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Introduction to the papers of TWG01: Argumentation and proof

Samuele Antonini¹, Orly Buchbinder², Kirsten Pfeiffer³ and Gabriel J. Stylianides⁴

- 1 University of Pavia, Department of Mathematics, Pavia, Italy, samuele.antonini@unipv.it
- 2 University of New Hampshire, Department of Mathematics and Statistics, Durham, USA
- 3 NUI Galway, School of Mathematics, Statistics and Applied Mathematics, Galway, Ireland
- 4 University of Oxford, Department of Education, Oxford, UK

INTRODUCTION

The role and importance assigned to argumentation and proof in the last decades has led to an enormous variety of approaches to research in this area. The 21 papers and 5 posters presented in TWG Argumentation and proof come from 15 countries, residing in 4 continents, and offering a wide spectrum of perspectives. These contributions intertwine educational issues with explicit references to mathematical, logical, historical, philosophical, epistemological, psychological, curricular, anthropological and sociological issues.

Taking into account this diversity, the contributions were presented and discussed in working sessions under seven themes: theoretical and philosophical issues; theoretical and philosophical issues including Habermas' rationality; argumentation and proof in teacher education; argumentation and proof in textbooks; argumentation and proof at the university level; proof representation; and performance, assessment and abilities.

THEORETICAL AND PHILOSOPHICAL ISSUES

The three papers presented in this session concerned various theoretical issues.

Knipping, Rott and Reid contrast three different perspectives when analyzing classroom argumentation (interactional, task analysis, and sociological). In particular, they state that multiple perspectives and levels of analysis are required in research on classroom argumentation, showing that each of these perspectives offers insights into students' argumentations but no single perspective is adequate to completely describe

the nature of students' argumentations and ways to support their development.

In the framework of Husserl's transcendental phenomenology, Moutsios-Rentzos and Spyrou present a reading of the genesis of proof in ancient Greece. The philosophical and historical analysis aims to set up a didactical framework to foster students' need for proving.

The paper by Raman-Sunström and Öhman focuses on the notion of mathematical 'fit', with the goal of identifying some of its characteristics. In particular, their analysis leads to investigation of the relations between the 'mathematical fit', the notion of explanation and some issues related to the aesthetic aspects of proof.

Finally, two posters were presented. Pericleous discusses a study in a Cypriot classroom in the framework of Activity Theory. Vallejo and Ordoñez provide an example of proof-based teaching discussing knowledge construction in the field of natural numbers.

THEORETICAL AND PHILOSOPHICAL ISSUES, INCLUDING HABERMAS RATIONALITY

Three presentations were concerned with the 'scientific culture' in the classroom and in particular with analysis of different aspects based on Habermas' (1998) construct of rationality. Cramer investigates how Habermas' theory helps to explore obstacles and barriers to argumentation. Goizueta and Mariotti focus on the assessment of the validity of mathematical models in a problem-solving situation and underline the need for research to analyze epistemological aspects of the mathematical culture of the classroom.

Boero applies Habermas' theory to analyze a university student's attempt to prove an elementary theorem concerning continuous functions in epsilon-delta calculus.

The presentation of papers stimulated a rich discussion on Habermas' theory of rationality as a research tool which provides a 'dynamic vision' of mathematical activity, and as a general perspective for analyzing the epistemological dimension of classroom interactions and its socio-interactive roots.

The two posters presented in the session involve different frameworks that were discussed and compared with Habermas' rationality: the Toulmin (1958) model in the Ishii's poster, and a competence-based four step model set up by Süss-Stepancik and Götz.

ARGUMENTATION AND PROOF IN TEACHER EDUCATION

In recent years, the general interest in research in mathematics teacher education has stimulated many questions in research about argumentation and proof and teacher education. The presentations of papers in this session considered a variety of different tasks and activities.

Kempen and Biehler focus on perception of generic proofs in number theory and identify three different kinds of pre-service teachers' perceptions of proof: logical acceptance and psychological conviction, general acceptance of the concept and psychological uncertainty, and inappropriate understanding of the concept.

The paper by Buchbinder and Cook is concerned with learning opportunities for pre-service teachers. They suggest that proof construction can be fruitfully inspired by exploring unconventional computational algorithms presented through math-tricks.

From a different point of view, Erkek and Işıksal-Bostan's paper focuses on advantages and disadvantages of the use of GeoGebra in a study involving pre-service elementary mathematics teachers.

The poster by Modeste and Rojas discusses a research project that aims to build a model of mathematical activity that can be used in primary teacher education.

The main issues discussed in the session were the design of proving tasks with the goal to avoid cultivating misconceptions in the teaching of proof, and to foster positive attitudes towards mathematics.

ARGUMENTATION AND PROOF IN TEXTBOOKS

Textbooks play a major role in everyday mathematics practice in many countries around the world and many teachers rely heavily on their textbooks that influence their decisions of which tasks to implement in the classroom, and how to implement them. Using different theoretical frameworks and adopting different approaches to the analysis, three papers investigate aspects related to argumentation and proof in textbooks in four different countries: Israel, Spain, Sweden and Finland.

Silverman and Even characterize justification and explanation for mathematical statements offered in 7th grade Israeli textbooks. The analysis revealed that the textbooks commonly used several modes of reasoning in explanations for each statement. Nearly every justification was deductive or empirical, yet different modes of reasoning were used for geometric and for algebraic statements.

Bergwall presents a framework for analyzing generality in proving tasks in calculus in Swedish and Finnish textbooks. The author discusses the usefulness of framework in analyzing and comparing textbooks and states that there is not necessarily a correlation between the number of general proving tasks and the opportunities for students to engage in reasoning about arbitrary functions.

Finally, Conejo, Arce and Ortega present the evolution of the proof schemes shown in grades 11 and 12 textbooks related to the theorems of limits. In particular, they develop a framework based on Harel and Sowder's (1998) notion of "proof schemes" and show a case study applying the framework to Spanish mathematical textbooks from the 70s until today.

The discussion focused on methodological aspects related to the unit of analysis (e.g. task, lesson, chapter, etc.) and on the difficulties identifying proof and argumentation tasks in textbooks (e.g. looking for keywords like "prove" or "show" might not be enough).

ARGUMENTATION AND PROOF AT THE UNIVERSITY LEVEL

Mathematics education at university level and, in particular, the teaching of proof and proving require specific methodological approaches and theoretical considerations that take into account the specific goals and the modality of teaching in this academic setting.

The theoretical paper presented by Annie Selden and John Selden suggests a perspective for understanding university students' proof constructions based on the ideas of conceptual and procedural knowledge, explicit and implicit learning, behavioral schemas, automaticity, working memory, consciousness, and two systems cognition.

One technique that future mathematicians should master is proof by *reductio ad absurdum*. Alvarado and González focus on it and present part of a research study in which college students performed a task which required application of this technique.

During the session, the discussion focused on the need for development of tasks, sequences of tasks and courses, as well as specific didactical approaches to support university students' proof production and comprehension. In this vein, Pfeiffer and Quinlan presented a paper on proof evaluation tasks in a university mathematics course. The responses to the task, in which students were asked to evaluate and rank different proposed proofs, provided rich opportunities for students to attend to the nature and functions of mathematical proofs; the task also revealed some interesting features of students' thinking. The authors argue that proof evaluation tasks can afford rich learning opportunities as well as enable novice students to participate in authentic mathematical practice.

PROOF REPRESENTATION

Contributions in this session discuss aspects of representation of proof, with particular attention to oral and written modes of representation, which involve different cognitive processes, and require careful consideration when one attempts to interpret research findings on students' conceptions of proof, and when comparing findings from different studies.

In particular, Andreas Stylianides focuses on the role of the mode of representation in students' argument constructions. He discusses findings from a classroom-based design experiment suggesting that the use of an oral mode of representation may be more likely, compared to a written mode, to support the construction of an argument that approximates or meets the standard of proof. This raises concern about the validity of research findings reported in the literature on students' conceptions of proof, and creates difficulties in comparing findings across different studies.

Azrou's paper deals with the writing of a proof text as the final step of the proving process. She describes university students' difficulties to get a satisfactory product, which frequently result in an unclear text in a disorganized form, in particular when students are asked to answer open questions.

The aim of Moulin's and Deloustal-Jorrand's work is to explore potential functions of stories in the learning of Science and Mathematics with the focus on potential connections between the mathematical space and the rhetorical space during problem solving activity. They characterize theoretically a processes-transferring space between the narrative activity and the problem solving activity. By analyzing oral and written products of children work, they show that the narration act supports students' mathematical reasoning.

PERFORMANCE, ASSESSMENT, ABILITIES

The main issues discussed in this session concerned the importance of a priori analysis of assessment tasks in order to understand their requirements and compare to students' mathematical histories, and the influence of the type of curriculum on students' proof performance.

In particular, Sears and Chávez examine students' performance on a proof task about corresponding parts of congruent triangles. Using data from 1936 students, they show that, regardless of curriculum type, students experience difficulty with constructing this type of proof.

Luz and Yerushalmy examine the design principles of e-assessment of understanding of geometric proofs. In particular, they review various proving task-design studies, looking for a template that incorporates interactive sketching that can be checked automatically.

Finally, the participants were involved in a debate on the question about the possibility, in mixed-ability lower secondary school classrooms, to engage all students in proof without compromising the development of the proof abilities of the most “talented” students. In particular, Moya, Gutiérrez and Jaime present a study on the ability to make proofs of mathematically talented secondary students attempting geometry proof problems.

CONCLUSIONS

We think that TWG on argumentation and proof has offered the participants the richness of diversity in this research domain and the opportunity of fruitful discussions. It also seemed to stimulate not only the interest of comparison but also the curiosity of undertaking a possible integration of different perspectives and the need of enhancing the development of international collaborations.

REFERENCES

- Habermas, J. (1998). *On the pragmatics of communication*. Cambridge, UK: MIT Press.
- Harel, G., & Sowder, L. (1998). Students' proof schemes: results from exploratory studies. In A. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research on collegiate mathematics education*, vol. 3 (pp. 234–283). Providence: American Mathematical Society.
- Toulmin, S. (1958). *The Uses of Argument*. Cambridge, UK: Cambridge University Press.