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# Constructing validity in classroom conversations

Manuel Goizueta<sup>1</sup> and Maria Alessandra Mariotti<sup>2</sup>

1 Universitat Autònoma de Barcelona, Barcelona, Spain, [mgoizueta@gmail.com](mailto:mgoizueta@gmail.com)

2 Università degli Studi di Siena, Siena, Italy, [mariotti21@unisi.it](mailto:mariotti21@unisi.it)

*We account for different strategies used by a group of students to talk about and assess the validity of mathematical models while working in a problem-solving task. Two main competing strategies are described, one centred in ritualized uses of well-known mathematical constructs as a means to cope with perceived didactical expectations and a second strategy centred in the assessment of the representativeness of mathematical models when accounting for the proposed empirical situation. The interactions analyzed exemplify the difficulties students and teachers experience when dealing with epistemological aspects of knowledge being constructed in classroom conversations. Our findings point to the need for research to focus on epistemological aspects of the mathematical culture of the classroom.*

**Keywords:** Validity, argumentation, mathematics classroom epistemology, classroom culture.

## INTRODUCTION

It is widely accepted that argumentative competencies should be developed within the mathematical activity in the classroom, both as a product of this activity and as a means to support it. We share Boero's (2011) idea that a 'culture of argumentation' is to be developed in the classroom and that it should include practices on the production of conjectures, meta-mathematical knowledge about the acceptability of references advanced for the validation (acceptance/rejection) of claims and knowledge about the role of counter-examples and generality. It should include elements for evaluation of mathematical productions and general ideas about the use of all this knowledge within argumentative practices, along with the needed awareness to allow deliberate and autonomous control of the process.

In order to promote such a culture, the processes by which students actually construct and validate math-

ematical knowledge in classroom activities, as well as the meta-mathematical knowledge underpinning this processes, should be better understood. This report is part of a series in which we argue that it is worth characterizing this processes in terms of validity conditions emergence and fulfilment in order to account for the epistemological dimension of classroom interaction and its socio-interactive roots (see Goizueta, Mariotti, & Planas, 2014). Our main interest is to investigate the epistemological basis of argumentative practices in the mathematics classroom and, particularly, how is validity interactively negotiated in rich problem-solving mathematical activities. In this occasion, we compare different strategies for validation of mathematical models observed in a secondary mathematics classroom. We account for the epistemological complexity of these strategies and explain, from this point of view, the difficulties students face to discuss the validity of mathematical models.

## THEORETICAL FRAMEWORK

According to Habermas (1998), accepting a validity claim is tantamount to accepting that its legitimacy may be adequately justified, that is, that conditions for validity may be fulfilled. Validity conditions are not restricted to absolute standards; instead, they are contingent constraints that emerge to accommodate elements considered suitable for validity appraisal in a given context (Goizueta, Mariotti, & Planas, 2014). They embody what are intersubjectively considered good reasons in a particular context of justification. Thus, validity relates to acceptance based on contingent validity conditions fulfilment. This implies that validity is not a property of claims themselves, but emerges from the mode they are contextually dealt with. It should be stressed that whatever is considered as a good reason, must not necessarily be explicit or even stateable, nor the individuals must be aware of it in a conscious way, it might be enacted implicitly in successful social participation (Ernest, 1998).

According to Habermas' construct of rational behavior (Habermas, 1998) and its adaptation by Boero and colleagues (2010), in the students' argumentative practices we can distinguish an epistemic dimension (inherent in the epistemologically constrained construction and control of propositions, justifications and validations), a teleological dimension (inherent in the strategic decision-making processes embedded in the goal-oriented classroom environment) and a communicative dimension (inherent in the selection of suitable registers and semiotic means to communicate within the given mathematical culture). Such a distinction reveals useful to reconstruct the origin of specific validity conditions. For instance, focusing on the epistemological dimension and following Steinbring (2005), we assume that a "specific social epistemology of mathematical knowledge is constituted in classroom interaction and this assumption influences the possibilities and the manner of how to analyse and interpret mathematical communication" (p. 35). Within this socially constituted mathematics classroom epistemology a criterion of mathematical validity is interactively negotiated between the participants. A central consequence of these assumptions is the basic necessity for interpretative research to reconstruct the situated conditions in which (and from which) mathematical validity emerges as part of the interactive development of mathematical knowledge. Although 'conditions' might be considered in a broader sense, we are particularly interested in the epistemological assumptions at stake, the references (mathematical and not) that might be considered as relevant and the social environment in which the process is embedded.

In the context of the mathematics classroom, the general relation between classroom epistemology, mathematical activity and social environment must be considered under the light of a specific, content-related didactical contract (Brousseau, 1997). Mathematical acceptability of students' explanations is linked to the reciprocal expectations and obligations perceived within the didactical situation by the teacher and the students and to the mathematical contents at stake (or perceived as being at stake). Thus, when faced with a problem, the didactical contract may indicate relevant mathematical knowledge and references to the students, according to the proposed didactical situation. However, not all the emerging references are linkable to well established and intersubjectively shared mathematical knowledge. We might also need

to consider other references (statements, visual and experimental evidence, physical constraints, etc.) that are not part of institutionalized corpora, such as scholar mathematics, but (nevertheless) are used de facto as taken-as-shared, unquestionable knowledge (Douek, 2007). This corpus of references might be tacitly and operatively used by students to make sense of the task, semantically ground their mathematical activity and back their arguments. Accounting for the reference corpus at stake might be particularly relevant when considering problem-solving settings in which empirical references are to be considered as part of the proposed milieu.

### **PARTICIPANTS, TASK AND DATA COLLECTION**

The participants in our design experiment were thirty 14/15-year-old students and their teacher in two lessons in a secondary school mathematics classroom in Barcelona, Spain. It was a problem-solving setting, with time for small group work and whole-class discussion. The researchers suggested the following task:

Two players are flipping a coin in such a way that the first one wins a point with every head and the other wins a point with every tail. Each is betting €3 and they agree that the first to reach 8 points gets the €6. Unexpectedly, they are asked to interrupt the game when one of them has 7 points and the other 5. How should they split the bet? Justify your answer.

As we ascertained in a pilot experiment, this task can be approached and solved using arithmetical tools, without having been taught formal probability contents, which was the case of this group. The teacher was explicitly asked to avoid showing approval or disapproval to the students' numerical answers and proposed models. Instead of hint-guiding the students towards a correct answer, she was asked to foster the emergence of competing models and their discussion. By avoiding directive guidance, teacher's interventions were meant to foster autonomous processes of validation within the students' mathematical activity. Models observed during the pilot were discussed with the teacher and numerical variations of the problem were designed in order to help her problematize these models when and if necessary.

We were aware of the high complexity of the problem as a modelling task. The construction by the students of a situation model (in the sense of Blum & Borromeo-

Ferri, 2009) is related to some well-known epistemological obstacles regarding probability-thinking development (García Cruz, 2000; Wilensky, 1997). This circumstance, along with reports on the use of this problem in the classroom (García Cruz, 2000), suggested it might be a good candidate to promote a rich argumentative environment and to foster the discussion of mathematical, but also meta-mathematical issues. The novelty of the task was expected to prevent students from using mechanical approaches based on well-established solving strategies. Different situation models were expected to emerge and, consequently, a variety of mathematization processes. The need to compare competing mathematical models was expected to foster the emergence of different arguments to validate them.

For data collection, three small groups were videotaped and written protocols were collected. The data were analysed and coded with the aid of qualitative data analysis software. By constant comparison of similarly interpreted situations and triangulation with other research team members' perspective, key aspects of the classroom mathematical work were inductively inferred. This process was iterated for analysis refinement and confirmation.

## DATA ANALYSIS AND RESULTS

### Constructing a situation model: Initial validity conditions

Whether explicitly or implicitly students bring to the discussion taken as shared references about the proposed empirical situation. In so doing, they often describe it as a 'random game' and use words associated with this notion in informal ways. An example is the following intervention by a student, Anna, during the first session:

Anna      Obviously, because it's random, the game (...) A does have more chances of winning but B could win as well (...) From what we've got so far, A would have to get more money, because he's got more points.

These taken as shared references, a cluster of empirical information, beliefs and words to talk about the situation, conform part of the reference corpus the students resort to in order to semantically and empirically ground the task, understand the problem

and conform (often in a tacit way) an associated situation model. Observed in all groups and illustrated by Anna, a validity condition emerges expressing the need for the winning player to get more money. Any possibly valid mathematical model, within which an acceptable answer can be constructed, should fulfil this requirement, so we categorize it as a validity condition. Students' choices also indicate adherence to common clauses of the didactical contract: they tend to disregard and discard solutions that appear spontaneously but are not considered mathematics-related (e.g., 'each one gets his money back because the game did not come to an end'). A common clause of the didactical contract constrains them to actually do some mathematics in order to solve the problem. The need for the solution to be mathematics-related acts as a validity condition that students take into account to decide on the model's validity.

Often, the first numerical answer students propose roots on models associated to typical school problems about proportional costs, which tend to be solved by manipulating the numerical data appearing in the wording. A ritualized way of presenting this solution may be paraphrased as: 'if by winning 8 points a player gets €6, for each point won a player should get €0.75'. The students quickly realize that the amounts of money distributed according to this model, namely €5,25 and €3,75, do not add up to six euros, what, according to them, falsifies the model. The necessity for 'adding up to six' emerges as a validity condition that any numerical answer, and the model within which it is constructed, should fulfil.

The emergence of these validity conditions (namely the need for the winning player to get more money, the need for the distributed amounts of money to add up to six euros and the need for the solution to be mathematics-related) was observed in all groups, as well as the resorting to them as means for validity appraisal. While the need for the solution to be mathematics-related derives from expectations related to the didactical contract, the other validity conditions relate to the need for any possible valid model to account for the empirical references considered by the students. This stance accounts for both the epistemic and the teleological dimensions of the students' activity: a mathematical model that satisfies the validity conditions considered must be constructed; it should be acceptable in the context of the mathematics classroom and lead to an acceptable solution for the game.

### The proportional solution

In the following we focus on the case of the group of Lyn, Ely, Tim and Lucy.

After ascertaining that the initial model does not fulfil the validity conditions, Tim considers the twelve points won by both players and proposes a second model: give away the bet proportionally to the points won: €3.50 for the winning player and €2.50 for the other; what corresponds to 7/12 and 5/12 of the bet respectively. Let us call it, from now on, ‘the proportional solution’<sup>1</sup>.

- 41 Tim Look, we add the points, right? We take the money and divide it by the number of points, then we get how much is one point worth out of these twelve points that we have. Then we divide it and we get zero point five.
- 42 Lyn No, because seven plus five is not eight!
- 43 Tim Three point five plus two point five is six! One gets three point five and the other two point five.
- 44 Lyn OK, but listen: seven plus five is not eight. OK?
- 45 Tim It is twelve.
- 46 Lyn Right, and the six euros were for eight points.
- 47 Tim Yeah, but, because that was not right [referring to the ‘€0,75 per point’ model], we distribute it this way. ‘Cos they couldn’t finish, we must distribute it this way!

By focusing in a well-known procedure [41] Tim is suggesting that proportionality might be an adequate model to solve the problem. He infers from the numerical information the “correct numbers” to introduce in the model: it is not eight but twelve what they should use. Lyn [42] does not address Tim’s proposal in a straightforward manner; it is just by looking at her three utterances that we may try to understand what she is actually talking about. Lyn focuses [46] on the rules of the game: a player wins by winning eight points and by no other circumstances. We conjecture that Lyn objects taking twelve points into consideration because this action does not represent these

rules. This interpretation will be confirmed later (see [63] below). Tim responds [43] by focusing on a validity condition: the amounts of money distributed according to this model ‘add up to six’. We infer that, for him, this indicates some degree of certainty about the validity of the model and the numerical solution, although the epistemic status is not addressed in an intelligible way. We observe the emergence and fulfilment of validity conditions as a means for validation. The shift of foci in Tim’s utterances [41] and [43] and the disconnected answer in [45] might indicate that he is simply not understanding Lyn’s objection. It is just after Lyn’s clarification [47] that Tim seems to address it, but rejecting it in a rather authoritarian way. Noticeably, on the one hand Tim does not take care of Lyn’s objection about the representativeness of his model, on the other hand Lyn seems not be able to make her objection explicit or even clear. It is not possible to infer from these utterances whether Tim does not understand the objection or does not find it relevant.

### The teacher’s intervention

The teacher comes and the following conversation takes place:

- 54 Lyn Ok, if there are just six euros to distribute... and six euros are for eight points but there are twelve points in total... Do we have to forget that six euros are for eight points?
- 55 Teacher No! You should never forget. You are confusing things; six euros are for eight points of one single person. When you say twelve points, the twelve points were obtained by two persons. All right?
- 56 Lyn I know, but distributing means giving to both...
- 57 Teacher No. Because the first arriving to eight gets everything; the whole six euros (...) The problem is that, because the game couldn’t be finished, we have to distribute.
- 58 Tim I have given to the twelve points, the six euros, and then one gets three point five and the other two point five. And it would be fair.
- 59 Lyn But that doesn’t make sense, ‘cos...
- 60 Teacher Why it doesn’t make sense? That’s a possible solution. I agree. A criterion to divide the money is what Tim just

1 For the case of Anna’s group we describe the emergence of this model in (Goizueta, Mariotti, & Planas, 2014).

- said. Now Lyn says it doesn't make sense. You should give me an argument why it doesn't.
- 61 Lyn Because... I mean... No, no, no. Because the six euros are for eight points. And what he is doing is picking the six euros and dividing by all the points.
- 62 Teacher He's done that exactly.
- 63 Lyn No, no, no. I mean... That is not like the rules we have.
- 64 Teacher To see if our solution is consistent, what we can do is to change how the game ended up. Let's imagine six-two. They are six-two when the game is interrupted. Let's check how to split the money and check if it's fair or not.

Lyn refers in a highly condensed way [54] to what they have done to this point and to their current dispute. If we consider this intervention as continuing the students' conversation, it provides the group with new elements to understand Lyn's objection: dividing the money by twelve entails "forgetting" that it corresponds to eight points. The teacher, who is interacting with this group for the first time, seems not capable of inferring the precedents and thus of understanding the epistemological complexity of the discussion. She interprets [55] Lyn's intervention as a mere confusion. The following utterances [56] [57] confirm the ineffectiveness of the exchange. Tim interrupts with his explanation [58], focusing on a procedure and the numerical answer it yields, which surely reveals to the teacher the proportional solution. Lyn addresses [59] the problem of the representativeness of the model again, but still in a rather unintelligible way for the teacher, who reacts to Lyn's objection [60] by apparently supporting Tim and asking Lyn for a justificatory argument. Although it is not clear how the students might interpret the teacher's utterance, it is in line with the didactical planning of the session: fostering the emergence of competing models and students' discussion of them. By saying "a possible solution" and "a criterion" she is implying that other models and criteria could be considered as well. The following utterances by Lyn [61] [63] seem to reveal that the teacher interventions do not help her better frame the problem of the representativeness of the model. Even when she tries to reformulate the issue [63], the teacher does not address it directly. Instead, she sticks again to the planning and proposes [64] to consider a numerical variation of the problem along with a hint. It is difficult to decide how the students

might interpret the words 'consistent' and 'fair', but the teacher seems to suggest that the model's validity can be assessed by assessing the 'fairness' of the solutions it yields. The epistemological status of such a link is not made clear, but the teacher is tacitly indicating a new objective to the students: to explore this validity criterion for the situation model that they intend to exploit.

Lyn and Tim's actions suggest different epistemological and teleological stances underlining and shaping their arguments. Lyn seems to understand their immediate task as assessing the relation between the proposed mathematical model and the implicit situation model; thus her intention (as we interpret it) is to put in evidence the lack of representativeness of Tim's mathematical model. Tim seems to focus the solution of the task on the identification and skilful execution of well-known, suitable mathematical procedures; by evoking proportionality in a rather ritualistic way and proposing a numerical result, he copes with that demand. While Lyn's activity suggests the need for a reflective approach, Tim's suggests a reproductive one.

#### **A shift of attention: Towards a probabilistic model**

After working for some minutes alone, Lyn proposes a new idea:

- 87 Lyn Look, I don't know where I want to get to with this, but to arrive to where you get six, this needs just one and this needs three. So, based on that... let's see how to distribute the eight. But I don't know how.

Lyn shifts the focus of attention from the actual score, the base of the proportional solution, to the potential scores, the base of a probability related solution. Although we have observed this shift in other groups, Lyn's case is exceptional in that it is not the response to teacher's regulative interventions (in fact it involves disregarding them); hence it is not a case of perceived expectations fulfilment. It seems to be a genuine, autonomous attempt to investigate elements of the empirical situation in order to construct a situation model and look for a representative mathematical one.

- 95 Lyn If we divide six euro by this, that is what is missing [writes down '3 + 1 = 4' and '6/4 = 1,5']. We get one point five. [writes down

- ' $3 \times 1,5 = 4,5$ ' and ' $1 \times 1,5 = 1,5$ '] But we should give more to this [winning player], 'cos he's closer than this [losing player]. So, what I would do is to give this [' $4,5$ '] crossed to this [winning player] and this [' $1,5$ '] crossed to this [losing player]. I don't know why!
- 96 Ely That looks OK!
- 97 Lyn But, why?
- 98 Ely 'Cos it's proportional. I think it's OK.
- 104 Lyn I think it makes sense. But I don't know why!
- 105 Ely If you add them up you get six.

Lyn's argument [95] develops through relating the data '3' and '1' (points needed to win) to the losing and winning players respectively. By simple extension, the product ' $4,5$ ' ends up to be associated to the losing player while the product ' $1,5$ ' does to the winning one, what does not conform to what she considers a validity condition. She proposes a 'crossed association', but recognizes it as arbitrary by stressing that she does not find a reason to do so. Instead, Ely supports this solution [96] [98] [105] by referring it to well-known validity conditions: the soundness of proportionality as an adequate mathematical construct to solve the task and the two distributed amounts of money adding up to six. However, such support does not fit Lyn's fundamental need for validating the mathematical model according to its representativeness of the empirical situation [95] [97] [104]. We observe again different epistemological and teleological stances underlying students' arguments. Lyn subordinates the validity of the mathematical model, she proposes establishing a relation between it and the implicit situation model. Instead, Ely positively assesses the validity of the mathematical model on the basis of the soundness of proportionality, to which she associates it, and the fulfilment of previously considered validity conditions.

## CONCLUDING DISCUSSION

As observed throughout the analysis, the students seem to be unable of explaining to each other the base on which they claim or challenge the validity of the models they propose. This may be interpreted referring to the difference between the teleological and epistemological stances they adopt. Tim and Ely seem to understand the task as the need for identifying some known, relevant mathematical construct to

produce a sound answer that is consistent with shared expectation of the didactical contract. Accordingly, the goal they are pursuing is that of producing a numerical solution through a proportional model, tacitly linked to the problem through its ritualistic use in situations of distribution. It is this link and the fulfilment of the related validity conditions what seems to confer validity to the model. However, the evaluation of the model always remains implicit. Lyn's activity reveals a different stance: the mathematical model must represent the empirical situation if it is to be considered valid, thus her immediate goal when discussing Tim's model and her own is assessing their representativeness.

Both cases illustrate how different epistemological perspectives may influence students' teleological stances and thus students' attitudes towards validating the models they produce, but also how undeveloped is this attitude and far from a desirable culture of argumentation. In the case of Lyn, although she clearly focuses on validating the model with respect to how accurately it represents the proposed situation, she seems unable to share her ideas with her peers in an intelligible way. Meanwhile, Tim and Ely prove incapable of interpreting her stance and seemingly unaware of the necessity of linking the validity of the model to its representativeness. Even the teacher seems unable to perceive the tension between the different stances and help the students fruitfully discuss it.

From our point of view, this experience illustrates a more general situation, namely that students and teachers do not always have the tools to deal with, or talk about, the epistemological complexity of mathematical knowledge in the classroom. If a desirable culture of argumentation is to be fostered in the classroom to allow students to autonomously construct and validate mathematical knowledge, some awareness about one's and other's epistemological stances should be developed as a condition for a critical, reflexive account of one's and other's actions. It should not only be about producing valid arguments that satisfy conventional communicative requirements, but also about being aware of and able to discuss the grounds for their validity in their context of justification. Suitably fostering such discussions in the classroom should be the teleological counterpart of such epistemological stance. We suggest that more attention should be paid to the mechanisms that reg-

ulate the emergence of the mathematics classroom epistemology in order to provide teachers with adequate means to plan and control its development as part of a desirable culture of argumentation. To this regard, we claim that the interpretation of classroom mathematical activity in terms of validity conditions emergence and fulfilment might be worthwhile in order to reveal epistemological aspects of classroom interaction and their socio-interactive roots. This interpretation might help researchers describe and model the advocated development of a culture of argumentation.

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