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# Argumentation below expectation: A double-threefold Habermas explanation

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*Teaching mathematical argumentation is a challenging task, and how to teach argumentation to students from all backgrounds remains an open question. It is especially difficult to say why some situations evoke the vivid exchange of arguments, while other situations completely fail to engage students in argumentation. This paper points out how two related concepts from Habermas' theory of communicative action may help to explore possible barriers and obstacles to argumentation: The rules of discourse ethics for argumentation processes, procedures and products and the consideration of rationality from the epistemic, the teleological and the communicational perspective. Both approaches help to recognize that mathematical argumentation requires more than mathematical content knowledge.*

**Keywords:** Argumentation, Habermas, rationality, discourse ethics.

## INTRODUCTION

Argumentation is highly relevant for the learning of mathematics, and may even be seen as a prerequisite for learning (Krummheuer, 1995). Besides argumentative learning, Knipping and Reid (2015) point to learning argumentation as an important research interest. In order to learn argumentation and proving, Boero states that (2011, p. 120, italics in original) “the best didactical choice is to exploit *suitable mathematical activities* of argumentation and proof”. Learning argumentation is a good starting point in school, because as Douek (1999) highlights: if an emphasis is put on proving this can be a restraint to argumentation. But beyond the question of how to find suitable mathematical activities for fostering mathematical argumentation there remains the open question of how to ensure that all students may participate in argumentation, regardless of their socio-economic or linguistic background. In the light of

research results for problem solving and modelling activities, Knipping (2012, p.1) warns that: “classroom argumentation could become a social filter” if this question does not remain in focus. Gaining a better understanding of argumentation in general is a good first step towards identifying possible filtering effects.

Knipping and Reid (2015) point out that analysing argumentation in the mathematics classroom is important in order to gain a better understanding of its characteristics. Situations in which argumentation remains below expectation in that it either does not happen at all or not in the way anticipated by the teacher are equally interesting to consider if we want to shed a light on the difficulties argumentation entails for students. In this paper I consider a lesson from my research work in which argumentation did not happen in the intended way. Two related parts of Habermas' theory of communicative action are used to identify obstacles and barriers to argumentation: Habermas' view on argumentation as a process, procedure and product from his work on discourse ethics as introduced in Cramer (2014a) and the threefold perspective on communicative, teleological and epistemic rationality as introduced to mathematics education by Boero (2006). In the first part of the analysis, I consider moments in which a student does not engage in argumentation. By means of Habermas' discourse ethics I analyse the situational conditions and present a possible explanation by identifying subjectively perceived barriers. In a second analysis I focus on rudimentary arguments, which either broke off or did not provide a contribution to the solution of the task. Communicative, teleological and epistemic aspects are considered to identify obstacles within the argumentation process. Benefits and limits of the double approach are discussed in the end.

## THE IMPORTANCE OF (LEARNING) ARGUMENTATION

Proof lies at the heart of mathematics as an academic discipline, and argumentation is vitally important for the development of mathematical understanding. Krummheuer (1995) even considers participation in collective argumentation processes as the social precondition for learning. Argumentation, proof and their “complex, productive and unavoidable relationship” (Boero, 1999) have been central to mathematics education research for several decades. In my research, I adopt Habermas’ (1981, p.38) definition of argumentation as a type of speech in which participants discuss controversial claims and support or criticise these by arguments containing reasons which are rationally connected to the claim. Proving processes lie within this definition of argumentation. Proof as the goal of classroom activity can be a restraint for the development of argumentation, according to Douek (1999). Therefore, argumentation in general is the focus of my research, including, but not limited to, deductive reasoning as the path towards proving.

The learning of mathematics is tightly interwoven with, if not even dependent on, student participation in argumentation processes. Taking this into consideration, the emphasis on mathematical arguments in NCTM and other national standards documents is a welcome development. It is, however, unclear how to achieve involvement of all students in argumentation. Lubienski (2000) has shown that children from lower socioeconomic backgrounds benefit less from the reform-based emphasis on problem solving. Concerning the achievement of children in high-stakes tests, Prediger and colleagues (2013) have pointed out that language proficiency is the main predictor for success. Knipping (2012) highlights the necessity of decontextualized language in argumentation. To avoid the looming filtering effect of argumentation she envisages that a thorough analysis of unsuccessful attempts at including argumentation in class could be helpful. In the following, I choose the term “barrier” for occasions in which argumentation did not develop, whereas “obstacle” is used when argumentation is begun but breaks off. I distinguish these two terms because they are different in nature. I will clarify the distinction by exploring two different threefold concepts from Habermas’ theory of communicative action for the analysis of situations when argumentation remained below the teacher’s expectations. By

the analysis, I hope to better understand what makes access to argumentation difficult.

## A TWOFOLD HABERMAS RESEARCH TOOL

The foundation for my analyses is laid in Habermas’ theory of communicative action (1981) and related works. Within this framework, Habermas (1983) describes communicative action and strategic action as two opposed forms of social conduct. Whereas in strategic action the speaker’s intention is to enforce a claim regardless of means, speakers who act communicatively seek consent by supporting their claim with suitable arguments. These arguments are exchanged in discourse. To foster vivid discussions and active argumentation in the mathematics classroom, discourse opportunities must be created.

### Barriers arising from argumentation as process, procedure and product

Discourse is described by Habermas (1983) as a communicative situation shaped by argumentation. New knowledge is inferred from shared common knowledge. Habermas (1983) describes rules for argumentation from the interwoven perspectives of processes, procedures and products. These rules need to be subjectively fulfilled for speakers to engage in argumentation. In the following I present a translated adaptation of these rules for the mathematics classroom (cf. Cramer 2014a).

The first of the three interwoven perspectives Habermas presents is the view on argumentation as a *process*, traditionally considered by the science of *rhetoric*. Characterizing features of argumentation processes are the exclusion of force and the reliance on nothing but the best argument. The process rules are:

- (R1) Everyone may participate in discussions.
- (R2) The topics to be discussed are conjointly determined.
- (R3) There are equal rights and no compulsion to participate in communication.

Argumentation can also be seen as a *procedure* of hypothetically checking claims by giving reasons, free from any immediate pressure to action. This view on

argumentation is traditionally rooted in the science of *dialectics*. Its rules are:

(D1) A speaker is only allowed to claim what he or she believes to be true.

(D2) Shared knowledge may not be attacked without reasons.

Arguments are the *products* of argumentation. Their structure is governed by the rules of *logic* and Habermas gives three rules from this perspective:

(L1) No speaker may contradict himself.

(L2) Who uses a warrant in one situation must be willing to use it in analogous cases.

(L3) Expressions need to have shared meanings.

These three perspectives must not be regarded as separate entities. According to Habermas, they are all equally important preconditions for argumentation. For participation, all of the rules have to be subjectively fulfilled. Of course, perfectly equal positions and the conjoint determination of topics in school are virtually impossible to achieve. However, I have explored before (Cramer, 2014b) how the *subjective* fulfilment of these rules can be shown in the situation of a logical game in which argumentation was successfully evoked. The focus of this paper is on communicative situations where argumentation unexpectedly did not develop. My interpretations of the students' subjective interpretations of the situational preconditions are based on their contributions and the detailed analysis of the situation as a whole. Within these, I try to identify barriers resulting from the subjective non-fulfilment of the rules for argumentation processes, procedures or products, as Habermas claims that subjective fulfilment of these criteria is a prerequisite for engaging in argumentation.

### **Obstacles arising from epistemic, teleologic and communicative rationality**

Boero (2006) introduced Habermas' threefold perspective on rationality into mathematics education research to account for students' rational behaviour in proving activities. As Boero and Planas (2014) remark, epistemic, teleological and communicative perspectives on rationality gradually became a toolkit for many different aspects of research in mathematics

education. I use Habermas' elaborations on rationality to account for obstacles that students may encounter in argumentation processes, leading to breakdowns or failures in reaching the pursued goal.

According to Habermas (2009), a person's behaviour can be described as rational if that person can account for his or her behaviour from three different but interwoven perspectives. *Epistemic rationality* is concerned with the propositional structure of knowledge. In order to act rationally from the epistemic point of view, an interlocutor needs to be aware not only of whether he or she holds certain statements to be true, but also of the reasons he or she has to justify this belief. In discourse, epistemic rationality leads to the possibility of negotiating and transferring knowledge, as the speakers are not only capable of sharing their convictions and beliefs, but also their justifications. Rational action is furthermore characterized by the actors' conscious choices of strategies and tools to help them arrive at mutual or even shared understanding. This conscious choice of strategies according to the aim the actor is following is called *teleologic rationality*. It implies that there are reasons for the actor why he prefers one tool to another, and that these reasons justify the actor's belief that he can achieve his aim under certain preconditions. In mathematical argumentation, actions are usually expressed with words. Language serves as the link between knowledge, aims and the communicative situation. Language is not, however, rational in itself (Habermas, 1996). *Communicative rationality* is located within the use of speech in discourse to develop a common understanding. This uniting power of speech secures the continuity of shared knowledge and the frame within which all interlocutors can refer to it.

The practice of argumentation unifies and requires all three forms of rationality (Habermas, 2009, p. 17). Obstacles may be encountered on the epistemic, teleologic or communicative rationality layer. In contrast to the aforementioned barriers, these obstacles occur after a student has begun to engage in argumentation. They can cause the argumentation to break off, which makes them an interesting analysis approach.

### **GREY AND WHITE BOXES**

The analyses in this paper are based on data from my dissertation project in which I worked as a teacher-researcher with a group of five 15-year-old girls, all of



whom are non-native German speakers from different schools. The lesson took place one week into the project; it was the first content lesson after an introductory interview in the preceding week. Before the project, the girls knew neither me as their teacher, nor each other. In the lesson considered, four girls were present. They worked in pairs with very little teacher guidance in order to foster the creation of their own arguments. The following problem was given to the students:

In a square of grey boxes you are supposed to place white boxes. The grey boxes must no longer touch afterwards. Example for 16 squares:

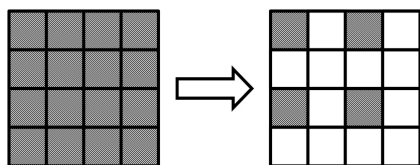


Figure 1

How many white boxes do you need if the square consists of 36, 100, 1024 grey boxes?

The tasks allows for many different solutions (splitting the square into  $2 \times 2$  squares with three white boxes each, recognizing that every second column and half of every second row are filled with white boxes, etc.). The desired outcome was that the students find a pattern that allows them to deduce that for even side lengths, 0.75 times the number of grey boxes is the solution. In the following, I describe episodes from Ayla and Jawahir's pair work process to illustrate how the two approaches from Habermas' work may help to identify barriers and obstacles. A combination of descriptions and transcript excerpts is used for clarity.

### Getting started: argumentation barriers

After distribution of the problem, some introductory teacher comments and further clarification of the task, the girls start to work on the task individually and in silence. Only few comments are exchanged; both girls are visibly engaged in drawing and counting activities. Some minutes after Ayla instructed Jawahir to draw a  $6 \times 6$  square for the 36 boxes task, they exchange results. Both arrived at 27 white squares as the correct answer by drawing and counting. Hardly any other verbal exchange takes place. Four minutes after their comparison and 17 minutes after the problem was given, the first longer conversation takes place. The

transcript of this episode about the number of white boxes for 100 grey boxes is given in the following.

- 3 Ayla: Did you just do one hundred? (...)  
 4 Jawahir: I have six rows here, here.  
 5 Ayla: But why six?  
 6 Jawahir: I wanted five, I did f/, I did six. (...)  
 7 Ayla: But look, (...) you did six here, didn't you? Do //four//  
 8 Jawahir: //Mh// (affirmatory).  
 9 Ayla: f/, no.  
 10 Jawahir: It is supposed to be hundred, ehm  
 11 Ayla: Yes yes but look, you can just do it like this (takes piece of paper) simply ten here (points in one direction with a pen) and ten here (points to another direction with a pen). Then you have hundred in here.  
 12 Jawahir: (...) I hate maths.

In this exchange, Ayla inquires about Jawahir's approach to the task. She wants to know why Jawahir chose six as the side length of her "square". Jawahir does not respond with an argument for her approach, but states that she had originally planned to work with a side length of five (6). Ayla questions her approach by pointing to the preceding task where Jawahir had used six as the side length (7). It is not clear if she names four as the side length of the example on the worksheet or if she wants to tell Jawahir to enlarge the number of rows and columns by four. Jawahir responds by recalling the task (10). Finally, Ayla takes over and gives the argument that a ten by ten square contains 100 grey boxes (11). Jawahir neither reacts to justify her solution, nor does she articulate agreement with Ayla's justification. Instead, after a short pause, she claims to hate maths (12). Two minutes after this exchange, Jawahir says, "I am stupid" in the middle of her work.

In the situation at hand, Ayla is trying to enter into a social process of argumentation with Jawahir, while Jawahir does not enter into the discourse. Regarding Habermas' preconditions we can deduce from Ayla's behaviour that she does not meet any barriers that prevent her from requiring or giving arguments. She justifies her choice of ten as the side length of the square, she questions Jawahir's approach to the task and substantiates her doubts by referring to the former task where six already served as the side length for the 36 boxes square (7). Jawahir, on the other hand,

does not justify her solutions nor does she ask for reasons when Ayla presents her approach.

In the following, I speculate on possible barriers for Jawahir's participation in argumentation. From the *process* perspective on argumentation, there are several possible subjective limiting factors. Jawahir does not participate in the discussion. It is therefore possible that she does not feel welcome to contribute (R1). Her statement that she "hates math" makes mathematical content an unlikely choice of discussion topic for her (R2). Furthermore, her claim "I am stupid" suggests that she does not perceive herself as an equal discussion partner (R3). The same claim also shows a lack of confidence in her mathematical abilities. It is well possible that this presents a barrier inherent in the *procedure* precondition that speakers are only allowed to claim what they believe to be true (D1). No knowledge is exchanged, so nothing can be deduced about shared knowledge (D2). As no arguments are produced, the barriers arising from the *product* perspective can hardly be assessed. Speculations are that mathematical insecurity might lead to self-contradiction (L1), deficiencies in the knowledge of mathematical structures and concepts may lead to problems with identifying analogous situations where warrants can be transferred (L2), and a lack of conceptual knowledge could lead to differing usages of expressions (L3).

Although it is difficult to nail down exactly what it is that kept her from entering into discourse, it is clear that Jawahir in contrast to Ayla did not engage in argumentation in the situation at hand. A lack of mathematical content knowledge probably contributes to the problem, but self-perception seems to be equally relevant. A deficiency in deductive reasoning skills can be excluded as the cause of her silence, as Jawahir produced highly sophisticated arguments in other situations (cf. Cramer 2014b). Within the complex interplay of situational constraints, Habermas' discourse ethics rules provide a tool that can account for some barriers for participation.

### **Spontaneous breakdowns: argumentation obstacles.**

Several minutes later, the girls are still engaged in drawing boxes and counting. No exchange about patterns or generalities has been observed so far. The teacher directs the attention towards the side length of the 1024 square. Following some hints, Ayla finds out that the solution is connected to the calculation

of square roots. She is used to working with a table of square roots from her text book (she calls it "clever book"). After she has looked up the square roots of 36 and 100, this exchange takes place:

- 40 Teacher: So why does that match the side lengths?  
41 Ayla: Because it is the same, isn't it, this way (hand from bottom to top) and this way (hand from left to right).

The table in her book only contains numbers below 1000, so the teacher uses a calculator and shows 32, the square root of 1024 to Ayla and Jawahir. Afterwards, Jawahir starts conversation by asking the teacher for a way to solve the task:

- 60 Jawahir: How does the thousand work, I mean this (points at worksheet) here?  
61 Teacher: Think about that together. (...) Ayla already found the side length of that thing. (...) And maybe you see something HERE (points to the worksheet) that you can (...) carry on (...) somehow. Something this and this here (...) have in common. If you look at them all next to each other, this and this and this.  
62 Ayla: Yes, there always is one, and then not, and then one, and then not.  
63 Teacher: Yes, exactly. And then there for example is a row where there is nothing.  
64 Ayla: Mhm. (Affirmatory)  
65 Teacher: And that is actually kind of the same everywhere. And maybe you find something general (...) how you can find it out WITHOUT COUNTING.

The teacher leaves the table. 50 seconds later, Ayla starts talking:

- 66 Ayla: So if thirty (...) times thirty is nine hundred (...) nine hundred thirty, (...) nine hundred thirty, thirt/ (5 sec), nine hundred sixty. It has to be nine hundred sixty. See (...) write that down. (...) I will now draw these boxes, if nine hundred sixty comes out I was (...) right.

After this monologue, Ayla and Jawahir start to create a 32x32 square by gluing together various pieces of

paper. Some interesting comments during this activity:

- 67 Ayla: Wow, how many boxes ARE THERE? (..) I think this (.) is this one thousand-thingy, isn't it, isn't it, isn't it?

Due to the limited time, their drawing remains incomplete. During the whole-group comparison, Ayla presents 960 as their solution. She justifies her answer as follows:

- 81 Ayla: We first (.) calculated the square root with you. And then ehm, we arrived at thirty-two. Then we tried (.) to do thirty-two boxes, top and bottom. We didn't manage to. Yes, and we got to nine hundred sixty, because we calculated it.
- 82 Teacher: So what did you calculate to get to the nine hundred sixty?
- 83 Ayla: Yes well, because the book said ehm (.) thirty by thirty is nine hundred, and then we calculated a bit.

This longer episode is very interesting from the perspective of rationality. From the *epistemic* perspective, it is observable that Ayla has understood the concept of square root as possibility to calculate the side length of a square (41). She does, however, appear surprised at discovering the huge number of boxes in the 32x32 square they create (67). A possible explanation for her surprise is that she does not see squaring as a reverse operation to calculating the square root. This can be seen an epistemic obstacle. It leads both to difficulties in her calculation attempt for 32 squared (66) and to her disbelief faced with the enormous number of boxes in the 32 by 32 square.

Her abovementioned lack of understanding square roots can also be considered a *teleologic* obstacle, as it goes hand in hand with her strategy of multiplying seemingly random numbers. Ayla multiplies 30, the highest number in her multiplication table, by 32, the result for the square root of 1024 given by the calculator (66). Even when asked (82), Ayla does not give any reasons for the appropriateness of her strategy. Ayla's identification of the pattern (62) sheds light on another obstacle in her teleologic rationality. Although she recognizes an overarching structure for the differently sized squares, she cannot transfer this insight into a useable tool towards finding a mathematical solution.

Finally, Ayla's justifications "because the book said" and "we calculated a bit" (83) show obstacles in her *communicative* rationality. Ayla does not seem to know how to justify her solutions in a mathematically acceptable manner. Furthermore, she does not or cannot use algebra as a tool to represent the pattern she discovered (62), "one, and then not, and then one, and then not".

Possible obstacles can be identified on all three levels of rationality and provide a possible explanation for why Ayla did not arrive at a mathematically correct solution to the task despite her willingness to engage in argumentation. Identifying these obstacles for argumentation can however provide teachers with a tool to work on better support for their students. In Ayla's case, working on calculating square roots and squaring as reversing each other could be beneficial, but also a fostering of using algebra to express patterns might prove helpful.

## OUTLOOK AND DISCUSSION

In this paper, I have elaborated on two different but related concepts from Habermas' theory of communicative action that can help to cast a new light on barriers and obstacles for argumentation. The subjective fulfilment of Habermas' rules for argumentation as process, procedure and product can help to identify barriers leading to non-participation in argumentation. The threefold perspective on epistemic, teleologic and communicative rationality on the other hand provides a tool for the detailed analysis of obstacles leading to unsuccessful argumentation.

Both approaches are united in that they take a perspective that simultaneously highlights and limits the importance of mathematical content knowledge for argumentation. A lack of knowledge can be both a barrier to engaging in, and an obstacle for being successful at argumentation. However, among other influences, the social situation, feelings of inequality, availability of suitable strategies and knowledge about communicative practices are equally important to consider if we want all students to engage in argumentation, especially in the light of looming effects of social and linguistic disparities. Habermas' considerations underline the importance of perceived equality, shared meanings and the absence of force for engaging in argumentation. He also shows that the awareness of justifications, suitable tools and

means of communication is essential both to start and to continue argumentation. More research on barriers and obstacles for argumentation may help to finally overcome them and enable participation in argumentation to all students.

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