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The impact of a teaching intervention on sixth grade students' fraction understanding and their performance in seven abilities that constitute fraction understanding

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In a previous study, we found that students' abilities in fraction recognition, definitions and explanations, argumentations and justifications, relative magnitude of fractions, representations, connections and reflection constitute fraction understanding of sixth grade students. In the present study, we examine the impact of an intervention comprising of lessons for developing the seven abilities on students' fraction understanding and their performance in the seven abilities. The sample comprised of 343 sixth grade students. Repeated measures analysis showed that the students of the experimental group outperformed those of the control group in their level of fraction understanding and their ability in fraction recognition, definitions and explanations, argumentations and justifications, connections and reflection.

Keywords: Fraction understanding, repeated measures analysis, sixth grade, students' abilities, teaching intervention.

INTRODUCTION

Fraction complexities and students' difficulties have led a number of researchers in the past to study fraction understanding carrying out research programs and teaching experiments (e.g. Cramer, Post, & delMas, 2002). In the present study, we examine the impact of a teaching intervention which is based upon a different perspective compared to the studies already carried out. More specifically, in a previous study conducted by the authors (Nicolaou & Pitta-Pantazi, 2011), we confirmed a theoretical model with seven abilities that constitute fraction understanding at elementary

school. The seven abilities were: (a) fraction recognition, (b) definitions and mathematical explanations for fractions, (c) argumentations and justifications about fractions, (d) relative magnitude of fractions, (e) representations of fractions, (f) connections of fractions with decimals, percentages and division, and (g) reflection during the solution of fraction problems. Five of these abilities, definitions and mathematical explanations, argumentations and justifications, representations, connections and reflection correspond to the mathematical processes suggested by NCTM (2000) that are important for understanding mathematical concepts. We consider that apart from the various issues related to fraction understanding that were already investigated, students' ability to engage in processes such as reflection during the solution of fraction problems, explanations for fractions, argumentations and justifications about fractions, representations of fractions and connections of fractions with decimals, percentages and division are essential for fraction understanding. Additionally, we included fraction recognition and the relative magnitude of fractions in the abilities required for fraction understanding at elementary school (the reasons for this decision are provided in the next section "Theoretical Background").

In the present study, we examine the impact of a teaching intervention comprising of lessons for developing the seven abilities on sixth grade students' fraction understanding and their performance in the seven abilities. The aim was twofold: (a) To examine the impact of the intervention on sixth grade students' fraction understanding, and (b) To examine the impact of

the intervention on students' performance in each of the seven abilities.

THEORETICAL BACKGROUND

The seven abilities

According to NCTM (2000), mathematical processes are very important for understanding mathematical concepts. Based on this rationale, we considered that students' abilities in these processes are also important for fraction understanding at elementary school. We also included two more abilities as explained below. In this section we will briefly refer to these seven abilities (for more details see Nicolaou & Pitta-Pantazi, 2011).

Reflection during the solution of fraction problems refers to students' ability to reason their thinking and their answer while solving fraction problems, to support the reasonableness of their answer and verify a given answer. In order to solve fraction problems, students should be able to use the various fraction sub-constructs; they should be able to think of fractions embedded in mathematical problems corresponding to these sub-constructs and utilize them accordingly to solve the problem. Moreover, students should be able to carry out operations with fractions. Argumentations and justifications about fractions refer to a kind of "informal proof" at elementary school. For the purpose of the present study, argumentations and justifications refer to students' ability to judge statements about fractions as true or false, justifying at the same time their choice. Definitions and mathematical explanations for fractions refer to students' ability to define in their own words what a fraction is and also to explain in various ways (verbally, by using drawings, examples etc.) other issues concerning fractions (e.g., fraction equivalence, comparison, density). Connections refer to students' ability to connect fractions to other forms of rational numbers (decimals and percentages) and division. Representations of fractions refer to students' ability to translate to visual, verbal and symbolic representations and their ability to construct drawings for fractions.

In order to propose a model that would provide a sufficient description of fraction understanding, we also included fraction recognition and relative magnitude of fractions in the abilities required for fraction understanding. Fraction recognition refers to students' ability to recognize structural characteristics of frac-

tions and detect similarities and differences between fractions. It also includes students' ability to categorize fractions on the basis of a common characteristic. The relative magnitude of fractions refers to students' ability to compare and order fractions and is crucial for fraction understanding, since in the case students are not able to compare and order fractions, then they probably do not understand the meaning of fractions.

METHODOLOGY

The rationale of the intervention – Its principles

The intervention comprised of activities which aimed at developing the seven abilities. The design of the intervention was based on some principles. According to the first principle, the activities and the problems should be interesting, arise from everyday life and attract students' interest (Elbers, 2003). The second principle referred to the sequence of the activities from the easiest to the more difficult ones. The third principle was about the way students worked; either individually or in small groups favoring discussion and exchange of ideas (Elbers, 2003; Martino & Maher, 1999; Terwel, Van Oers, van Dijk, & van den Eeden, 2009). After working individually or in small groups, students discussed their ideas in the whole classroom and this procedure helped them to share their views and interact (Elbers, 2003; Martino & Maher, 1999; Terwel et al., 2009). Additionally, in every activity students had to explain their thinking and provide adequate reasoning for their decisions. The fourth principle referred to the modification of the lesson plans according to the strengths and weaknesses of the students and their previous knowledge of fractions. The fifth principle referred to the role of the teacher as a facilitator of learning. While students worked individually or in small groups, the teacher moved around the classroom, provided support and feedback. During the whole class discussion and exchange of ideas, the teacher was expected to guide the discussion and pose questions that would stimulate further inquiry (Martino & Maher, 1999).

Participants, instruments and procedure of the study

Participants in the present study, which was quantitative in nature, were 343 sixth grade students (age range 10.8–11.8 years old). A test comprising 37 tasks for measuring the seven abilities was developed. The tasks of the test were content and face validated by four experienced primary school teachers and two

Fraction recognition	One of the following fractions differs from the others. Find that fraction and circle it. $\frac{2}{7}$ $\frac{3}{2}$ $\frac{14}{49}$ $\frac{10}{35}$ $\frac{4}{14}$
Definitions and mathematical explanations for fractions	Imagine that your teacher asked you to explain to one of your classmates what a fraction is. Use as many different ways you can.
Argumentations and justifications about fractions	If I double both the numerator and the denominator of a fraction, then the formed fraction has twice value compared to the initial one. <div style="border: 1px solid black; display: inline-block; padding: 2px 10px;">T</div> <div style="border: 1px solid black; display: inline-block; padding: 2px 10px;">F</div> Justify your answer:
Relative magnitude of fractions	Order the fractions $\frac{1}{2}$, $\frac{4}{3}$, $\frac{2}{3}$, $\frac{1}{4}$ starting from the smallest one.
Representations of fractions	Marinos ate $\frac{1}{2}$ of a cake and Marinas $\frac{3}{8}$ of the same cake. Construct a drawing to show what part each child ate and what part the two children ate together.
Connections of fractions with decimals, percentages and division	Convert the following fractions to decimals. a) $\frac{1}{4} =$ b) $\frac{2}{5} =$ c) $\frac{3}{10} =$ d) $\frac{1}{20} =$
Reflection during the solution of fraction problems	In order to prepare a cake, I need $\frac{1}{10}$ L milk. If I have $\frac{4}{15}$ L milk, then how many cakes can I prepare? You should definitely reason about your answer.

Table 1: Tasks used to measure each ability

university tutors of mathematics education before their administration. Tasks 1–3 were used to measure fraction recognition, tasks 4–7 definitions and mathematical explanations for fractions, tasks 8–12 argumentations and justifications about fractions, tasks 13–15 and 32–37 were about reflection during the solution of fraction problems, tasks 16a–16c and 17 were about the relative magnitude of fractions, tasks 18–27 measured representations, while tasks 28–31 examined students' ability to link fractions to decimals, percentages and division. Examples of tasks used to measure each one of the abilities are presented in Table 1.

The first measurement was conducted at the beginning of the school year in the first two weeks of October. After the first measurement, the participants were split into experimental and control group. The experimental group comprised of eight classes (144 students), while the control group comprised of eleven classes (199 students).

The intervention comprised of nine lessons, four of which had a duration of 80 minutes, two 60 minutes, while the other three 40 minutes. The total time devoted was 14x40 minutes. The time devoted to the

development of each of the seven abilities was about the same (2–3 40 minute periods) with the exception of the relative magnitude of fractions, which was devoted less time (about 40 minutes). The reason for this is that similar activities for developing this kind of ability are included in school textbooks. It should also be stressed that some lessons aimed at developing more than one ability and comprised of activities that served towards this goal.

The implementation of the intervention started at the end of October after the first administration of the test for measuring the seven abilities (pre-test). Its duration was about nine weeks until the end of January before the second administration of the test (post-test). The teachers that participated in the study were asked to teach one lesson every week. The students of the control group during this time were taught according to the Cyprus National Mathematics Curriculum which included topics such as recognizing representations of fractions, some activities of explaining what a fraction is, recognizing fractions as the division of the numerator by the denominator, equivalent fractions, fraction comparison and ordering, improper fractions and mixed numbers, decimals and percentages and their conversion to fractions. It

must be noted that the total time devoted to fraction teaching was about the same for the two groups.

After the completion of the intervention, all students (including those of the experimental group) were taught according to the Cyprus National Mathematics Curriculum until the end of the sixth grade. They practiced simplifying and comparing fractions, adding and subtracting fractions with the same or different denominator, adding and subtracting mixed numbers, solving fraction problems, fraction multiplication and division, solving problems of mixed numbers and multiplication and division of mixed numbers. About three months after the second measurement, a third measurement took place (retention-test) for examining the duration of the impact of the intervention.

The lessons

The activities designed for implementing the goals of the intervention shared a scenario that was challenging, attractive and pleasant for students. The scenario referred to the trip of the "Mathematician" to the land of fractions. The trip comprised of various activities and the participants were called to answer questions and solve some problems. The scenario was designed in order to please students and motive them to engage in the activities with enthusiasm.

The first lesson with duration 80 minutes aimed at developing students' ability in "defining" what a fraction is, recognizing fractions in various representational systems, placing fractions on number line and linking the concept of fractions to the division numerator \div denominator. The goal of the second 80 minutes lesson was the development of students' ability to connect fractions to the other two forms of rational numbers, decimals and percentages and was complementary to the first lesson. During the lesson, the teacher raised questions: "What are decimals?", "What are percentages?" that would lead to the definition of decimals and percentages. Students were also asked to convert fractions to decimals and percentages and the reverse. Emphasis was placed on understanding the conversion and not just applying the rules. Students also worked on textbook activities. The third 40 minutes lesson cultivated students' ability to construct visual representations of fractions and acquire a feeling of the relative magnitude of fractions. Students worked in pairs and were encouraged to construct various representations. Students also worked on textbook activities that referred to fraction comparison and

ordering by utilizing visual representations. The fourth lesson with duration 80 minutes referred to the development of students' ability to convert verbal and symbolic representations to visual ones and vice versa. The fourth lesson was complementary to the first and third lessons regarding the development of students' abilities in representations. In some activities, problems of fractions were presented to students and they had to write the equation and construct drawing/drawings in order to solve them (translation from verbal to symbolic and visual representations). Other activities asked students to translate from symbolic to visual and verbal representations. In these activities, the equation was given and students had to write a problem that could be solved by this equation or construct a drawing. Finally, students were called to write problems on the basis of visual representations (from visual to verbal representation). Lessons 5 and 6, with duration 60 minutes each, aimed at developing students' ability in reflection during the solution of fraction problems. Five carefully selected problems were presented to students and they had to solve them and were encouraged to reason their thinking, explain the strategy they used, express their confidence about their solution, examine whether the path they followed was correct or not and what they did correct and what wrong. Furthermore, they were called to think about the reasonableness of their answer and verify their answer. The seventh and eighth lessons with duration 40 minutes each aimed at developing students' abilities in fraction recognition, mathematical explanations for fractions, justifications about fractions and reflection. The two lessons included activities requiring students to detect fraction similarities, fraction differences and write fractions that share a common property. The ninth 80 minutes lesson aimed at developing students' abilities in argumentations and justifications about fractions and reflection.

Statistical analyses

To answer the two research questions about the impact of the teaching intervention on students' fraction understanding and their performance in the seven abilities, the z-scores that emerged from the Confirmatory Factor Analysis utilized in a previous study carried out by the authors (Nicolaou & Pitta-Pantazi, 2011) were used. Descriptive statistics (means and standard errors of estimate) for students' fraction understanding and the seven abilities were first found for the control and the experimental group respec-

tively. Afterwards, Repeated Measures Analysis was used with dependent variable the score for fraction understanding and then for each of the seven abilities separately and independent variables the kind of condition (control or experimental) and the time of the measurement (pre-test, post-test, retention-test). The independent samples t-test was also used to test for the equality of the two groups in the pre-test.

RESULTS

Table 2 shows means and standard errors of estimate of fraction understanding for the experimental and the control group for each of the three measurements.

Table 2 illustrates that before the intervention, the two groups of students were equal with respect to their level of fraction understanding. This was also confirmed by the application of t-test ($t=.182$, $p=.856>0.05$). In the period of the implementation of the intervention, both groups improved, but the improvement of the experimental group was greater. In the period three months after the intervention the two groups continued to improve and the difference found in the post-test was maintained in the retention test. Repeated Measures Analysis showed that the means of fraction understanding differed significantly between the three measurements (Pillai's $F_{(2,340)}=.516$, $p<0.01$), and additionally, there was a statistically significant interaction of fraction understanding with the kind of the condition (Pillai's $F_{(2,340)}=.029$, $p<0.01$). Therefore, the differences found between the two groups of students could be attributed to the implementation of the intervention which gave the experimental group superiority over the control group.

Descriptive statistics for students' performance in each of the seven abilities are shown in Table 3.

From Table 3, we observe that the experimental and the control group had about equal means for all the seven abilities at the pre-test. This was also confirmed by the application of the t-test, which showed the equal-

ity of the two groups of students before the conduction of the intervention (for fraction recognition $t=-0.603$, $p>0.05$; for definitions and mathematical explanations $t=0.170$, $p>0.05$; for argumentations and justifications $t=0.600$, $p>0.05$; for relative magnitude of fractions $t=-0.203$, $p>0.05$; for representations $t=0.901$, $p>0.05$; for connections $t=0.827$, $p>0.05$; for reflection $t=-0.641$, $p>0.05$). Regarding the evolution of students' abilities, the experimental group showed continuous improvement during the period of the three measurements in the relative magnitude of fractions, representations, connections and reflection, whereas this was not the case for fraction recognition, definitions and mathematical explanations and argumentations and justifications where there was improvement from the pre-test to the post-test and then a decline in the period three months after the intervention. Concerning the control group, students' ability in the relative magnitude of fractions and reflection showed continuous improvement in the period of the three measurements. For representations, a very small improvement was observed during the period from the pre-test to the post-test, but this improvement was much greater in the period from the post-test to the retention test. Students' ability in fraction recognition and connections had an increase during the period from the pre-test to the post-test and then a decline in the period from the post-test to the retention-test. For definitions and mathematical explanations and argumentations and justifications there was improvement in the period between the pre-test and the post-test and then stabilization. The improvement of the students of the control group in some of the abilities can be attributed to maturity and teaching. Nevertheless, both at the post-test and at the retention-test the experimental group outperformed the control group in all the abilities with the exception of the relative magnitude of fractions where the two groups were about equal. The application of Repeated Measures Analysis revealed that the means for all the seven abilities differed significantly between the three measurements. Additionally, for five abilities: fraction recognition, definitions and mathematical explanations, argumentations and justifications, con-

	Pre-test		Post-test		Retention test	
Group		SE*		SE		SE
Experimental ($n=144$)	11.48	0.49	17.04	0.51	18.56	0.60
Control ($n=199$)	11.37	0.42	15.28	0.44	16.45	0.51

*SE: Standard Error of Estimate

Table 2: Means and standard errors of estimate of fraction understanding for the experimental and the control group for each of the three measurements

	Fraction recognition					
	Pre-test		Post-test		Retention test	
Group		SE		SE		SE
Experimental	1.91	0.05	2.43	0.05	2.17	0.05
Control	1.95	0.05	2.22	0.04	2.01	0.04
	Definitions and mathematical explanations for fractions					
	Pre-test		Post-test		Retention test	
Group		SE		SE		SE
Experimental	0.50	0.04	0.84	0.04	0.70	0.04
Control	0.49	0.03	0.59	0.04	0.59	0.03
	Argumentations and justifications about fractions					
	Pre-test		Post-test		Retention test	
Group		SE		SE		SE
Experimental	0.99	0.07	1.60	0.07	1.42	0.07
Control	0.94	0.06	1.25	0.06	1.26	0.06
	Relative magnitude of fractions					
	Pre-test		Post-test		Retention test	
Group		SE		SE		SE
Experimental	1.70	0.08	2.27	0.08	2.49	0.08
Control	1.73	0.06	2.27	0.07	2.41	0.07
	Representations of fractions					
	Pre-test		Post-test		Retention test	
Group		SE		SE		SE
Experimental	3.06	0.10	3.29	0.10	3.87	0.12
Control	2.95	0.09	2.98	0.09	3.63	0.10
	Connections with decimals, percentages and division					
	Pre-test		Post-test		Retention test	
Group		SE		SE		SE
Experimental	4.67	0.31	7.25	0.32	7.51	0.33
Control	4.59	0.26	6.43	0.27	6.18	0.28
	Reflection during the solution of fraction problems					
	Pre-test		Post-test		Retention test	
Group		SE		SE		SE
Experimental	1.02	0.08	1.85	0.11	2.05	0.13
Control	1.08	0.07	1.52	0.09	1.69	0.11

Table 3: Means and standard errors of estimate of students' performance in each of the seven abilities for the experimental and the control group for each of the three measurements

nections and reflection there was an interaction with the condition (experimental or control) (the results are presented in Table 4). Therefore, the higher means of the experimental group at the post-test and retention-test can be attributed to the implementation of the intervention, which according to the results of the Repeated Measures Analysis was effective in improving students' abilities in fraction recognition, defini-

tions and mathematical explanations, argumentations and justifications, connections and reflection.

DISCUSSION

The results of the present study revealed that the intervention was effective in developing sixth-grade students' fraction understanding and their abilities

	Between the three measurements	Interaction with the condition
Ability	Pillai's $F_{(2,340)}$	Pillai's $F_{(2,340)}$
Fraction recognition	0.248**	0.031**
Definitions and mathematical explanations	0.188**	0.065**
Argumentations and justifications	0.322**	0.046**
Relative magnitude	0.316**	0.002
Representations	0.289**	0.008
Connections	0.239**	0.021*
Reflection	0.342**	0.038**

* $p < 0.05$, ** $p < 0.01$

Table 4: Results of Repeated Measures Analysis for each of the seven abilities

in fraction recognition, definitions and mathematical explanations for fractions, argumentations and justifications about fractions, connections with decimals, percentages and division and reflection during the solution of fraction problems. Therefore, the content of the intervention and the principles followed were effective in improving students' fraction understanding and their performance in five out of the seven abilities.

The conduction of the retention test added value to the findings of the present study, as it permitted the extraction of conclusions about the duration of the effects of the intervention. The conduction of three measurements also provided some evidence for the evolution of each of the seven abilities for the students of the control classes that did not receive the intervention.

For two of the abilities, the relative magnitude of fractions and representations of fractions, the intervention did not have significant effects. Concerning the relative magnitude of fractions, the results might be attributed to the fact that the curriculum provided much emphasis in developing this kind of ability. For representations, the results can also be due to the moderate emphasis given by the curriculum towards developing students' ability in representations of fractions. However, in the case of representations it must be stressed that the experimental group was superior to the control group both in the post and the retention test, but this superiority was not statistically significant.

The contribution of the present study in the area is situated in that the intervention focused on the abilities that are essential for fraction understanding at elementary school, while the focus of other studies was different (for example, Lamon (2012) examined fraction

understanding addressing fraction sub-constructs). The present study also has implications for teaching fractions with understanding as teachers can utilize the content of the intervention for developing students' fraction understanding and their performance in the five abilities applying at the same time its principles.

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