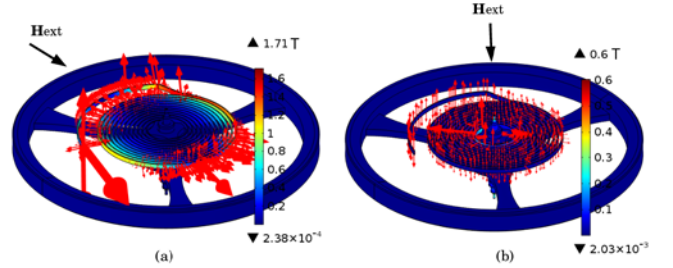


Fig. 4. Magnetic flux density (color) and magnetic forces (arrows) for (a) horizontal field (b) vertical field

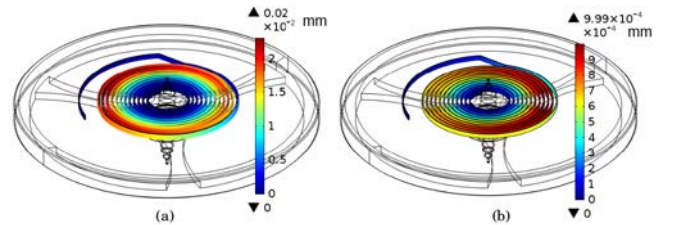


Fig. 5. Total displacements (a) horizontal field (b) vertical field

The contribution of the magnetostatic forces and the magnetostriction effect to the displacement in horizontal direction is detailed in fig. 6. It is shown that the magnetostriction effect seems negligible for such application. In fact, for dominant air-gap geometry, the magnetostatic forces are generally more important than magnetostriction

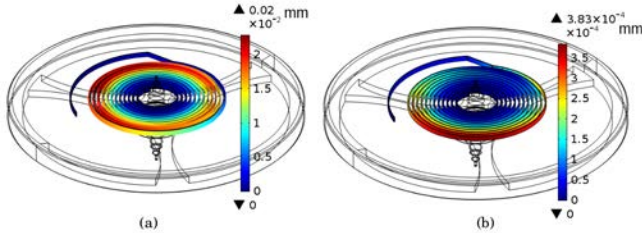


Fig. 6. Displacement of the balance-spring for horizontal field due to (a) magnetostatic forces (b) magnetostriction effect.

B. Dynamic Modeling

The second study deals with the oscillations of the balance-spring in the presence of the magnetic field. Solving this dynamic mechanical problem becomes difficult since rotational displacement is large. Thus, a geometric non-linearity has to be taken into consideration [9] in the strain/displacement relation. In this case, the mechanical stiffness \mathbf{K}_{uu} is a function of displacement \mathbf{u} and must be evaluated at each time step. However, the problem is simplified by assuming that the shape (section, area, thickness, length) does not change (small strain assumption). In this case, the relation strain/stress can still be evaluated using linearized expression (Hooke's law).

Considering the complexity of the problem, the original geometry of the balance-spring is simplified into a 2D geometry as shown in fig. 7. To keep the oscillation frequency, we have to consider an equivalent moment of inertia with an increase of the mass density of the central part of the balance in spring plane.

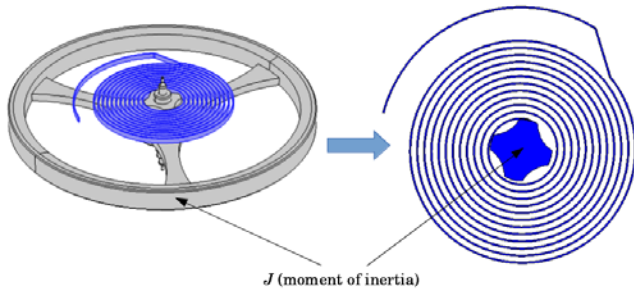


Fig. 7. Simplification of the balance-spring geometry

In the oscillation, the outer spring end is held in a clamped position and the inner spring end rotates with the center mass. Fig. 8 shows the evolution of the spring geometry during the oscillation obtained with only mechanical part of finite element model (without magnetic perturbations). The spiral diameter becomes smaller for clock-wise rotation and becomes bigger for counter-clock rotation. Another aspect to be considered in the simulation is the meshing technique.

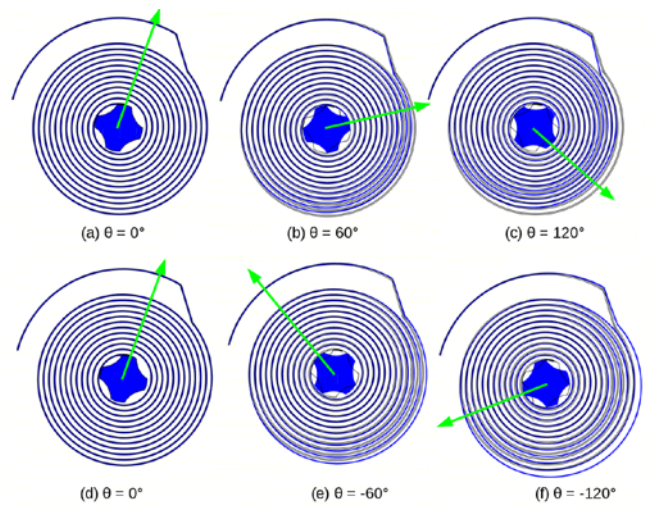


Fig. 8. Spring evolution during oscillation (black) initial geometry (blue) geometry in oscillation

In this only-mechanical problem, Lagrangian mesh algorithm in which each mesh node follows the motion is used [10]. However, it can generate mesh distortion for magneto-mechanical coupling study, especially for inner air mesh elements, and the problem is then not converging. To tackle this problem, one can combine Lagrangian mesh with Eulerian mesh algorithm (fixed node). With this combination algorithm, called ALE (Arbitrary Lagrangian-Eulerian), the mesh nodes on the boundary and on the interface follow the motion. Inside the domain, the mesh nodes can move or be held arbitrary to optimize the shape. Fig. 9 shows the meshing of the simulation geometry and its evolution in a zoom area

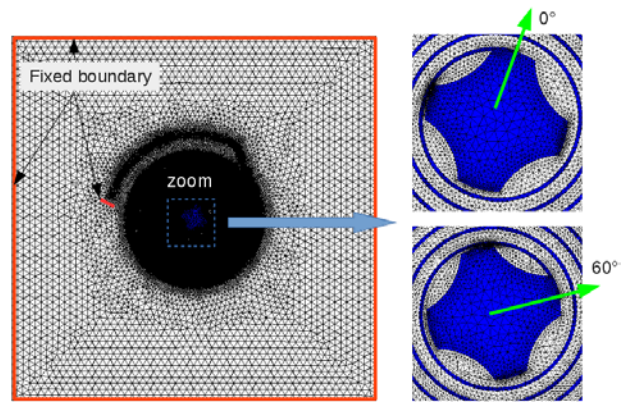


Fig. 9. Meshing of the simulation geometry

The complete algorithm of the modeling considering the presence of magnetic field is given in fig. 10. The simulation starts with a static mechanical-only computation that gives an initial position to the balance-spring by imposing a chosen rotational displacement. This allows the spring being in a pre-stress configuration. The balance-spring is then released and oscillates around its equilibrium position. The second part of the simulation follows an iterative process including for each time step a magnetostatic computation, the expression of the corresponding magnetostatic forces, the non-linear mechanical computation, and the mesh update.

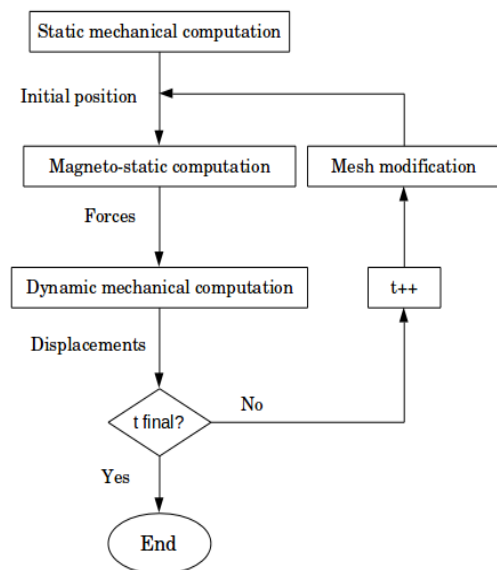


Fig. 10. Complete algorithm of the modeling

The algorithm is implemented to the system in fig. 9. The spring is placed in a uniform magnetic field of 2400 A/m and a small initial angle (10°) is set up in order to help for the convergence (limited displacement between two time steps). The observation is performed during 1 second with the step time of 0.1 milliseconds (10000 iterations). Fig.11 shows the magnetic flux lines and the magnetic force at equilibrium

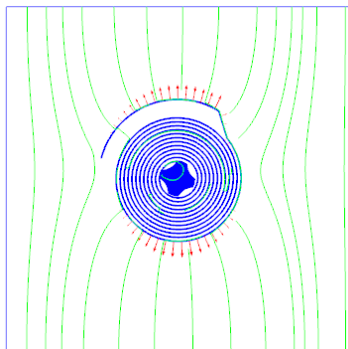


Fig. 11. Magnetic flux (green lines) and force density (red arrows)

Fig. 12 shows the oscillations of the balance-spring for $H_{\text{ext}}=0$ A/m and 2400 A/m. We note that the magnetic field modifies both the amplitude and the frequency of the balance-spring oscillations. To analyze clearly the change of the frequency, a FFT with zero padding technique is used. The frequency of resonance under the considered magnetic field is decreased by 0.02 Hz. The watch placed in such magnetic field will consequently lose 7.1 minutes per day. From discussions with watch manufacturers, this result is in accordance with experimental observations.

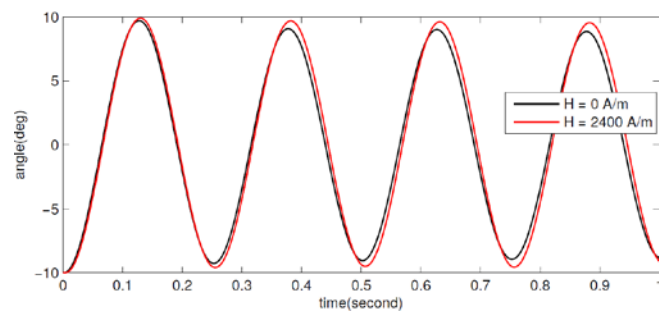


Fig. 12. Oscillations of the balance-spring with and without the presence of magnetic field

V. CONCLUSION

The magnetic perturbations on the balance-spring are one of the main problems for the watchmakers. Unfortunately, there is no accurate numerical modeling dealing with this challenge. This contribution provides information about how a magnetic field perturbs the watch: The simulations have shown that the magnetic field modifies the geometry and the resonance frequency of the balance-spring. Although for a short period this difference seems negligible, it will induce an inaccuracy which exceeds the standard norm.

Because of the complexity of the problem (due to the coupled effects and the geometric non-linearity), the dynamic modeling in this work has been limited to a 2D geometry. In future work, it will be interesting to study a realistic 3D geometry. More efficient meshing technique can be applied by separating the magnetic and mechanical mesh. In this case, the forces obtained from magnetic computation are projected into mechanical computation.

The size of the balance-spring (a few millimeters) makes the experimental validation highly challenging. Collaboration with watchmakers is necessary to treat this aspect.

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