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# Is the use of GeoGebra advantageous in the process of argumentation?

Özlem Erkek and Mine Işıksal-Bostan

Middle East Technical University in Ankara, Faculty of Education, Ankara, Turkey, [ozdursun@metu.edu.tr](mailto:ozdursun@metu.edu.tr)

*The purpose of this study was to investigate the advantages and disadvantages of using GeoGebra in an argumentative application. For this purpose, the data were collected from eight prospective elementary mathematics teachers via video recordings, reflection papers and interviews. Data were analyzed through qualitative data analysis methods. According to the results, making accurate drawings, creating confident justifications based on exact measurements, dragging objects to see relationships and saving time were the main advantages of GeoGebra in argumentation. On the other hand, not attending class discussion and not reasoning the relationships after measuring with GeoGebra were the disadvantages for argumentation.*

**Keywords:** Argumentation, GeoGebra, geometry, advantageous, disadvantageous.

## INTRODUCTION

Argumentation is defined as “a process of establishing or validating a conclusion on the basis of reasons or the act of proposing, supporting, evaluating and refining the process, context, or products of an inquiry (Sampson & Clark, 2011, p. 66). Toulmin (1958), the pioneer of this multidisciplinary trend in literature, proposed the Argumentation Model, which is used to determine arguments in numerous scientific studies. In this model, an argument comprises three elements which are claim, data and warrant (Toulmin, 1958). Claim is a statement to be supported. The other element data corresponds to evidence presented for supporting the claim. Warrant is an inference rule enabling data to be connected to the claim. In addition to these elements, Toulmin (1958) also asserted that qualifier, rebuttal and backing may also be needed to describe an argument. While qualifier refers to the strength of the argument, the rebuttal expresses a counter-argument. Backing corresponds to addition-

al support for the warrant of an argument. Although, Inglis, Mejia-Ramos and Simpson (2007) asserted that the full version of the Toulmin model need to be used in mathematics, backings, modal qualifiers and rebuttals are less often used in the studies related to mathematics education corresponding to the nature of mathematics. Toulmin’s (1958) argumentation scheme was represented in the Figure 1.

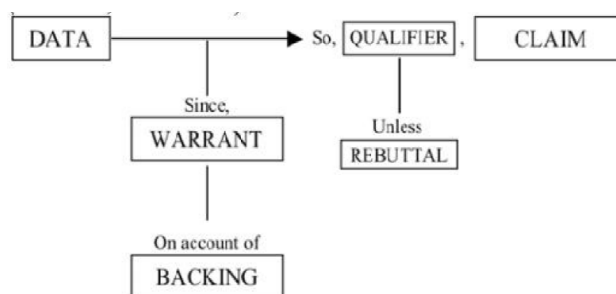


Figure 1: Toulmin’s (1958) Model of Argumentation (p. 104)

The importance of argumentation has increased gradually in learning and education and many researchers implied that the students should participate in discussions for mathematics learning (Krummheuer, 1995; Stein, Engle, Smith, & Hughes, 2008). Inglis and colleagues (2007) pointed out that the first researcher who integrated Toulmin’s argumentation scheme into the mathematics field was Krummheuer (1995) who analyzed classroom-based mathematical arguments. Krummheuer (1995) used the term collective argumentation and defined it as “a social phenomenon when cooperating individuals tried to adjust their intentions and interpretations by verbally presenting the rationale of their actions” (p. 229). After this first attempt, the number of studies related to the applications of argumentation into mathematics has increased (Inglis et al., 2007; Pedemonte, 2007; Wentworth, 2009). For instance, Wentworth (2009)’s study include technology allowing online synchronous and asynchronous discussion to explore argumentation of prospective teachers and secondary

school students. The results indicated that it is difficult for participants to write all their arguments and it is difficult to detect and follow their reasoning process only from their writings (Wentworth, 2009). In the same way, mathematics educators are aware of the fact that logical inferences and formal justifications are not always convincing and comprehensible so they propose that additional informal justification ways, like dynamic geometry tools, are necessary (Prusak, Hershkowitz, & Schwarz, 2012). Thus, mathematics educators have developed dynamic geometry tools allowing inquiry-based environment and encourage argumentation in geometry (Prusak et al., 2012). Furthermore, researchers emphasized the importance of dragging in conjecturing in many studies (Arzarello, Olivero, Paolo, & Robutti, 2002; Baccaglioni-Frank & Mariotti, 2010). For instance, Arzarello and colleagues (2002) stated that dragging encourages conjecturing and exploring since individuals have the chance of observing the invariant properties after changing the shapes. Obtaining immediate feedbacks was claimed to be helpful for discovering and proving invariant properties of drawings (Arzarello et al., 2002). Dynamic geometry software allows students to construct and experiment with geometrical objects to make conjectures and interpretations (Healy & Hoyles, 2001). Since conjecturing is a crucial action in argumentation, it is worth to study the advantages and disadvantages of dynamic geometry program (GeoGebra) in argumentation process. It is claimed that it is not only the technology that brings an educational change but also the teacher (Arzarello et al., 2002). However, few studies analyzed the prospective elementary mathematics teachers' argumentation in geometry. The prospective teachers' argumentation skills are important to be searched because they are the people who will facilitate and manage the whole class discussions, ask questions, listen to students who will develop mathematical understanding in the future (Stigler & Hiebert, 1999). Their own proficiency in developing arguments has also great importance for their future performances in argumentation applications. Therefore, the investigation of the argumentation process with dynamic geometry program, GeoGebra, will be beneficial to determine the critical issues to be considered by prospective teachers.

This study is significant since the results would provide insight to prospective teachers and teachers who want to integrate GeoGebra and argumentation into their geometry lessons. In addition, the results

would provide a clear picture of possible benefits and problems that prospective elementary mathematics teachers may confront while experiencing argumentation with GeoGebra. Based on the rationale mentioned above, the purpose of the present study was to investigate advantages and disadvantages of using GeoGebra in argumentation while solving geometry tasks. The following research question has guided the study: What are the advantages and disadvantages of using GeoGebra in argumentation process?

## METHODOLOGY

This study is a qualitative case study. Purposeful sampling was used to identify the participants. The researchers selected 8 prospective teachers enrolled in Elementary Mathematics Education program of a public university in Ankara, Turkey. All participants were 4<sup>th</sup> grade students who have completed almost all courses related to elementary mathematics education program. The four participants in GeoGebra group were voluntary students who have taken the elective course called "Exploring geometry with dynamic geometry applications". In this way, there is no need to teach them how to use GeoGebra program. The other four participants in Paper-pencil group were the communicative students who have not taken the course. The application was done in one session for each group and it lasted 2 hours to solve three geometry tasks.

To triangulate data, we utilized multiple data sources. Data collection took place in the spring semester of 2012–2013 education year through video and audio recordings of the two applications, reflection papers and interviews. Interviews were also recorded with a camera.

The researcher who was the participant observer in the classroom settings started the application by giving some information about argumentation in each group. The purpose of this was to give an idea to the participants about the application process and to encourage them to justify their claims as much as possible. Also, the researcher asked them to think aloud in pair-work to share their ideas with their partners. Moreover, the researcher asked participants to concentrate on the discussion on the board in order to encourage all students' contribution to the classroom discussion. There were 3 geometry tasks in the application. The first geometry task was like a warm-up

**Geometry Task 1**

1. A recreational park has a rectangular shape. At each vertex of the rectangular park there is an attraction. The manager of the park decided to locate the ticket booth at an equal distance from the four attractions. Find the point in the rectangle which fits for the ticket booth.
2. Another park has the shape of an equilateral triangle. At each vertex of this park there is an attraction. The manager of the park decided to locate the ticket booth at an equal distance from the three attractions. Find the point in the triangle which fits for the ticket booth.
3. What if the recreational park has the shape of a scalene triangle? Find the point (if any) in this triangle which fits for the ticket booth.

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**Geometry Task 2**

In any  $ABC$  triangle  $D$  and  $E$  points are midpoints of the sides  $|AB|$  and  $|AC|$ .  $F$  and  $G$  points are placed on side  $|BC|$  so as to be  $|BG|=|CF|$ .  $|DG|$  and  $|EF|$  intersects at point  $H$ .  
 When does  $|AH|$  become angle bisector of the angle  $A$ ? (Think about all triangle types)  
 Explain your reasoning and justify your solutions.

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**Geometry Task 3**

Let  $P$  be any point on the median of  $|AG|$  of a triangle  $ABC$ . Let  $m$  and  $n$  parallel lines through  $P$  to the sides  $|AB|$  and  $|AC|$  of the triangle.

1. What relation is there between the segments  $|EG|$  and  $|GF|$ ? Explain your reasoning.
2. What if the triangle  $ABC$  is equilateral or isosceles triangle? Can any generalization be made for the relation between the segments  $|EG|$  and  $|GF|$ ? Explain your reasoning.

3. Where must be the point  $P$  positioned such that  $|BE|=|EF|=|FC|$ . Justify your solution.

Figure 2: Geometry tasks

task which was discussed in pairs and then the class discussion began. After getting all different solutions, the researcher moved to the second task's pair discussion. In the same way, the third task was discussed in pairs and later with the whole class. The researcher reworded the claims of students in order to provide a collective argumentation environment and to make it clear for other participants. In addition, the researcher asked probing questions such as "Why do you think so? Are you sure about that? What is your justification for that? Do you all agree with your friend's claim?". That is, the researcher motivated students to justify why a conjecture is true or false and convince others. The subsequent class discussions focused on student reasoning and argumentation.

In GeoGebra group there were two pairs of students and each pair had one computer and one worksheet to study together. In Paper-pencil group there were two pairs of students and they had a ruler, a compass

and a protractor to be able to draw accurate shapes and one worksheet to study collaboratively. After the participants solved the tasks in each group, one voluntary pair was interviewed; the other pair wrote a reflection paper in order to collect more information about their experiences regarding the benefits and the drawbacks of using GeoGebra in argumentation.

For the data analysis, the recorded dialogues of the participants were transcribed. Toulmin (1958)'s Argumentation Model was used to determine the arguments of the participants. By using the Toulmin's (1958) predefined argument elements, the researcher had the chance of organizing the arguments of the GeoGebra and the Paper-pencil groups in such a way that they are comparable because they discussed the same geometry tasks. Two experts assisted with the researcher determining the argument schemas from video recordings. An inter-rater reliability agreement was 90%. In addition, the reflection papers and inter-

views were also open coded for triangulation of the data to detect advantages and disadvantages of using GeoGebra in argumentation.

**Data collection tools**

Three geometry tasks about triangles (see Figure 2) were selected for the argumentation application based on the criteria such that the tasks should be suitable for the discussion and argumentation, have alternative solutions, and be able to be solved via GeoGebra and paper-pencil. After getting permissions from the authors, the tasks were translated into Turkish. Expert opinions were taken for the statements to be clear for the participants. The first task was about the circumcircle and taken from the study of Prusak and colleagues (2012, p. 28). The second task was prepared by Ceylan (2012) and it was adapted by the researcher to be able to be suitable for argumentation. The third task was prepared by Domenech (2009) for high school students.

The researcher prepared interview questions for the voluntary pair just after watching the video of the application. The day after the implementations the interviews were conducted. If the participants did not remember some details about their arguments, the video record of that part was shown to make them remember what they did in the application. The aims of implementing interview were to obtain the ideas of participants about the advantages and disadvantages of GeoGebra while solving geometry tasks through argumentation and to clarify their arguments. Some components of arguments may be missing in discussion when the discussion flows to different directions. In some situations, the participants may not explain their reasoning but imply it somehow. The researchers asked those parts to the participants through interviews in order to make correct interpretations.

**RESULTS AND DISCUSSION**

After the applications, the schemas of all the arguments of the participants were drawn and then the arguments of GeoGebra group and Paper-pencil group were compared in terms of the number of arguments, the existence of argument elements and the contents of argument elements. Considering Inglis and colleagues’ (2007) suggestion the elements of the full model of Toulmin were searched but it was detected that few modal qualifiers and rebuttals and no backings were expressed by the participants. In addition to this, some arguments were generated by more than one student during the class discussion. That is, claim was stated by one student while the warrant of that claim was stated by another student. By comparing these arguments, the researcher concluded that GeoGebra was both advantageous and disadvantageous in argumentation depending on the educational goals of teachers and the nature of tasks.

**Advantages of using geogebra in argumentation**

One of the advantages was that the participants in GeoGebra group were able to made accurate and dynamic drawings for the tasks. In this way, they could see the relationships easily, became more confident in their arguments, and went on forming justifications and inferences. However, in Paper-pencil group, they were not sure about their conjectures or made wrong conjectures because of their inaccurate drawings. The conversation and argument schema below in paper-pencil group exemplifies such a situation.

Teacher: How can we find the circumcenter of any triangle?  
 S2: We could not remember the rule so we could not determine the lines which intersect and generate circumcenter.

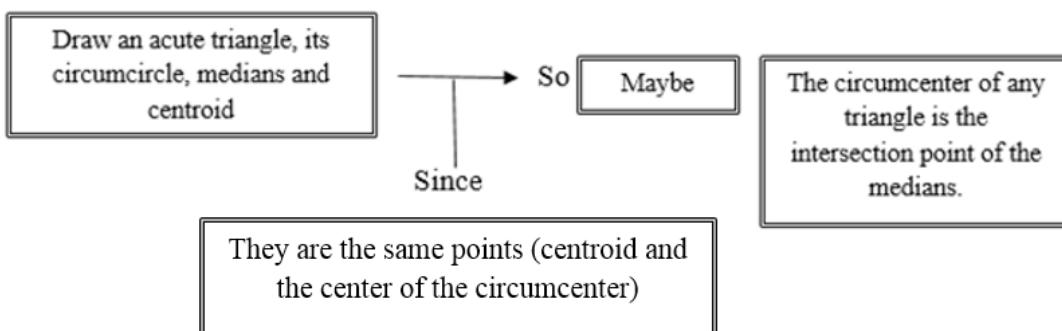


Figure 3: A Sample Argument Schema for the conversation



Maybe it can be the intersection point of medians. Let me try it.

(S2 drew an acute triangle, a circumcircle and medians of the triangle on the board and saw that the centroid approximately seems to be the center of the circumcircle)

S2: Oh they are the same points. I think we can find circumcenter by intersecting medians of a triangle.

In the conversation above, S2 concluded that the circumcenter of a triangle may be the intersection point of the medians, centroid. This was a wrong argument and the reasons for the mistake were approximate drawing and not thinking all triangle types. S2 thought that centroid and circumcenter are the same point but in fact they were very close to each other in an acute triangle that she draw. The other students were also satisfied with the drawing and they did not discuss the solution further. Then the instructor asked whether that was valid for an obtuse triangle or not? This question helped participants to think other triangle types. However, in GeoGebra group, the students drew the medians and found the centroid and compared it with the circumcenter in different triangle types. In this way they did not generate wrong arguments. Therefore, they formed further arguments and found different solutions.

Another advantage of the use of GeoGebra in argumentation can be the dragging option which enables maintaining some geometrical properties of a figure and exploring the relationships. The students

in GeoGebra group used dragging option effectively and made generalizations easily which support conceptual understanding. Moreover, they were more motivated to discuss and to find alternative solutions to the problem. For instance, when they saw any relationship in an equilateral triangle, they could check whether the relationship was valid for other triangles by dragging the vertices of the triangle and changing its properties. However, in Paper-pencil group, participants did not have such an opportunity. They got stuck while they were drawing the new shape again and again on the paper with materials such as ruler, protractor and compass. Therefore, the participants had difficulties in focusing on the relationships between the unchanging properties of different shapes, because of repetitive drawings and their motivation for discussion decreased. For instance, in GeoGebra group while participants were discussing the geometry task 2, they noticed that  $|AH|$  is angle bisector when  $|DE|$  is perpendicular to  $|AH|$  (see Figure 4). In addition, they investigated that  $|AH|$  is angle bisector when  $ADHE$  is deltoid. Then they analyzed whether  $|AH|$  is angle bisector when  $ADHE$  is any other type of quadrilateral. They used dragging option and see the relationships in a short time. In the end, they concluded that  $|AH|$  is angle bisector when  $ADHE$  is deltoid and rhombus. However, in Paper-pencil group, participants concluded that  $|AH|$  is angle bisector when  $ADHE$  is only deltoid since they could not move F and G points to obtain different quadrilaterals on the paper.

It can be said that dragging option in GeoGebra was beneficial in argumentation because the participants could see the relations that they could not see on fixed shapes drawn on the paper and they justified willingly

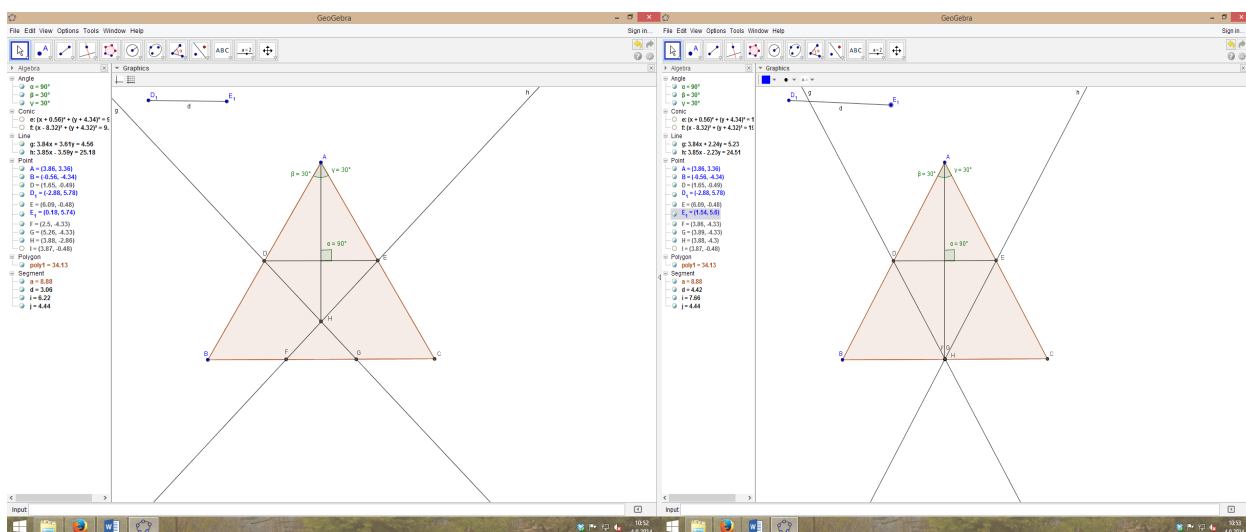


Figure 4: A drawing for task 2 in GeoGebra group class discussion

their reasoning based on the properties of the different shapes composed by dragging.

Based on the findings, the other advantage of GeoGebra is time issue. The participants drew shapes easily and quickly with GeoGebra options such as measuring angle, measuring side, drawing circle, drawing perpendicular or parallel line etc. In GeoGebra group, the participants did not spend too much time on drawings but they used most of their time for reasoning and argumentation. However, it was time consuming for Paper-pencil group to measure angle, to draw angle bisector, to draw circle with compass... etc. They sometimes had to draw the shape again because of wrong conjectures and the argumentation paused very often. In a similar situation, the GeoGebra group members had the chance of going back on the screen with “Ctrl+Z” and continue to draw on the same shape. Thus, GeoGebra was useful in order to save time and to retain the argumentation.

Dynamic geometry software were developed in order to engage inquiry based strategies and argumentation in geometry (Prusak et al., 2012). In argumentation, the participation of all students is crucial because all students should evaluate others’ arguments and contribute to their solutions. At the same time, they should convince others by justifying their claims. As for the advantages, Geogebra allowed the participants to use dragging option, to construct accurate drawings such as segments, angles, angle bisectors, parallel lines, perpendicular lines. In line with the results of Prusak and colleagues’ (2012) study, dragging option gave students the opportunity to see relationships visually (Healy & Hoyles, 2001) and check whether their conjectures are valid. Thus, they could obtain immediate feedback and enable rethinking and making new conjectures and geometrical analysis to solve problems (Prusak et al., 2012) which promoted their argumentation. In this way, the reasoning process of participants could be followed by the instructor clearly with the help of their arguments in classroom interactions and their products with GeoGebra. Moreover, GeoGebra was time saving for drawings and motivating for students to discuss the task. When the teachers do not want to spend much time for drawing shapes but to promote reasoning on the task, they can integrate Geogebra into their lessons so that students will draw the accurate shapes quickly and discuss on the properties of the shapes more.

### **Disadvantages of using GeoGebra in argumentation**

The results indicated that there were some disadvantages of the use of GeoGebra in argumentation. One of them was that all students were not at the same pace in using GeoGebra and interpreting the drawings. Therefore, it was difficult to control all participants and to make them participate in the class discussion. When one of the students came to the board to show her solution, some of the students who were thinking on their own drawings did not follow the discussion and did not make comments for the solution. However, all participants should have followed and contributed to the arguments of others in collective argumentation process in order to reach a conclusion. When all students could not follow the discussion, there may be some missing points in their understanding. In the collective argumentation, the arguments are developed through two or more students’ participation (Krummheuer, 1995). In order to critically make sense of the arguments developed, the interaction with other students was stated to be crucial in collective argumentation (Yackel, 2002). Thus, teachers should promote student interactions by rewording their claims and asking probing “Why” questions during the argumentation.

Another disadvantage was that the task may not be compatible with the GeoGebra usage so students who obtain feedbacks from GeoGebra immediately may generalize easily without reasoning. De Villiers (2003) asserted continuous transformations of objects with dynamic geometry software made students convinced easily with general validity of a conjecture. For instance, in the second task, they said that “I measured with GeoGebra and dragged point A to check for other triangles. The lengths are equal for all triangles.” In such a situation the teacher asked the reason why these lengths are equal and made them justify their solution. In the class discussion, the teacher directed participants to think about the reasons for this equality to obtain their justifications. However, in Paper-pencil group, participants started to reason whenever they see the worksheet and their discussion was more productive and they formed more arguments. This is because the geometry task 2 was not suitable for GeoGebra use since the measurement tool in GeoGebra program stopped the argumentation of the participants. It is stated that conjecturing and justifying has a great importance in argumentation (AEC, 1991). However, the tasks

should be suitable for conjecturing and the use of Geogebra to find different solutions. Otherwise, the participants will solve the task on their worksheet and use GeoGebra only for checking the solution by dragging or measuring. That is, the tasks that can be solved with some measurements via GeoGebra may terminate the argumentation. This may not promote their argumentation and they may not justify their claims so that they do not state warrants. When the geometry task requires conjecturing and dragging to recognize the relationships and teacher needs to hear students' justifications to decide whether they conceptually understand the topic or not, the GeoGebra might be advantageous for students. Otherwise, the students might not think critically on the task so the argumentation might not be effective. Thus, it was suggested that the teachers should select GeoGebra tasks carefully and determine whether Geogebra should be used or not in their argumentation based geometry lessons considering the objectives of the lessons and the needs of their students.

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