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This descriptive study examines students' performance on a proof task about corresponding parts of congruent triangles. We collected data from 1936 students, 59.8% used a subject-specific curriculum and 40.2 % used an integrated curriculum. Our findings indicate that, regardless of curriculum type, students experience difficulty with constructing the proof. Additionally, we observed that although students from the integrated curriculum were more likely to obtain partial credit when compared to their subject-specific counterparts, only 28 students in the sample (21 from subject-specific curriculum and 7 from the integrated curriculum) were able to obtain full marks.

Keywords: Proof, geometry, US textbooks, high school.

OVERVIEW

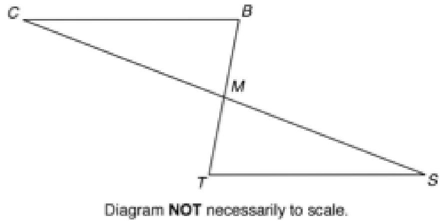
Although there is a consensus that students should have experiences with proof in all areas of mathematics (NCTM, 2000), proof has traditionally been part of only the geometry curriculum (Zaslavsky et al., 2012). Proof plays a central role in mathematics, and yet for many high school students it remains an alien concept, as suggested by weak performance of secondary students in proving (Harel & Sowder, 2007). For example, Senk (1985), studied 1520 students doing geometrical proofs and found that only 30% of students enrolled in a geometry course were able to demonstrate mastery of proof. Despite the low performance across the various proof tasks, Senk noted 47% of students were able to prove that a pair of congruent triangles was congruent. Similarly, Healy and Hoyles (1999) noted 19% of students were able to write a complete proof for familiar geometry statements, and 4.8% of students were able to write complete a proof for an unfamiliar geometry statements. Moore (1994) noted that students who are challenged to write proofs, may have difficulty with language and notation, may lack understanding of the concept, cannot state appropriate definitions, have difficulty structuring the proof, or may not know how to begin the proof.

Furthermore, large-scale assessments have documented that student performance in geometry is relatively poor. In the 1996 NAEP mathematics assessment, geometry was identified as a strand where student performance was low, particularly for 12th grade students (Martin & Strutchens, 2000). Moreover, extended constructed response items in the NAEP assessment had a much lower rate of satisfactory responses than multiple-choice items or short constructed responses for grades 8 and 12 (Silver, Alacaci, & Stylianou, 2000). Furthermore, in its report on evaluation of curriculum effectiveness, the National Research Council (2004) analyzed nine evaluation studies of NSF-supported curricula, and geometry was one of the strands where these curricula failed to show strong favorable results. On the other hand, two of four studies of commercial materials showed favorable results in geometry. Nevertheless, neither the analysis of NAEP data nor the NRC report specifically included results regarding proof in geometry.

Textbooks convey a mathematical progression for curriculum objectives and cognitive developmental structures for learners (Van Dormolen, 1986). The majority of secondary schools in U.S. follow a curriculum built around a sequence of three full-year courses, Algebra 1, Geometry, and Algebra 2 or Algebra 1, Algebra 2, and Geometry (Dossey, Halvorsen, & McCrone, 2008). Integrated curriculum materials developed since 1990 have been adopted in some high schools. These materials integrate algebra and geometry content, together with functions, data analysis, and discrete mathematics each year of the secondary mathematics curriculum (Hirsch, 2007).

7. Given the following facts, show that *TS* is congruent to *BC*.

Fact: *S*, *M*, and *C* lie in a straight line.
Fact: *M* is the midpoint of *TB*
Fact: *TS* is parallel to *BC*



Show that *TS* is congruent to *BC*.

Figure 1: Proof task relative to corresponding parts of congruent triangles

Researchers have been interested in the different student outcomes that can be attributed to the use of these different types of textbooks. For example, at the fifth and sixth grade levels, Carroll (1998) found that students who used the UCSMP (an integrated curriculum that is designed to align with NCTM *Standards* (NCTM, 2000) outperformed on geometrical reasoning activities their counterparts that used a subject-specific curriculum, which are publisher-developed (Stein, Remillard, & Smith, 2007). Otten, Males, & Gilbertson (2014) documented that reasoning and proof tasks are visible in high school geometry subject-specific textbooks and integrated textbooks; however, few studies have documented high school students performance of proof in respect to the organizational structure of the curriculum used. Chávez and colleagues (2013) found that students using an integrated curriculum scored significantly higher than those using a subject-specific curriculum on a common objectives test. In this paper we examine students' performance on a geometrical proof task from this common objectives test with consideration to the curriculum type used. We investigated the following research questions:

- How successful are students on a proof about congruent parts of congruent triangles?
- What differences are there between curriculum types for students' performance on a geometrical proof task about congruent parts of congruent triangles?

METHOD

The mathematical task used and students' data were drawn from Test E in the Comparing Options in Secondary Mathematics: Investigating Curriculum (COSMIC) study. Test E is one of 5 tests developed for the COSMIC study (Chávez et al., 2011). This longitudinal study examined the impact of different content organizations on students' learning. The mathematics emphasized on Test E was *functions* and other topics that were common to both types of textbooks and were considered important, namely: symbolic algebra, Pythagorean Theorem and a proof for congruent triangles. Test E had 10 items: 1 multiple-choice item, 1 matching item, and the 8 remaining items were constructed response. For this paper, we will discuss students' performance on the proof task.

Mathematical task

In Test E, question 7 (Figure 1) required students to prove that two sides of two triangles were congruent. The given information was enough to prove that the triangles were congruent and therefore the corresponding sides were congruent. Figure 1 includes the task, as presented to students in the assessment instrument. The purpose of this task was to assess students' ability to write a proof for a very typical geometry problem. Our curriculum analysis had shown that tasks similar to this were found on both sets of textbooks.

This item was scored on a 9-point scale. The scoring rubric is shown in Figure 2. According to this rubric, the item was divided into four subitems, corresponding to the four expected components of a correct answer. Students were awarded points for identifying a pair of congruent triangles, pointing out at least two pairs of congruent angles, stating the correct theorem as to why the triangles were congruent, and for providing a concluding reasoning that the sides of congruent triangles were congruent. These four subitems are referred to as 7.1, 7.2, 7.3 and 7.4. As described elsewhere (Chávez et al., 2011), the rubric and the problem itself were submitted to external reviewers. The problem was also piloted before being use with the study participants. From these rounds of review, revision, piloting, and revision, we were confident that the problem was appropriate and that the scoring rubric was suitable. In particular, the scoring rubric enabled us to document student understanding, and also was

Student argues that one pair of sides is congruent:	
Student states that $\overline{TM} \cong \overline{MB}$ because M is the midpoint of $\overline{TB}$ (or equivalent statement)	2 points
Partial Credit — 1 point	
Student states that $\overline{TM} \cong \overline{MB}$ without a valid supporting reason	
Student argues that <u>two</u> of the three following pairs of angles are congruent:	
1. Student states that $\angle TMS \cong \angle BMC$ because they are vertical angles (or equivalent statement)	4 points
2. Student states that $\angle STM \cong \angle CBM$ because $\overline{TS} \parallel \overline{BC}$ or alternate interior angles	
3. Student states that $\angle TSM \cong \angle BCM$ because $\overline{TS} \parallel \overline{BC}$ or alternate interior angles	
Partial Credit — 3 points	
Student provides two of the above statements, but only one appropriate supporting reason	
Partial Credit — 2 points	
Student provides two of the above statements, but no valid supporting reasons	
OR	
Student provides one of the above statements, and one appropriate supporting reason	
Partial Credit — 1 point	
Student provides one of the above statements, and no valid supporting reasons	
Student argues that the two triangles are congruent:	
Student states that $\triangle TMS \cong \triangle BMC$ by ASA	2 points
OR	
Student states that $\triangle TMS \cong \triangle BMC$ by AAS	
Partial Credit — 1 point	
Student states that $\triangle TMS \cong \triangle BMC$ with no valid supporting reason	
Student argues that the required two sides are congruent:	
Student states that $\overline{TS} \cong \overline{BC}$ because $\triangle TMS \cong \triangle BMC$	1 point
OR	
Student states that $\overline{TS} \cong \overline{BC}$ because corresponding sides of congruent triangles are congruent (or equivalent statement)	
Total points	9 points

Figure 2: Rubric for proof task relative to corresponding parts of congruent triangles

sensible enough to allow us to measure variation in student's responses.

Participants

As described in previous papers (Grouws et al., 2013; Tarr et al., 2013), the schools in our study were high schools that offered both an integrated mathematics sequence (Course 1, Course 2, Course 3, Course 4) and a subject-specific sequence (Algebra 1, Geometry, Algebra 2, Pre-calculus); and that gave students the choice between these two alternatives without tracking them by ability. The students in the study ages ranged from 15–18 years old. Our sample included 10 high schools in six United States (US) school districts located in five geographically diverse states. *Core-Plus Course 3* (Coxford et al., 2003) was the textbook series used by all of the teachers teaching an integrated curriculum. Teachers of subject-specific curriculum used Algebra 2 textbooks produced by several different publishers.

The textbooks had similar content (Chávez, Papick, Ross, & Grouws, 2011; Chávez, Tarr, Grouws, & Soria, 2013). For example, in most subject-specific geometry textbooks Chapter 4 focuses on Congruent Triangles (Sears & Chávez, 2014). Teachers that used subject-specific geometry curriculum acknowledged that the chapter on congruent triangles is used to develop students' conceptions about proof (Sears & Chavez, 2014).

We collected data from 1936 students from these 10 high schools. Of these, 1157 students (59.8 %) used a subject-specific textbook; 779 (40.2 %) used an integrated textbook. The students in the study were in the third course for their respective curriculum sequence (Course 3 and Algebra 2, respectively). There is a slight variation among the subject-specific textbooks on their attention to proof (Sears & Chávez, 2014; Otten, Males, & Gilbertson, 2014). Given our focus on the impact of content organization on students learning, in this

Score	Frequency	%
Did not attempt	223	11.5
0	1092	56.4
1	190	9.8
2	166	8.6
3	89	4.6
4	51	2.6
5	36	1.9
6	28	1.4
7	20	1.0
8	13	0.7
9	28	1.4
Total	1936	100.0

Table 1: Scores for Question 7, Test E

paper we make no distinction between the different subject-specific curricula.

Results

Score	Frequency	%
0	1329	68.6
1	227	11.7
2	157	8.1
Did not attempt	223	11.5
Total	1936	100.0

Table 2: Scores on Test E Question 7.1

Most students did not attempt the task or obtained no credit for the task if they did attempt it. Table 1 shows the frequency of students who were awarded the different possible scores in this task. As indicated in Table 2, 8.1% of the students stated that segments *TM* and *MB* were congruent and offered a justification.

Score	Frequency	%
0	1331	68.8
1	119	6.1
2	150	7.7
3	36	1.9
4	77	4.0
Did not attempt	223	11.5
Total	1936	100.0

Table 3: Scores on Test E Question 7.2

Score	Frequency	%
0	1503	77.6
1	110	5.7
2	100	5.2
Did not attempt	223	11.5
Total	1936	100.0

Table 4: Scores on Test E Question 7.3

In Table 3 we can see that, although partial credit was given for correct statements (without justification), about pairs of angles that are congruent, very few students received partial or full credit. Furthermore, few students stated that the two triangles were congruent (Table 4). Only half of those who did offered any justification.

Score	Frequency	%
0	1562	80.7
1	151	7.8
Did not attempt	223	11.5
Total	1936	100.0

Table 5: Scores on Test E Question 7.4

The vast majority of students did not conclude that congruent sides of congruent triangles were congruent (Table 5).

Frequency by curriculum type

The following tables (Tables 6–10) show the scores by curriculum type. A larger percentage of students using subject-specific curriculum did not attempt to solve the problem. Among those students using subject-specific curriculum who did attempt the problem, a larger percentage received no points compared to those using integrated curriculum that attempted the problem. On the other hand, comparatively fewer students using the integrated curriculum received full credit for the problem. The mean score for question 7

	SS		INT	
Score	Frequency	%	Frequency	%
Did not attempt	139	12.0	84	10.8
0	680	58.8	412	52.9
1	116	10.0	74	9.5
2	83	7.2	83	10.7
3	39	3.4	50	6.4
4	26	2.2	25	3.2
5	20	1.7	16	2.1
6	16	1.4	12	1.5
7	11	1.0	9	1.2
8	6	.5	7	.9
9	21	1.8	7	.9
Total	1157	100.0	779	100.0

Table 6: Scores for Question 7 Test E, by curriculum type

	SS		INT	
	Frequency	Percent	Frequency	%
0	801	69.2	528	67.8
1	132	11.4	95	12.2
2	85	7.3	72	9.2
Did not attempt	139	12.0	84	10.8
Total	1157	100.0	779	100.0

Table 7: Scores on Test E Question 7.1 by curriculum type

	SS		INT	
	Frequency	Percent	Frequency	Percent
0	821	71.0	510	65.5
1	62	5.4	57	7.3
2	76	6.6	74	9.5
3	14	1.2	22	2.8
4	45	3.9	32	4.1
Did not attempt	139	12.0	84	10.8
Total	1157	100.0	779	100.0

Table 8: Scores on Test E Question 7.2, by curriculum type

	SS		INT	
	Frequency	%	Frequency	%
0	891	77.0	612	78.6
1	67	5.8	43	5.5
2	60	5.2	40	5.1
Did not attempt	139	12.0	84	10.8
Total	1157	100.0	779	100.0

Table 9: Scores on Test E Question 7.3, by curriculum type

	SS		INT	
	Frequency	%	Frequency	%
0	930	80.4	632	81.1
1	88	7.6	63	8.1
Did not attempt	139	12.0	84	10.8
Total	1157	100.0	779	100.0
0	930	80.4	632	81.1

Table 10: Scores on Test E Question 7.4, by curriculum type

was significantly different in these two groups, after controlling for prior achievement (F = 6.355, p = 0.012). Nevertheless, it is important to consider these differences in the context of the full test and taking into account all the relevant variables (Chávez et al., 2013). The results in these tables are simple summaries of the points scored by students in one assessment item.

DISCUSSION

Our results are consistent with others in showing that students, regardless of curriculum type, have difficulties writing proofs. There is evidence, however, that the organizational structure of curriculum can have implications on students' opportunities to prove. In particular, students in subject specific curricula were less likely to complete the proof task or to receive no points than students using integrated textbooks. Most students were not able to obtain full credit on the task. According to the results, only 1.8% of students in subject curricula, and 0.9% of students in integrated curricula were able to obtain full credit, which suggests that students continue to struggle with writing proofs. Otten et al (2014) noted that, although both subject-specific and integrated curriculum provides opportunities for students to differentiate between deductive and inductive reasoning, students were seldom required to construct complete proof arguments. This can be problematic, because it deprives students from viewing proofs in their entirety and can potentially contribute to students experiencing difficulty in starting proofs, as well as conceptualizing the structure of proofs (Moore, 1994).

Students should experience proofs more often. They should learn to expect that posing conjectures and proving statements is central to mathematics. Classroom observations for the COSMIC study indicated that teachers using integrated curriculum placed a greater focus on reasoning than teachers using subject-specific curriculum, "although it was not a strong focus for either group of teachers" (Grouws et al., 2013, p. 442). Complete proofs are seldom included in assessments, and more often they appear as fill-in-the-blank tasks. Emphasis on ritualistic aspects of proof may explain why students have difficulties writing proofs on their own.

CONCLUSION

Without a doubt, the results summarized here show that students in US high schools, regardless of the curriculum used in their schools, have difficulties writing proofs in geometry. Work done by Sharon Senk in the 1980s, among others, shows that this is an old problem. It was beyond the scope of the COSMIC study to conduct a fine-grained analysis of this specific topic. Nevertheless, it is clear that the evidence presented here points to grave deficiencies in how geometry is taught in schools. The case of proof is a particularly delicate one, because teachers attempt to teach proof as a topic that must be taught among other topics in the curriculum, rather than as a way to communicate mathematics. Therefore, these results may be interpreted as a lack of certain skills. It would be more appropriate, however, to consider them as an indication that when proof is taught as a set of rules that should be applied to specific problems in geometry rather than as a way of communicating mathematics, students learn neither. Hence, curriculum developers, and educators, must find ways to introduce experiences with proof in their materials and lessons, so that students develop sound habits of justification and proof.

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