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Research in teacher education and innovation at schools: Cooperation, competition or two separate worlds?

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The plenary lecture builds on the plenary lecture from ICME10 (Survey team 3). The lecture defined areas that had attracted little attention of researchers but were crucial (not only) for teacher education. It also comes out of discussions on ICMI Study 15 and recommendations formulated in these discussions. The lecture is also significantly informed by the work of Working Groups at CERME conferences since CERME1 until CERME8, partially also taking into account information from CERME9. The field of research in mathematics teacher education has changed considerably over the years since ICME10, which asks for a new definition of issues and trends. The goal of the lecture is to point out some trends in this area of research, especially in the field of cooperation between teacher education and innovations at school.

The first part of the text focuses on trends in current research into teacher education and practice. The goal of this part is not an exhaustive overview but indication of the main trends in the research domain. The second part of the text presents a more detailed discussion of several current research areas, their theoretical backgrounds as well as applications of their findings in teacher education and everyday school practice.

Keywords: Teacher education, cooperation of teachers and researchers, changes in teachers' knowledge, beliefs and approaches, problem solving.

INTRODUCTION

Let me begin the text by my personal confession: When the Programme Committee of CERME9 offered me to give a plenary lecture on the topic “Research into teacher education and practice“, I felt that this was great honour and I was excited, even thrilled

by the ideas of starting work on the plenary lecture. However, my initial enthusiasm slowly lessened. The reason for this faltering were not any doubts on the relevance of the topic. However, the deeper I emerged into the issue, the more aware I grew of the immense scope of research I could get access to. I realized that my lecture would never be and could not be exhausting and that I would have to focus on selected aspects of the issue only. I decided to build the plenary lecture on three important resources to which I had personally contributed:

- The plenary lecture from ICME10 (Adler, Ball, Krainer, Lin, & Novotná, 2004) in which the areas that had attracted little attention of researchers but were crucial (not only) for teacher education were defined. However, the field of research in mathematics teacher education has changed considerably over the years since ICME10, which asks for a new definition of issues and trends.
- ICMI Study 15 “The Professional Education and Development of Teachers of Mathematics” (Ball & Even, 2009). This study confirmed the great variety of research in this area. All this research attracts a lot of attention worldwide and brings new and interesting results.
- CERME conferences, where teacher education has always been paid much attention to. The focal point has been shifting with respect to the development of research in the area. However, it has always been based on the interaction between practices in teacher education and requirements of everyday school practice.

The focus of the first part of the text is on developments in research into teacher education and practice

until present. The goal of this part is to point out main trends in the research domain. It is important not only to list the topics addressed by research since CERME1 but also to show the used methodologies and posed research questions.

The aim of the second part of the text is to illustrate some of the trends in research in teacher education and innovations at schools, focusing mainly on the further development of teachers' knowledge, beliefs about and approaches to mathematics education resulting from cooperation with researchers. When selecting from the many, the attention was paid to those areas of research that the author is familiar with and in which she has been involved.

RESEARCH INTO TEACHER EDUCATION AND PRACTICE UNTIL PRESENT

Survey Team 3 at ICME10. Research on mathematics teacher education: Mirror images of an emerging field

The Survey Team 3 (ST3) consisted of the following members: Jill Adler, Deborah Ball, Konrad Krainer, Fou-Lai Lin and Jarmila Novotná.

As a member of ST3 at ICME10 in Copenhagen, I was involved in collecting information on research focusing on mathematics teacher education in the years 1999–2003 (Adler et al., 2005). This work clearly showed it was inevitable to delimit the areas and issues we would come out of. The survey included published research in international mathematics education journals, international handbooks of mathematics education and some international mathematics education conference proceedings. Some regional sources from various parts of the world were also included. The survey was restricted to 1999–2003, covering the period between ICME9 and ICME10. More than 200 papers were analysed.

The central question for the survey was: *Is research in the field contributing to the improvement of the education of teachers of mathematics?*

The work was framed by the following considerations: What is the state and status of research in mathematics teacher education, within and across contexts? Which problems have been constructed as central in this field in the recent past, and how have these been approached? What shifts – theoretical and method-

ological – can be discerned and how might they be explained? Who does the research? Where? What progress has been made, empirically, theoretically, methodologically? Are there evident gaps, and if so where? What kind?

We investigated “the *who* (who was writing/doing the research, and from where), the *how* (what methods were used) and the *what* (what was being studied, theoretical orientations, assumptions and outcomes)” (Adler et al., 2004). We also examined the range of findings and conclusions in these studies. These helped to identify four areas that asked for further investigation.

Where is the centre of the field? The investigated publications were divided into two groups. The first includes publications focusing on theorising and understanding teacher learning. The second concerns aspects of curriculum reform, the goals of teacher education initiatives, i.e., evaluation. We identified the shift from studies that tended to tell success stories about teacher education initiatives, and advocacy in the initial phases of curriculum reform, to deeper reflective research that is more convincing in the scholarly sense. Teacher educators' learning was paid much less attention to. It was noted that: “We do not understand well enough how mathematics and teaching, as inter-related objects, come to produce and constitute each other in teacher education practice. We lack adequate knowledge about what and how this happens inside a teacher education program, and then across ranging or contrasting programs, contexts and conditions.”

What are the theories and methods in the field? We stated that the field is emerging and needs to increase rigour. The vast majority of this research is case study research, where at least one of the researchers is also a teacher educator, and often the educator(s) whose programme is under study. It certainly makes sense if we want to study teachers' learning and teaching practices. The emergence of theories of situated learning, and attempts to theorise learning of professional practice were identified. In many papers, theoretical frameworks are left implicit. Small-scale qualitative research predominates.

Contexts of mathematics teacher education research: Who, where, and with whom? Most teacher education research is conducted by teacher educators studying

the teachers with whom they are working. Attention has mainly been paid to showing that particular programmes of teacher education ‘work’; a large number of papers were dealing with reform processes, particularly in the USA, and with teachers in professional communities and in other institutional settings.

Dominance of English-speaking world. This dominance was remarkable (e.g., 80% of the papers published in JMTE have been written by authors from, and report research done in, English-speaking countries). Obviously, the situation is different if we focus on national or regionally focused conferences and journals. The influence of this situation on the orientation of research was not analysed but it certainly has a great impact.

ST3 also formulated domains of interest that were underrepresented in the analysed resources. It was noted that there were fewer studies on:

Teacher working outside of “reform” contexts: Many teachers make effort to develop their teaching skills in environments where reform is not the dominant issue but where they are assisting a wide range of learners in learning mathematics.

Teachers’ learning from experience: We do not know enough what teachers learn from experience, whether they learn from experience at all, and what actually supports learning from experience. Teachers spend most of their time doing teaching; we do not understand enough about what helps some teachers to learn from their own teaching while others do not.

Teachers’ learning to directly address inequality and diversity in their teaching of mathematics: We do not know enough about teachers’ learning to directly address inequality and diversity within their teaching of mathematics (culture, gender, language, socio economic status and mathematical background).

Comparisons of different opportunities to learn: We lack comparisons in the field that compare different opportunities to learn. How does one approach to helping teachers to learn mathematics compare with another?

“Scaling up”: We do not know enough about what happens when programmes spread to multiple sites, what it means to scale up or what it means to extend a programme that has worked in one setting to another

setting – what works, what goes wrong, what designers need to know and think about.

Education of teacher educators: Despite their important role in the system of teacher education, educators’ education, professional background, etc. was not studied in the analysed publications.

This was the situation in 2004 perceived through the analyses of ST3. Approximately at the same time, another important event focusing on teacher education, The Fifteenth ICMI Study, was launched. The study was designed to offer an opportunity to develop a cross-cultural conversation about mathematics teacher education in mathematics around the world. The Study Volume is described below (Even & Ball, 2009).

ICMI Study 15: The Professional Education and Development of Teachers of Mathematics

ICMI Study 15 focused on mathematics teacher education practice and policy around the world. As stated in (Even & Ball, 2009), its premise was that the education and continued development of teachers are keys to pupils’ opportunities to learn mathematics. What teachers of mathematics know, care about, and do is a product of their experiences and socialization both prior to and after entering teaching, together with the impact of their professional education. It was claimed that systems of teacher education, both initial and continuing, are built on features that are embedded in culture, the organization and nature of schooling, and too rarely is there cross-cultural exchange of knowledge and information about the professional development of teachers of mathematics. Learning about practices and programmes around the world can provide important resources for research, practice, and policy in teacher education, locally and globally.

The contributions accepted to ICMI Study 15 were divided into two Themes: Theme 1 – *Initial mathematics teacher education*, and Theme 2 – *Learning in and from practice*. In several aspects, both Themes brought new ideas in the issues considered by ST3 as less studied in 2004. It is evidenced by the list of main questions discussed in the ICMI Study 15.

Theme 1 focused on the following main questions:

Structure of teacher preparation: How is the preparation of teachers organized – into what kinds of institutions, over what period of time, and with what

connections with other post-secondary study? Who teaches teachers, and what qualifies them to do so? How long is teacher preparation, and how is it distributed between formal study and field or apprenticeship experience? How is the preparation of teachers for secondary schooling distinguished from that of teachers for primary and middle levels of schooling?

Curriculum of teacher preparation: What is the nature of the diversity most pressing within a particular context – for example, linguistic, cultural, socio-economic, religious, racial – and how are teachers prepared to teach the diversity of pupils whom they will face in their classes? How are teachers prepared to know mathematics for teaching? What are the special problems of content preparation in different settings, and how are they addressed?

Recruitment and retention: Who enters teaching, and what are the incentives or disincentives to choose teaching as a career in particular settings? What proportion of those who prepare to teach actually end up teaching, and for how long?

Most pressing problems of preparing teachers: Across the initial preparation and early years, what are the special problems of teaching mathematics within a particular context and how are beginning teachers prepared to deal with these problems?

The early years of teaching: What are the conditions for beginning teachers of mathematics in particular settings? What supports exist, and how effective are they, for what aspects of the early years of teaching? What are the special problems faced by beginning teachers, and how are these experienced, mediated, or solved? What is the retention rate of beginning teachers, and what factors seem to affect whether or not beginning teachers remain in teaching? What systems of evaluation of beginning teachers are used, and what are their effects?

Mathematics educators' activities and knowledge: It concerns one of the underrepresented domains mentioned by ST3. These contributions focused mainly on models of educators' development, their quality, national support, their own practice and research.

During the Theme 1 sessions at the ICMI Study 15 conference, additional important questions emerged that had not been included in the Study Volume: What

is the role of didactics of mathematics (mathematics education) in teacher education? What is the place of ICT in teacher education? How is the practical part of this preparation (the teaching practicum) integrated? What do we know about the construction of professional knowledge of teachers in relation to teacher education programmes?

The collection of papers in Theme 2 provides a range of approaches to studying teachers' learning. The papers focused on four main domains:

Development of teaching in and from practice: What are the characteristics of the process of developing professional expertise in the teaching of mathematics in and from practice? What are the beliefs, experiences and structures that are significant as far as the development of mathematics teachers and teaching are concerned? What are the conceptual, institutional, cultural, etc. structures that enable and constrain research into teacher development?

Process of learning in and from practice: What are the changes and approaches to professional development? How is the new organization of professional development initiatives for teachers conceived and implemented?

Models, tools and strategies to support learning in and from practice: What are the tools, dynamics, tasks, contexts, and learning settings that can be mobilized for pre- and in-service mathematics teacher education? What are the tasks for mathematics teacher education that are offered to teachers for deepening their knowledge of what and how to teach their pupils? What can be learned from analysing instructional episodes? What is the role and advantages of forming teachers' learning communities where they can share experiences, meanings, knowledge, lessons, etc. from their school practice?

Balance of teachers' mathematical content and pedagogy knowledge: How can we overcome the difficulties in practising teacher education and professional development that are caused by the complexity of the knowledge required for teaching? What is the relationship between teachers' content knowledge and pedagogical practices, considering it from various perspectives?

The Study Volume contains one chapter summarising key issues for research in education and professional

development of teachers of mathematics. It focuses on the goals of education, the role of mathematics education, understanding of practice-based professional development for mathematics teachers and the future of strengthening practice in and research on professional education and development of teachers of mathematics.

Examples of more recent work

Research in the area of teacher education and in the area of the potential and consequences of cooperation between teachers and researchers has undergone turbulent developments over the decade since ICME10 and ICMI Study 15. This can be documented by the variety of publications in the area – monographs, articles and special issues of renowned journals as well as various conferences focusing on research in this area. Let us present examples of some more recent work that do not focus narrowly on one aspect of research in the field but try to relate this area into a wider context of mathematics education. Considerable attention is paid to involvement of teachers in research, albeit in the form communities between teachers and teacher educators (see, e.g., Jaworski, 2005; Novotná et al., 2006) or in the form of independent research conducted by teachers themselves (see, e.g., Kincheloe, 2012). All these research studies stress the benefit of teachers' participation in them, despite some limitations.

Teacher education also attracts attention of the International Group for Psychology of Mathematics education. Every year the area is addressed by a significant number of research reports, short oral presentations, posters, working sessions, discussion groups and other components of the programme and is frequently addressed in plenary lectures, panels and research forums. The importance that IGPME pays to teacher education is highlighted also by publishing Handbook of Research on the Psychology of Mathematics Education (Gutiérrez & Boero, 2006) where the fifth section includes two chapters summarizing the PME research on teacher education and professional life of mathematics teachers (Llinares & Krainer, 2006; da Ponte & Chapman, 2006).

Third International Handbook of Mathematics Education (Clements, Bishop, Keitel, Kilpatrick, & Leung, 2013)

The book offers an overview of past, present and future aspects of all areas of mathematics education (social, political and cultural dimensions in mathe-

tics education; mathematics education as a field of study; technology in the mathematics curriculum; and international perspectives on mathematics education). Four out of the eight chapters of the second section focus on mathematics teacher education; they present research methods in mathematics teacher education, teachers as researchers, teachers' learning from teachers and developing mathematics educators.

The chapter *Developing mathematics educators* discusses different types of mathematics educators including teacher educators. It addresses cooperation between teachers and researchers. The concept of teachers as researchers is discussed from different points of view. It contributes to the area described as underrepresented in research in the material prepared by Survey team on ICME10.

Note: The issue of teacher educators has been addressed increasingly in the last ten years. An important step was Volume 4 of the Handbook of Mathematics Teacher Education (Jaworski & Woods, 2008). Recently, the proceedings of the international conference on "Educating the Educators" were published (Maaß, Törner, Wernisch, Schäfer, & Reits-Koncebovski, 2015).

Encyclopedia of Mathematics Education (Lerman, 2014)

This reference work covers all topics in the area of mathematics education. The entries offer theoretical background, summary of important findings and results in the area and provide references to important publications where more detailed information can be found.

One section coordinated by Mellony Graven addresses research in teacher education. The entries cover both the areas of pre- and in-service teacher education and the area of teacher educators, i.e. an area described as underrepresented in research on ICME10. More than twenty entries address directly teacher education and teacher practice and many other are somehow connected to the areas. The consequence of this effort to describe fully and comprehensively all aspects of mathematics education is that also topics described as underrepresented in research on ICME10 were paid due attention. Very valuable are the references to other literature and publications dealing with the topics but also focusing on development of research in the area over years.

ZDM, Mathematics Education (special issue, 47(1), Rösken-Winter, Hoyles, & Blömeke, 2015)

This special issue of the journal focuses on scaling up sustainable interventions through evidence-based CPD. In the articles, four perspectives are considered: crucial aspects of teacher learning, different CPD frameworks and their influence on developments in CPD, the meaning of developing CPD in an evidence-based way and crucial aspects of spreading CPD on a large scale. As Roesken-Winter, Hoyles and Blömeke state in their introductory survey paper, they “draw on Coburn’s four dimensions characterizing the process of scaling CPD interventions, depth, sustainability, spread, and shift in reform ownership to discuss how the challenge of scaling high-quality CPD might be successfully addressed”. The articles help to fill in some gaps in areas identified by Adler, Ball, Krainer, Lin and Novotná (2004) as underrepresented in research.

CERME CONFERENCES

An immense amount of work on the topic of teacher education and professional development has been done during the CERME conferences, from their early beginnings in Osnabrück, Germany in 1998. Teacher education has always been paid much attention. One Thematic Working Group has always focused on the issue, both on pre-service and in-service levels. Table 1 contains a more detailed look at the development of

the issue at CERME conferences. Proceedings from CERMEs are available online at <http://www.mathematik.uni-dortmund.de/~erme/index.php?slab-proceedings>. It shows that even if the programme components did not have the same focus and followed contemporary trends in research in the area of teacher education in the corresponding period, they always paid attention to interactions between practices in teacher education and requirements of everyday school practice.

The significance of the issue of teacher education since the beginnings of CERME conferences is confirmed by the publication of a separate third part of CERME1 proceedings: *On Research in Mathematics Teacher Education. From a Study of Teaching Practices to Issues in Teacher education* (Krainer, Goffree, & Berger, 1999). The book builds on the work done by Working Group 3 *Theory and practice of teaching from pre-service to in-service teacher education*. It is divided into six parts with respect to the topic that is addressed: Teacher education and investigations into teacher education; Teacher education and investigations into teachers’ beliefs; Teacher education and investigations into teachers’ knowledge; Teacher education and investigations into teachers’ practice(s); Teacher education through teachers’ investigation into their own practice; Investigations into teacher education: Trends, future research, and collaboration.

CERME	WG	Other programme type
1	Theory and practice of teaching from pre-service to in-service teacher education	
2	Theory and practice of teaching from pre-service to in-service teacher education	
3	Inter-relating theory and practice in mathematics teacher education	Plenary panel <i>Theory and Practice: Facilitating teachers’ investigation into their own teaching</i>
4	From a study of teaching practices to issues in teacher education	
5	From a study of teaching practices to issues in teacher education	
6	Mathematical curriculum and practice	
7	From a study of teaching practices to issues in teacher education	Plenary lecture <i>Research into Pre-service elementary teacher education courses</i>
8	From a study of teaching practices to issues in teacher education	
9	Mathematics teacher education and professional development	Plenary lecture <i>Research in teacher education and innovation at schools – Cooperation, competition or two separate worlds?</i>

Table 1: Development of the topic at CERME conferences

As far as the focus of this plenary lecture is concerned, the most interesting is the last of the above listed areas. The title of the plenary lecture speaks of collaboration, not competition or two separate worlds. Let us recapitulate here the main ideas presented in CERME1 proceedings. They remain topical for research in teacher education and innovation at schools despite being published in a book from 1999 (i.e., 16 years ago).

In the area of *Research in the perspective of teacher education*, the following questions, substantial for the area, are studied: To what extent do mathematics teachers' general beliefs relate to local beliefs (e.g., to specific topics as teaching algebra)? What are the conditions and constraints that influence teaching practice? How do teachers manage the connection between pupils' activities and the acquisition of mathematical knowledge? (The term "acquisition" used here is worth attention, it is broader than the term learning.) What is the interplay between mathematical knowledge and ability, self-confidence, personal history and conceptions of mathematics teachers? How do internal factors interplay with external factors concerning the professional development of teachers? How can problem solving be used as a tool to find out of mathematics teachers' beliefs in order to improve teachers' mathematical knowledge and mathematics teaching?

In the area of *Research in the context of teacher education*, authors study, for example, the following questions: Considering the professional development of teachers, what is the interplay between cognitive processes and cultural, social, affective processes? How do (student) teachers construct (what) knowledge? What is the role of discourse and collaboration? What kind of knowledge do teachers bring to in-service education and how does it grow? Is the gap between what teachers learn at the university (pre-service education) and their practice at schools evident and how could we explore it? How do student teachers develop their understanding of children's ways of thinking during school practice? Why and how do mathematics teachers from one school (want to) further develop their teaching practice using alternative learning and teaching methods?

In the contributions, serious attempts to find bridges between theories and practices of teacher education are present. In particular, the idea of viewing learning environments for (student) teachers at the same time

as a meta-learning environment for teacher educators who investigate into (student) teachers' growth and at the same time reflect on their influence within the interaction process is obvious.

The following are the *major trends* sketched in the texts. A broader understanding of research in teacher education is needed; it covers investigations focusing on teachers including their beliefs, knowledge and practice, and engagement of (student) teachers in investigating their own practice. There is an increasing importance of action research as the systematic reflection of practitioners into their own practice. There is an increasing importance of "stories" (narratives, curricula vitae, cases, ...). It seems more attention should be paid to cultural, situated, and organizational aspects of processes in classroom and teacher education courses. Moreover, looking for integration and interconnections is crucial.

Pupils' learning, (student) teachers' learning and researchers' and teacher educators' learning are considered as three domains of strongly interconnected learning. The attention is paid to learning from investigations (learning from research questions, from research methodologies, from elaborating the data and from presenting the research).

The Working Group continued its work also at CERME2. Its focus was on "teacher education between issues and practical realization". The contributions were based on teachers' knowledge, investigations into teachers' practices, their attitudes; research on the impact of the use of information technologies was also included.

The work in the WG was characterized as follows by its coordinators: "More than in other fields, the researcher in the field of teacher education subject has to balance what is suggested by the theoretical considerations and what is possible to realize in practice. The discussion reflected this position and the themes touched fluctuated between the two poles."

The WG formulated *perspectives for the future*: To investigate professional growth of pre-service teachers, qualified teachers and teacher educators, relationship between theory and practice, teacher development in the classroom, connection between pre-service and in-service education, development of teachers' subject knowledge.

At CERME3, two components of the programme were devoted to teacher education: The WG *Inter-relating theory and practice in mathematics teacher education* and the Plenary Panel *Theory and Practice: Facilitating teachers' investigation into their own teaching*.

The topic of the WG attracted an increasing number of authors. In order to keep the discussions efficient, the participants were divided into five subgroups: Teaching approaches in particular curricular areas; Teaching approaches and their development; Elements of reflection in teacher education; Role and nature of collaborative work in teacher education; Inter-relating theory and practice.

WG formulated *issues emerging from discussions*: Situations and problems in teaching are complex and need particular solutions that can only be developed in the specific context of their appearance. There are no general solutions that might be transferred from theory to practice; also at schools, improving and understanding one's own practice is important. More teachers who reflect critically on their teaching, exchange their experiences, and read theory-driven papers in order to broaden their understanding of educational processes are needed. More teacher educators who take their teacher education practice as an object of evaluation and research are needed. Also more collaboration between teacher educators and teachers in order to promote teacher education – as a field of practice and research is essential.

The Plenary Panel, chaired by B. Jaworski, focused on the relationship of theory and practice in mathematics education. Besides Jaworski's introductory and final thoughts, three panellists presented examples from their own country. Bartolini Bussi (2004) briefly presented one special national project for education in Science and Technology SeT Project (1999–2002). Krainer (2004) investigated the relationship theory-practice in the theoretical perspective of four di-

mensions of “learning systems”: *Action; Reflection; Autonomy; Networking*. Bergsten (2004) showed the theory-practice relation in mathematics education as multifaceted.

At CERME4 and 5, WGs related to teacher education were focusing on the same main topic: *From a study of teaching practices to issues in teacher education*. In both cases, the work was organized in subgroups; see Table 2 where corresponding topics are in the same row.

It is also of interest to compare emerging issues from the discussion in WGs at both conferences, see Table 3.

Much attention was paid to communities of practice and collaborative work in them. Cooperation between teachers and researchers was evaluated as important. It attracted much more attention at CERME5. At CERME4, attention was also paid to the assessment in mathematics teaching and implementation of ICT. At CERME5, these topics became so common in the discussion that they needed no special emphasis. Moreover, they were also discussed in other WGs focusing on ICT in mathematics education or assessment.

At CERME6, teacher education and development were included in WG *Mathematical curriculum and practice* where teacher education was directly linked with school practices. Its subtitle *From study of teaching practices to issues in teacher education* evoked its close link with the corresponding WG at CERME4 and 5. The call for papers asked for theoretical, methodological, empirical or developmental papers on teachers' practices, professional knowledge and teacher education. The work of this WG was organized in the following subgroups: Mathematical curriculum and practice; Professional knowledge (similar but different terms used: knowledge base for teaching; pedagogical content knowledge; competence; subject didac-

CERME4	CERME5
Understanding practice, understanding and promoting the mathematics teacher's development	Models to analyse the practice
Process of becoming a mathematics teacher	Knowledge for teaching (or professional knowledge).
Means, resources and methodology to research on and promote the mathematics teachers' development	Tasks and resources in pre-service teacher education
	Approaching reflection in mathematics teachers' professional development

Table 2: Subgroups at CERME4 and 5

CERME4	CERME5
Demand for theories, perspectives and methods capturing or approaching the flavour and the essence of the classroom activity (various theoretical frameworks)	Discussion on theories, perspectives and methods to approach the flavour of classroom activity
Incompleteness of current models to give an account of the real teaching-learning process	Confrontations of frameworks and models by means of analysing some corpus of a classroom teacher practice observation
Relationship between researchers and teachers	The nature and conditions of collaborative work. Particularly the role of the experts, and the necessity of making it possible that teachers meet together in order to reflect on their practices
Knowledge, pedagogical content knowledge, teachers' competence (including communities of practice and the socio-cultural theory)	Notion of community of practice and related notions
	Different notions about reflection
Assessment instruments as a tool to support learning	
The role of teacher when using ICT	

Table 3: Emerging issues formulated at CERME4 and 5

tical competence; practical knowledge – beliefs and knowledge); Professional development; Approaching reflection and collaboration in mathematics teachers' professional development (Reflection is a privileged way for professional enhancement. Collaboration is a means for professional development and for research strategy.); Models to analyse the practice (The practice of teachers includes classroom teaching, as well as education and other professional development contexts; How can we manage to make research results and instruments useful for teachers as means in their professional development, and for educators in education contexts?).

One of the *main conclusions* formulated in this WG was the following: "As for primary teachers, also for

secondary teachers, mathematical content knowledge and pedagogical content knowledge must be interrelated in teacher education." (Durand-Guerrier, Soury-Lavergne, & Arzarello, 2010, p. 1690).

At CERME7 and 8, WGs returned back to the previous title *From a study of teaching practices to issues in teacher education*, which clearly expresses the main focus of the work. In Table 4, the subgroups at both conferences are summarized; again, the allied topics are in the same row.

At both conferences, *critical issues* instead of emerging issues were formulated. The descriptions of these issues at both conferences differ substantially in their number as well as details in their formulations.

CERME7	CERME8
Mathematical content knowledge for teaching	Resources for teaching: Teacher knowledge and teacher beliefs
Professional content knowledge for teaching	
Reflection in mathematics teachers' professional development	Teacher reflection
Professional development	Teacher education and professional development)
Collaboration in mathematics teachers' professional development	Teacher collaboration
Conceptions and practices	Studying mathematics teaching
Interaction in the classroom	

Table 4: Subgroups at CERME7 and 8

CERME7:

- Recognition of the value and complementarities of different approaches to the professional development of teachers
- Recognition that there are constraints and affordances for different approaches, which vary between cultural contexts; working across cultures on teacher development projects, which employ different strategies, was considered to be a useful way of moving forward our understanding of different approaches
- Considerable work to be done in understanding how different frameworks relate to one another and in supporting researchers in selecting elements of different frameworks that will enable them to answer specific research questions

CERME8

- Working with multiple frameworks
- Suitable model for teacher knowledge
- The purpose for developing new theoretical models or for modifying/revising the existing ones
- Analyses of the influence of different types of knowledge
- Ways for promoting teacher knowledge
- Study of the mutual relation between teachers' knowledge and practice
- The role of context
- Role of teacher educators in helping students/teachers to develop different components of their knowledge – differences for prospective teachers and in-service teachers

Sierpińska's (2011) plenary lecture was devoted to research in teacher education and practices. In the talk, results of an ongoing research, focusing on a framework for analysing the "Teaching Mathematics" courses were presented. "Teaching Mathematics" courses were designed and implemented by the author in cooperation with her colleague. Sierpińska

presented her research conducted within the frame of implementation of these courses. The framework of this research might be useful for other researchers wishing to contribute to professionalization of elementary mathematics teacher educators' work.

At CERME9, for the first time, teacher education was the theme of three TWGs: *Mathematics teacher education and professional development*, *Mathematics teacher and classroom practices* and *Mathematics teacher knowledge, beliefs and identity*. For the detailed information about all three TWGs discussions and results see the corresponding chapters in CERME9 proceedings.

However, research on teacher knowledge can be come across not only in the corresponding TWGs, it pervades all TWGs, whichever area of mathematics education they deal with. In every TWG, one can come across papers that focus on the topic of the working group but at the same time are related to teacher education. And this is why I decided to call my plenary lecture *Research in teacher education and innovation at schools – Cooperation, competition or two separate worlds?* Are teacher education and innovation at schools closely related areas or are they two separate worlds that have very little or nothing in common? Is teachers' attitude to innovation in mathematics education influenced predominantly by the environment of the school they work at, their own experience with pupils, or by what they have learnt in their teacher education? In other words: Is teacher education an obstacle in the introduction of innovation at schools or does it support the process? Or are they independent of each other? It is very easy to understand the questions, to formulate them. However, it is far from easy to find answer to them and it seems the answers will not allow to be generalized.

Research studies – Summary

Research studies in the field of teacher education and innovation at schools on international level can be divided into at least two main areas: I. Focus on curricula of teacher education; II. Focus on the knowledge a mathematics teacher needs to teach well.

Area I usually includes issues of pre-service teacher education (primary and secondary) and the first years of their teaching practice: for example, structure of teacher education; admission of students into teacher education and their prospective career in the field;

curricula for pre-service mathematics teachers; conditions for novice teachers; preparation of teachers for overcoming obstacles they will come across in their practice; history and development of systems of education in various countries; international comparative studies of teacher education.

The fundamental question related to life-long learning of mathematics teachers and primary teachers is how they can learn for, during and from their teaching practice.

The areas in the spotlight are: What can mathematics teachers learn from their own and other teachers' practice? How do they further develop their knowledge of mathematics and of the ways of teaching mathematics if they work with recordings from teaching practice? How do they learn important information about variety, sociocultural and economic background of their pupils? How is teachers' life-long learning organized? How difficult is it for a teacher to get access to materials such as video recordings, journals, to come to lessons and observe them, etc.?

Research in *Area II* focuses on the knowledge prerequisite to successful teaching of mathematics. International community distinguishes between several types of prerequisite knowledge related to mathematics: the most prominent ones are mathematical content knowledge, MCK, and pedagogical content knowledge, PCK (Shulman, 1986).

There is a lot of discussion on whether MCK and PCK should be regarded as independent of each other or interlinked: For example, should pre-service teachers be taught pedagogical knowledge separately from content knowledge in different courses and seminars or should this be taught simultaneously as pedagogical content knowledge? Much attention is also paid to comparison of experience of novice and experienced teachers.

The turbulent developments in ICT has brought fast development of research focusing on the impact of ICT on teaching mathematics. Knowledge of the potential, advantages and possible risks of using ICT in teaching has become an important part of a teacher's knowledge. ICT supported mathematics education is a complex activity that requires a teacher's deep insight into mathematics, knowledge of a suitable ICT tool and understanding of pupils' thinking processes. That is

why the PCK model was amended by knowledge from the area of technology, the so called TPCK (technology pedagogical content knowledge) (Mishra & Koehler, 2006). Apart from the concept of TPCK, research also focuses on consequences of TPCK for teacher education programmes.

COOPERATION OF TEACHERS AND RESEARCHERS

In the second part of the text, we will focus on one aspect of the relationship between teachers' knowledge, approaches to teaching and beliefs on the one hand and innovation at schools on the other. It is connected to two areas: teachers as researchers and cooperation of teachers, teacher educators and researchers. This theme is not new, for example, a PME working group, *Teachers as Researchers*, first met in 1988, and then was meeting annually for nine years. Its work was based on the belief that classroom teachers could and should carry out research connected to the practice of teaching mathematics. The output of this work is the publication of a book (Zack, Mousley, & Breen, 1997). The attention paid at PME conferences to the topic did not end with this publication. The Plenary Panel at PME 27 focused on the issue "Teachers as researchers" (Novotná, Lebethe, Rosen, & Zack, 2003). The follow-up was organized in various forms: discussion groups, working groups and a research forum (Novotná et al., 2006).

In literature, a lot of attention is paid to the impact of teachers' contact with new educational trends in the development of their knowledge in a variety of ways: organisation of teacher education (pre- as well as in-service), opportunities to experience new approaches, access to appropriate resources, etc. Jaworski (2005) believes that one way to add to the body of knowledge is through 'co-learning partnerships':

The action research movement has demonstrated that practitioners doing research into their own practice [...] learn *in practice* through inquiry and reflection. There is a growing body of research which provides evidence that *outsider* researchers, researching the practice of other practitioners in co-learning partnerships, contribute to knowledge *of* and *in practice* within the communities of which they are a part. (p. 2)

The important issue of teachers as researchers, either cooperating in communities with researchers or doing their own research, is frequently analysed from the perspective of what it adds to the body of knowledge on mathematics education. Less investigated is the issue of what impact this type of teachers' activities has on their beliefs, teaching approaches, their knowledge.

There is no doubt that the cooperation of teachers and researchers is influenced by their pedagogical beliefs and mainly by teachers' reactions to innovative approaches. Hofmannová, Novotná and Hadj-Moussová (2003) investigated how in-service and pre-service teachers react to them.¹ The authors are convinced that without deep changes in teachers' beliefs and attitudes, major changes in pupil learning cannot occur. This corresponds to (Rogers, 1996): "The introduction of learning changes into the area of attitudes is perhaps the most difficult task that faces the teacher educator." The results of the presented research into affective barriers showed prevailing negative attitudes of participating teachers towards new educational trends. The following scheme of categories based on Rogers (1996) was created:

Inner barriers: fear of failing, fear of not meeting the requirements, fear of uncertain success. The identified causes of inner barriers were: changes caused by aging, negative self-concept, too high self-requirements and too positive perception of the others, fatigue.

Outer barriers: lack of time, personal and family problems. The identified causes of outer barriers: inability in time management, too much stress.

These findings had a major impact on the teacher education course because it enabled inclusion of new incentives into the course curricula. These new elements focus on work with teachers' motivation and attitudes. Barriers could thus turn into resources (Moschkovich, 2002).

¹ The innovative approach selected for this study was Content and Language Integrated Learning (CLIL): CLIL refers to any teaching of a non-language subject through the medium of a second or foreign language. CLIL suggests equilibrium between content and language learning.

In the following, one example of collaboration of teachers and researchers with an important input of participating teachers is described. The research project is presented from the point of view of the further development of teachers' beliefs and approaches to mathematics education resulting from the cooperation. It shows one form of research collaboration between university academics and teachers of mathematics. The question in the background is: What are the advantages and limitations of such cooperation?

Impact of teachers' participation in research (Eisenmann, Novotná, Příbyl, & Břehovský, 2015)

This study is a part of a three-year research project GAČRP407/12/1939 *Development of culture of problem solving in mathematics in Czech schools*. The goal of the research project is the development of a theory of mathematics problem solving with a focus on the role heuristic strategies play in the development of *pupils' culture of solving problems* (CSP). CSP is understood as a structure of internal factors that influence a pupil's performance and success in problem solving (Eisenmann, Novotná, & Příbyl, 2014).

In short-term (3 months) and long-term (18 month) experiments, lower and upper secondary pupils were introduced by their teachers to heuristic strategies that they rarely or never came across in usual lessons but that are very effective and useful in problem solving. The pupils were led systematically to the use of a suitable heuristic strategy when they come across a problem they cannot solve using "school solving algorithm" (Eisenmann, Novotná, & Příbyl, 2014; Novotná, Eisenmann, & Příbyl, 2015). The research focused on a number of research questions two of which are connected to the area of teacher education and teacher pedagogical beliefs: Will the experiments have impact on the teachers involved? And what will this impact be?

The research team developed sets of problems that can effectively be solved using one heuristic strategy. All these problems were carefully elaborated and commented upon and can be solved in several ways. Selected problems were also subject to a priori analysis (Nováková, 2013) and were piloted on a one-time basis in non-participating classes.

All participating teachers can be described as committed teachers who invest a lot of energy into their teaching and who had attended in-service teacher

education courses. They were introducing their pupils to the use of heuristic strategies through solving problems for the period of the experiments.

During both types of experiments, the impact on the participating teachers was analysed. The following changes are reported based on interviews and observation data from the collaborative work with the teachers over the period of the whole experiment and on the basis of the analysis of the structured interviews. The teachers (Novotná, Eisenmann, & Příbyl, 2015)

- lowered their demands on accuracy and correctness in their pupils' communication and recording in favour of understanding the problem solving procedures (which does not entirely correspond with the commonly accepted characterization of mathematics as a domain where accuracy and correctness of communication is an important issue),
- showed more tolerance to variety in pupils' solutions,
- acknowledged a change in their teaching towards constructivist and inquiry-based approaches,
- grew more interested in pupils' solving processes while solving problems;
- one of them reported that she started to think how to eliminate the pervasive pupils' sense of failure (e.g., she decided to use group work more often).

One of the most important results is that most of the participating teachers started to pose their own problems with the aim of making the pupils understand the various strategies better.

How is this research study linked with the results presented in (Hofmannová, Novotná, & Hadj-Moussová, 2003)? It contributes to the discussion on the conditions of cooperation between researchers and teachers and on its benefits for the involved teachers. Deeper understanding of the conditions for successful participation of teachers in research offers

teachers the scientific background by providing information about research results which is stimulating and which influences their work.

Subsequently, the movement reversed and teams of researchers and teachers worked together, either in order to create and disseminate tools for improving education (curriculum, materials, recommendations) or to answer the ongoing needs of certain researchers. (Novotná, Brousseau, Bureš, & Nováková, 2012, p. 326)

The cooperation of teachers and researchers in mathematics education represents a broad and relevant topic. The focus is mostly on the improvement of the quality of mathematics teaching and learning (Brown & Coles, 2000). The above presented studies focus on another research area which is a change in behaviour and practices of teachers involved in research in the area of mathematics education.

CONCLUDING REMARKS

The plenary lecture only covers a small part of research on teacher education and its relationship to innovative teaching strategies. Its ambition was not to be (and considering the scope of the issue could never be) exhaustive. As it was already mentioned, there is an immense number of individual and collective monographs on the issue, a great number of national and international conferences, seminars, summer schools, there are journals specializing on mathematics teacher education, for example, the renowned *Journal of Mathematics Teacher Education* (JMTE).

Teacher education is the topic of a number of international projects, for example, the completed project *Teacher Education and Development Study in Mathematics TEDS-M* (Tatto et al., 2012; <http://teds.educ.msu.edu/>) focusing on pre-service teacher education or the currently running project *FIRSTMATH – The First Five Years of Mathematics Teaching* (first-math.educ.msu.edu/) focusing on the first five years of teaching practice of novice teachers. One of the recent events, *ICMI Study 23 Primary Mathematics Study on Whole Numbers*, whose conference took place in June 2015 (Sun, Kaur, & Novotná, 2015) and the Volume is now under preparation, pays considerable attention to research in teacher education – two chapters in the volume focus on this topic.

The presented survey in various resources implies that some of the issues that seemed to be underrepresented at ICME10 in Copenhagen are now much more fully developed (e.g., the issues concerning teacher

educators). However, there are still areas that deserve more attention and research work.

To conclude, let us recapitulate the main and most frequent areas of study in research into teacher education since ICME10: Much attention has been paid to the balance between mathematical content knowledge (MCK) and pedagogical content knowledge (PCK). This area also covers works on knowledge prerequisite to cross curricular teaching of mathematics (inside mathematics or between mathematics and other subjects). Another important issue is the field of professional seeing of (student) teachers of mathematics and the “ability to notice” as integral part of PCK.

Also technology pedagogical content knowledge (TCPK) is one of the subjects of research in mathematics education that have become more prominent. The use of e-learning and b-learning (blended learning) environments in teacher education and teaching practice belong to this field.

School mathematics is based on problem solving. Therefore it comes as no surprise that much attention of research is paid to a teacher’s knowledge prerequisite to the efficient use of various solving strategies when solving mathematics problems (see e.g., Novotná, Brousseau, Bureš, & Nováková, 2012). This is closely related to a teacher’s competence to pose problems.

Another research area that got more attention is the issue of mathematical literacy and numeracy. This research studies the relations between mathematical literacy and mathematics education and tries to define the requirements on the teacher and their knowledge. Important here is the teacher’s mathematical culture and the potential for its development. Some researchers also focus on knowledge and skills prerequisite to the development of pupils’ mathematical culture which has important consequences for research into assessment in mathematics and into didactic approaches to possible learners’ difficulties. All this is in a narrow relationship with the cooperation between researchers and teachers and the further development of mathematics teachers’ beliefs.

The paper started with the author’s personal confession. Let it be concluded in a similar spirit: The three presented examples come from research studies in which the author was involved. They illustrate

different views on the topic of this plenary lecture. I am convinced teacher education and innovation at schools are related to each other, they influence each other and if they are separate, the conditions for their development are much worse. Researchers in mathematics education should always bear in mind whom the research concerns and how to make the teaching community interested in the findings. Either by inviting teachers to participate in the research, its design and activities, or by communicating the findings well to practising teachers, giving them support and getting from them feedback on how the innovation works in real school conditions.

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