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A Game Theoretic Approach for EV Recharge Pricing Under Competition: Analysis and Simulation

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Abstract

Electric Vehicles (EV) are a key element of future smart cities, providing a clean transportation technology and potential benefits for the grid. Nevertheless, limited vehicle autonomy and lack of charging stations are preventing EVs to be broadly accepted. To address this challenge, French GreenFeed project is working to develop an interoperable and universal architecture to allow EV recharge across multiple cities and countries. In this work, we consider such architecture and focus on price setting by its main actors. We show how a Stackelberg game models the market, and we study the outcomes when users choose a recharge station according to objective and subjective parameters. Simulation shows the different actors' revenues, and the social and user welfare for different scenarios.

I. Introduction

Electric Vehicles (EV) and Hybrid Electric Vehicles are expected to dominate the automobile industry in the near future [12]. They present the great advantage of being environmentally friendly, dramatically reducing greenhouse gases emissions with respect to fossil-fuel vehicles [8], while almost eliminating noise pollution. Moreover, EVs are nowadays part of a whole evolutionary energy context. Energy transition is taking place in several countries in order to introduce distributed and renewable energy resources into the grid. Electricity market is also changing into a deregulated market, where time-variant tariffs are introduced, making demand side management solutions possible.

In this context, EVs become also attractive because of the ancillary services they can offer to the grid. They can provide flexibility, by the possibility to shift the battery recharge. They can also provide the grid with the energy stored in their batteries through Vehicle to Grid (V2G) technologies, when energy production is lower than demand, and can store energy when supply exceeds demand.

In spite of the aforementioned advantages, EVs are facing some barriers to their large adoption, such as the so-called range anxiety. This term refers to the fear that the vehicle will not have enough range to reach the destination. With state-of-the-art batteries, vehicle's autonomy is on the average 50 km and it can reach up to 160 km with large batteries [10]. However, these figures may dramatically vary according to driving manner and particular circumstances (e.g. temperature, weight, etc.).

In this context, it is of paramount importance to have ubiquitous, easy and fast means to get the recharge service and to pay for it, regardless the EV model, without problems of interoperability or users' contract. In this sense, industry and research institutions, and standardisation bodies are carrying out efforts to develop electromobility and charging solutions, such as GreenFeed [3], green eMotion [2], standard ISO 15118 [5], the French initiative for EV roaming Gireve [1], or the platform Hubject [4].

Ongoing French project GreenFeed, aims to develop interoperable recharge solutions to foster EVs penetration. The project has defined an architecture, following the standard ISO 15118, that has the following main actors: EV Users (EVU), e-Mobility Provider (EMO), Charging Point Operator (CPO) and e-Mobility Operator Clearing House (EMOCH), as shown in Fig. 1a. Such architecture structures a supply chain market for EV recharge.

This work is part of the outcome of GreenFeed project, and focuses on the problem of setting EV recharge price at the different levels of the supply chain -one of the questions raised by the project. We assume variable recharge costs faced by the mobility providers (EMOs), but a fixed recharge price paid by the final client (EVU). Fixed prices are attractive from the point of view of the EVU, who is then shielded from electricity price volatility. We model the situation as a Stackelberg game, where CPOs play first, setting a price to be paid by the EMO, and where the EMO follows, setting a price for the recharge, which is paid by the final client. In addition, we take into account clients decision about where to get their EV recharged, considering subjective and objective parameters about the CPOs. Our results show interesting insights which could help CPOs and EMOs to set prices, and regulators to evaluate the market structure induced by GreenFeed's architecture. Simulation allow us to show in several scenarios the existence of a Nash equilibrium.

The reminder of this paper is organised as follows. Section II reviews related work. In Section III we introduce the GreenFeed architecture, formally explain the market structure and the problem under study. We then formalise the problem as a Stackelberg

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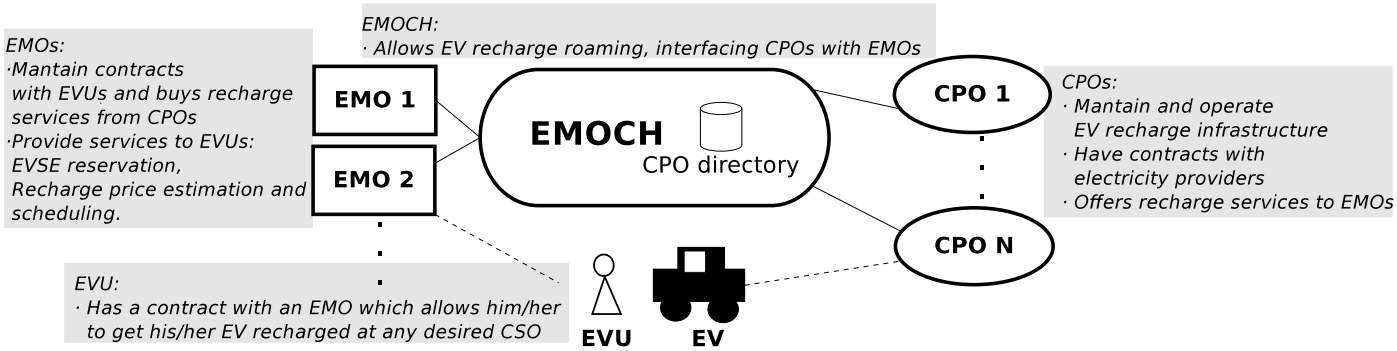
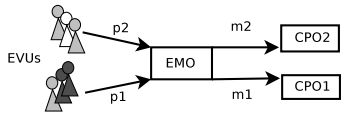
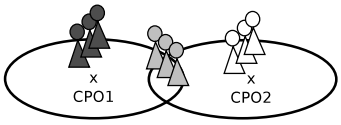


(a) EV-recharge-roaming-enabled architecture.



(b) Market structure: a supply chain model.



(c) Influence zone of each CPO.

Fig. 1: GreenFeed's architecture and market model.

Game (Section V), which we solved through a numerical approach (Section VI). Section VI also presents the results of exhaustive simulative studies in order to evaluate the outcome of the game and the influence of the different parameters of the model. Finally, we conclude in Section VII.

II. Related Work

Our model lays in the category of supply chain situations, where several results have been found in the literature. In particular, in [16] the author studies supply chain situations under Stackelberg game models,to analytically set the prices. This can be done in their situation since they consider a linear demand. In our case, we seek a more realistic demand model, by taking into account choices of the final clients (EV users) which implies subjective and objective parameters. Altogether, the demand expression obtained is not linear, and analytic results can not be derived. This same discrete choice model has been used in [7], where the problem of price setting for Internet access and backbone connection is studied. A similar model is studied in [9], where the authors also consider Internet Content Providers. These market structures are quite similar to ours, though in their case the retailer decides to whom to buy the service, while in our case, is the final user the one deciding to which provider (charging operator in our case) buy the service (EV recharge in our case).

With respect to pricing models in smart grids, several proposals have been done, seeking mainly to implement decentralised demand response actions, such as [14], [11] or [17]. In particular, [17], proposes a Stackelberg game to model the interaction between EV users and the smart grid. In that work, the smart grid is considered as an actor itself, and no other actors in the delivery chain are considered. To the best of our knowledge, none previous works address the case of supply chain nor competition in the context of EV recharge pricing.

III. System Model

We now introduce the architecture, the notations and the assumptions used throughout our analysis.

A. An EV-Recharge-Roaming-Enabled Architecture

As aforementioned, we consider the architecture proposed by GreenFeed project [3] and the nomenclature of standard ISO 15118 [5]. The system architecture is shown in Fig. 1a. The CPO maintains and operates the recharge infrastructure. The EMOCH is an entity mediating between EMOs and CPOs to allow EV recharge roaming. In particular, it standardises communication between different parties and exchanges EVU validation data and recharge pricing data. The EVUs are clients of the EMOs. At the same time, EMOs establish agreements with CPOs, allowing EV recharge at different geographical places. In this work, we shall consider a situation with two CPOs and one EMO, as shown in Fig. 1b.

B. The Competition Model

Figure 1b illustrates the scenario under study. We consider the case of two geographically close CPOs, and one EMO which maintains contracts with the EVUs and makes them possible to recharge their EVs both at CPO_1 and at CPO_2.

In order to model the influence of the distance of an EVU to the charging stations on his/her choice of CPO, we consider two different cases. EVUs that are far away from one of the CPOs will not really consider the further CPO as a possibility to get the EV recharged. EVUs that are close to both CPOs will consider both of them as possible recharge areas. In both cases we also consider that EVUs might choose not to be recharged at all. This logic determines three different areas, an area where EVUs choose either CPO_1 or not to be recharged -thus an area where CPO_1 has the monopoly-; an area where EVUs either choose CPO_2 or not to be recharged -thus CPO_2 has the monopoly-; and a common area, where EVUs consider both CPOs as possible recharge areas, thus where there exists a duopoly. We shall refer to these areas as area 1, 2, and 3 respectively, as illustrated in Fig. 1c. Without lost of generality, we consider that at each of the three differentiated areas there is a total density of EVUs equal to D_i, $i \in \{1, 2, 3\}$.

C. Pricing Schemes

The EMO charges any client (EVU) a fixed price p_k, $k = 1, 2$ for recharging at CPO k. In turn, a CPO k charges the EMO for a recharge o a price $m_{k,o}$, this price can enclose energy costs, infrastructure, operation and maintenance cost, and gains. The mean of all $m_{k,o}$ $\forall o \in O$ is referred as m_k. This model shields the EVUs from energy price volatility, and has the user-side advantage of being simple and predictable, since EVUs pay always the same amount [6].

D. EVUs Preferences

In the problem under study, EVUs at the duopoly (or monopoly) area decide whether to get the EV recharged at CPO $k = 1, 2$ ($k = 1$ or $k = 2$ for each monopoly area), or not to get it recharged at all. They make their choices based on some preferences, and we assume in a rational way. We model EVUs preferences -or utilities- as a two-term function, where one term depends on objective factors (price, quality) and the second term depends on subjective ones, which are modelled through a random variable.

Let n denote an EVU and $k = 0, 1, 2$ a CPO, where $k = 0$ means no recharge, then n's utility when choosing CPO k is given by Eq. (1)

$$u_{n,k} = f_k(x_k, p_k) + \kappa_{k,n}, \tag{1}$$

where $\kappa_{k,n}$ is a realisation of a random variable with Gumbel distribution. The Gumbel distribution is usually assumed in discrete choice modelling. In particular, it has the realistic property of independent of irrelevant (see e.g. [15]).

Regarding the objective term, $f_k(\cdot)$, we define it as a function of the price paid (p_k) and of the quality of the CPO (x_k), which models the type of recharge offered (normal, fast). In particular, following [7] we use a logarithmic relationship between these two parameters, which has been claimed to be quite representative of human perception. EVU n's utility when choosing CPO k is finally defined as in Eq. (2).

$$u_{n,k} = \alpha \cdot log\left(\frac{x_k}{p_k}\right) + \kappa_{k,n}, \tag{2}$$

with $\kappa_{k,n} \sim$ Gumbel (see e.g. [15]), and α a sensitivity parameter.

In the case of no recharge (i.e. $k = 0$), we shall assume an equivalent quality x_0 and an equivalent price of p_0. This could be interpreted as the direct price and quality of an at-home recharge.

IV. Demand and Revenues

The model being introduced, we are now able to compute the demand of recharges at each CPO and total demand at the EMO. Once demand is determined, and assuming the pricing schemes introduced in Subsection III-C, we compute the total CPOs' revenue and EMO's revenue.

Indeed, since EVUs are considered rational, each EVU n on the duopoly zone chooses provider j, $j \in \{0, 1, 2\}$ instead of provider $i \neq j$ if and only if $u_{n,j} \geq u_{n,i}$, and analogously for an EVU on a monopoly zone. We can then compute the probability that an EVU in a monopoly zone chooses CPO k, $k \in \{1, 2\}$ over no recharge, which can be shown to be as in Eq. (3) (see e.g. the appendix for specific calculations).

$$\sigma_k^M = \frac{(x_k/p_k)^\alpha}{(x_k/p_k)^\alpha + (x_0/p_0)^\alpha}. \tag{3}$$

Analogously, the probability that an EVU in the duopoly zone chooses CPO $k \in \{1, 2\}$ over $i \neq k$ or no recharge is given by Eq. (4).

$$\sigma_k^D = \frac{(x_k/p_k)^\alpha}{(x_k/p_k)^\alpha + (x_i/p_i)^\alpha + (x_0/p_0)^\alpha}. \tag{4}$$

Considering that densities of EVUs at each area are known, we can now compute the total demand at CPO $k = 1, 2$, which is given by Eq. 5:

$$\begin{aligned} d_k(p) &= D_k \cdot \sigma_k^M + D_3 \cdot \sigma_k^D = \\ &= D_k \cdot \frac{(\frac{x_k}{p_k})^\alpha}{(\frac{x_k}{p_k})^\alpha + (\frac{x_0}{p_0})^\alpha} + D_3 \cdot \frac{(\frac{x_k}{p_k})^\alpha}{\sum_{l=0}^{2} (\frac{x_l}{p_l})^\alpha}, \end{aligned} \tag{5}$$

and total demand at the EMO, given in Eq (6).

$$\Delta(p) = \sum_{k=1,2} d_k(p). \tag{6}$$

We now compute CPOs and EMO's revenues. CPO k's revenue is given by the sum of the prices paid by all the recharges that have been performed at k. This is the same as the total demand at k (number of served recharges) times the average price per recharge. Let us note CPO k's revenue as $\Gamma_k(p, m)$, $k = 1, 2$, its mathematical expression in terms of the parameters of our problem is shown in Eq. (7). We recall that for the sake of notation's simplicity recharge-associated costs are assumed to be already considered in m_k.

$$\begin{aligned} \Gamma_k(p, m) &= d_k(p_k) \times m_k = \\ &= \left(\frac{D_k \cdot (\frac{x_k}{p_k})^\alpha}{\sum_{j=0,k} (\frac{x_j}{p_j})^\alpha} + \frac{D_3 \cdot (\frac{x_k}{p_k})^\alpha}{\sum_{j=0,1,2} (\frac{x_j}{p_j})^\alpha} \right) \cdot m_k \end{aligned} \tag{7}$$

Analogously, EMOs revenue is given by the number of EVUs (total demand) times the fixed price charged to each of them, minus the sum of all the variable prices paid by the EMO to the CPOs, per recharge. This is the same as the number of EVs served times the difference of the fixed price paid by the EVUs minus the mean price paid by the EMO to the CPOs. Let us denote EMO's revenue as R, then its mathematical expression is given by Eq. (8).

$$\begin{aligned} R(p, m) &= \sum_{k=1,2} d_k(p_k) \times (p_k - m_k) = \\ &= \sum_{k=1,2} \left(\frac{D_k \cdot (\frac{x_k}{p_k})^\alpha}{\sum_{j=0,k} (\frac{x_j}{p_j})^\alpha} + \frac{D_3 \cdot (\frac{x_k}{p_k})^\alpha}{\sum_{j=0,1,2} (\frac{x_j}{p_j})^\alpha} \right) \cdot (p_k - m_k). \end{aligned} \tag{8}$$

V. Game Formulation

The market structure of our problem leads us to model it as a Stackelberg game. This kind of game has been widely used in the literature to model supply chain situations, see for instance [16].

In a two sided Stackelberg game, introduced by von Stackelberg in 1934 [18], there is one player (leader) which acts first selecting his/her action, and another player (the follower) that acts later, and selects his/her action knowing the leader's choice. Typically, a solution to the Stackelberg game is found by the backward induction method, meaning that the reaction of the follower is solved first, as a function of the leader's action, and then the leader's move is computed assuming he/she knows what the reaction of the follower would be. Both leader and follower are assumed to behave rationally, and thus seek to maximise their own utilities.

In our case we consider CPOs as leaders, and the EMO as a follower. In addition, on a third turn, EVUs act also as followers choosing a CPO where to get their EVU recharged at. The analysis could be carried out exactly in the same fashion if we were to consider the EMO as a leader.

A. The Pricing Game

Previous definitions allow us to formally define the Pricing Game, given by a Stackelberg game, with the following characteristics:

- The leaders are the CPOs and the EMO is a follower
- The leader k's set of available actions is $\{m_k : m_k \in \mathbb{R}^+\}$, $k = 1, 2$
- The follower's set of available actions is $\{p = (p_1, p_2) \in \mathbb{R}^2_+\}$
- Each leader's utility is given by Eq. (7) and the follower's utility is given by Eq. (8)

Please note that the EVUs actions are taken into account through the demand at each CPO.

B. Nash Equilibrium as a Solution Concept

We are interested in the Nash equilibrium as a solution to our pricing game, since it is a stable outcome of the market structure. In such situation, given by $S = (p_1^*, p_2^*, m_1^{eq}, m_2^{eq})$, no player can increase his/her revenue by unilaterally changing his/her own price. Following the backward induction method, in order to find the solution we first maximise EMO's revenue, finding $p_l^*(m)$, for $l = \{1, 2\}$. Secondly, we inject the result into CPO's revenue, and solve the equilibrium among CPOs, by finding a crossing point of their best responses. Unfortunately, our model can not be solved analytically, since it is not possible to find an explicit function $p_l(m)$. However, we provide a numerical solution, which is presented in next section.

The existence of the equilibrium in several scenarios has been found by simulation. The results of these simulative studies are presented in Section VI.

C. Users and Social Welfare

Let us now compute the different actors' welfare, which will allow us to evaluate the market structure. With respect to the EMO's and CPOs' welfare, they can be straightforward obtained as equal to their revenues ($\Gamma - 1, \Gamma_2$ and R). As for the users welfare, we define it, as usual, as the aggregated benefit the users get from the market (see e.g; [15]), with respect to a certain reference outcome. In particular, we consider as a reference outcome the no charge case, and proceed similar to [**?**]. In the no recharge case, user n's utility is given by:

$$u_{n,0} = \alpha \cdot log\left(\frac{1}{p_0}\right) + \kappa_{n,0}. \tag{9}$$

Since we assume EVU n is rational, he/she chooses CPO j if Inequality (10) and (11) hold.

$$u_{n,j} \geq u_{n,i},\ i \neq j. \tag{10}$$

$$u_{n,j} \geq u_{n,0}. \tag{11}$$

Thus, the surplus of EVU n in the duopoly area is:

$$S_3 \equiv \max(0, u_{n,1} - u_{n,0}, u_{n,2} - u_{n,0}). \tag{12}$$

Analogously, the surplus of EVU n in the CPO k monopoly area, for $k = 1, 2$ is:

$$S_k \equiv \max(0, u_{n,k} - u_{n,0}). \tag{13}$$

We define EVUs welfare as the expected outcome taken from the market structure by all EVUs (see e.g. [15]):

$$UW = D_1 \cdot E[S_1] + D_3 \cdot E[S_3] + D_2 \cdot E[S_2], \tag{14}$$

which is equal to Eq. (15). Calculations can be seen in the appendix.

$$UW = \sum_{k=1,2} D_k \cdot log\left(1 + \sum_{l=0,k} (\frac{x_l}{p_l})^\alpha\right) + \\ + D_3 \cdot log\left(1 + \sum_{l=0,1,2} (\frac{x_l}{p_l})^\alpha\right). \tag{15}$$

We can now define the social welfare as the aggregation of all involved actors' welfare. Since EVUs welfare is not necessarily expressed in a monetary unit, we introduce a conversion factor given by $\lambda > 0$. Finally, social welfare SW is expressed by Eq. (16).

$$SW = \Gamma_1 + \Gamma_2 + R + \lambda \cdot UW \tag{16}$$

VI. Simulation Studies

While it is not possible to analytically solve the pricing game, we describe in this section a numerical approach. We then use this approach to evaluate the market structure under different scenarios.

A. Numerical Approach

In order to obtain the Nash equilibrium of the pricing problem, we have implemented the backward induction method, for a discretized set of prices. The procedure is described in Pseudo-code VI.1, where the discretized set of prices for CPO k, $k = 1, 2$ is referred to as M_k.

Pseudo-code VI.1 Nash Equilibrium calculation.

1) Let M_j, $j = 1, 2$ be a finite set of real positive numbers of size $|M_j|$.
2) Let $m_j^{k_j} \in M_j$, for $k_j = \{1, \ldots, |M_j|\}$, $j = \{1, 2\}$.
3) Define a grid with elements $m^h = (m_1^{k_1}, m_2^{k_2})$, $h = 1 \ldots |M_1| \times |M_2|$.
4) For $h = 1 \ldots |M_1| \times |M_2|$ do
   a) Find the follower's optimal actions $p^*(m^h) = (p_1^*(m^h), p_2^*(m^h)) = \text{argmax}(R(p, m))$.
   b) Define $\Gamma_j^h = \Gamma(m^h, p^*(m^h))$, $j = 1, 2$.
5) Compute the leaders' optimal action $m^* = (m_1^{eq}, m_2^{eq})$.
6) Determine the follower's optimal action $p^*(m^{eq})$

Following the backward induction method, we first obtain $p^*(m^h)$, the maximizer of the EMO's revenue. For doing so, we use the simulated annealing method, where the objective function is given by Eq. (8). Then, the set of actions m^h, $h = 1 \ldots |M_1| \times |M_2|$, along with $\Gamma_j^h(m^h, p^*(m^h))$ determine a game in normal form (see e.g. [13]). The CPOs compete, and the output will then be the Nash equilibrium of that game. It is calculated as usual by searching the best response for each action, and finding a set of actions (m_1^{eq}, m_2^{eq}), where m_1^{eq} and m_2^{eq} constitute best responses to CPO_1 and CPO_2 respectively. Finally, the solution of the pricing problem is given by $(p^*(m^{eq}), m^{eq})$.

B. Numerical Results

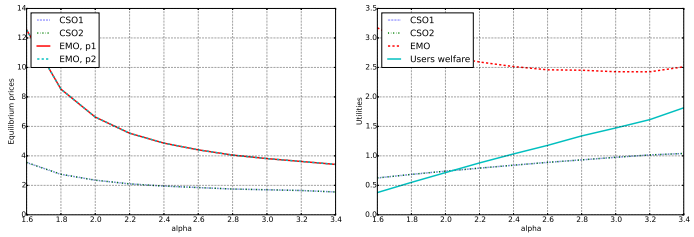
We now show the results of different simulation studies. We consider some symmetric scenarios, through which we can validate the correctness of our implementation, and some asymmetric ones in order to evaluate the market structure under different situations. We also perform simulations aiming to evaluate the impact of the different parameters of the model, as price sensitivity, no recharge price, and recharge quality.

The Influence of the Sensitivity Parameter α: Parameter α models the EVU's sensitivity to prices. We first consider a symmetric scenario, where density values are set to $D_i = 1$, $i = 1, 2, 3$ and quality is set to $x_k = 1$, $k = 0, 1, 2$ (both CPOs propose the same recharge quality) Parameter $p0$, we recall, models the price of not recharging the EV. We shall thus assume that in case of not recharging the EV, the EVU recharges it eventually at home, paying for the electricity and infrastructure. We first set $p0$ according to real data of electricity prices in France, and considering an amount of energy equal to $20kWh$ per recharge. This gives a value of $p0 = 3$.

For values of α smaller than or equal to 1.5 simulation results have shown no equilibrium exists. For values of α greater than 1.5 equilibrium prices and social welfare results are shown in Fig. 3a. We can observe, that at equilibrium both CPOs set the same mean recharge price, which is quite intuitive since scenario set-up is symmetric. As expected according to intuition, prices drop when EVUs price sensitivity increases. Each actors' welfare, as defined in Subsection V-C, at equilibrium is shown in Fig. 3b. Results show that the EMO receives a larger margin with respect to the CPOs welfare, and even more so when sensitivity is low. This can be explained by the fact that the EMO has the monopoly in the region, while CPOs have to share some part of the market.

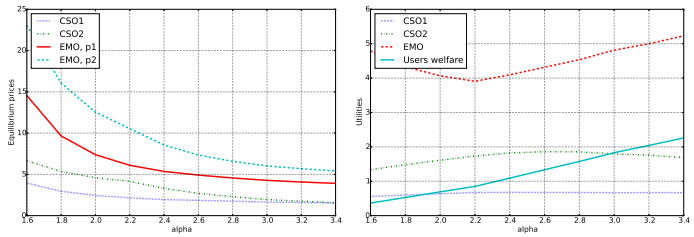
We now consider an asymmetric case given by one CPO that offers a higher quality than the other CPO. We thus set $x_1 = 1$ and $x_2 = 2$, $p0 = 3$ as before. Once again, for values of α smaller than 1.5, no equilibrium was found. For values of α greater than 1.6 equilibrium prices, shown in Fig. 3a, show that the higher the quality offered, the higher the price. Welfare results are shown in Fig. 3b. Once more the EMO gets more benefice from the market than the CPOs do. The CPO providing the greatest quality, i.e. CPO 2, obtains more revenue than the competitor CPO.

The influence of the quality parameter: We now set $p_0 = 3$ and $\alpha = 2.5$, and compute the equilibrium prices under an asymmetric scenario given by a CPO providing a normal recharge ($x_1 = 1$) and a CPO providing a better recharge ($x_2 = 1 \ldots 4$). Results can be seen in Fig. 4. Equilibrium prices show a linear increasing behavior with respect to the quality of CPO 2. CPO 1's equilibrium price remains constant. However, regarding welfare, results show that the welfare of CPO 1 decreases when CPO 2's quality increases. This could provide the right incentives to CPOs to remain competitive in the offered recharge quality.



(a) Equilibrium prices. (b) Welfare.

Fig. 2: Equilibrium outcomes for different values of α, symmetric scenario.



(a) Equilibrium prices. (b) Welfare.

Fig. 3: Equilibrium outcomes for different values of α, symmetric scenario.

The Influence of the no-recharge price parameter p_0: We now set $\alpha = 2.5$ and evaluate results under different values of parameter p_0. Simulation results are show in Fig. 5 for a symmetric case where both CPOs offer the same recharge quality (we set $x_i = 1$, $i = 1, 2$). Results show a symmetric outcome for both CPOs with increasing equilibrium prices. Welfare is as well increasing with $p0$ for the EMO and for CPOs. However, it is quasi-constant for users.

We now consider an asymmetric setting, where $x_1 = 1$ and $x_2 = 2$. Simulation results are shown in Fig. 6. Equilibrium prices increase with p_0, as well as EMO's and CPOs' utilities. User welfare appears constant with p_0. We observe the intuitive result that the CPO offering an higher quality (CPO 2) presents a higher equilibrium price than the CPO offering a lower recharge quality (CPO 1).

VII. Conclusion

In this work we have considered the EV recharge roaming enable architecture proposed by French project GreenFeed and studied it from the point of view of the market structure it defines. We have modelled the situation as a Stackelberg game, where electro-Mobility operators are the Followers and Charging Point Operator are the leaders. EV Users choice of recharge station has been modelled as a discrete choice, considering objective parameters such as price and quality of recharge as well as subjective ones, such as personal preferences. We have considered a logarithmic relation between quality and price in order to model users preferences, which has been claimed to be quite representative of human perception. However, this choice leads to non-linear demand functions and thus makes not possible to compute an analytic solution. Nonetheless, we have provided a numerical approach, which shows the existence of a Nash Equilibrium in different scenarios. Results of simulations show that the actor obtaining more utility from this market structure is the EMO, even being a follower. This can be due to the fact that the EMO is in a monopolistic situation. In future work, we would like to study the interdependence of subsequent choices, given for instance by EVUs sensitivity about CPOs' reputation. Repeated games are likely to model the problem. In addition, we would like to analyse situations with several EMOs, and the case where the EMOCH, charges for its services.

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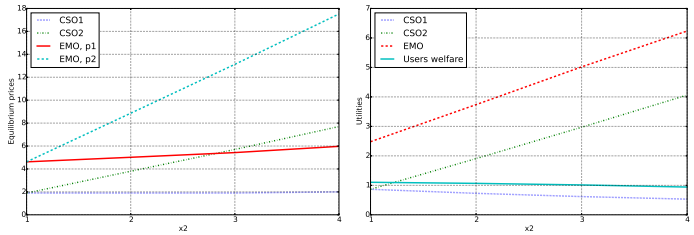
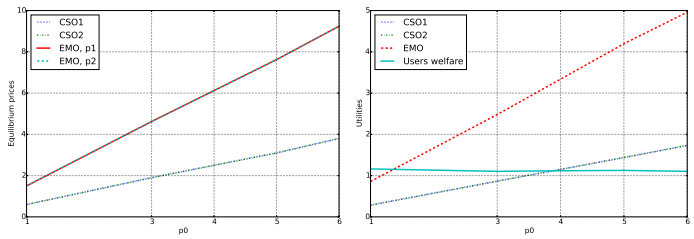


(a) Equilibrium prices (b) Welfare at equilibrium

Fig. 4: Equilibrium results for different values of CPO_2's quality.



(a) Equilibrium prices (b) Welfare at equilibrium

Fig. 5: Equilibrium results for different values of $p0$ for a symmetric scenario.

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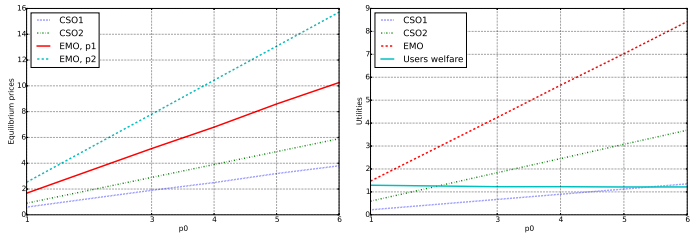

(a) Equilibrium prices (b) Welfare at equilibrium

Fig. 6: Equilibrium results for different values of $p0$ for an asymmetric scenario.

APPENDIX

Demand Calculations

To simplify notations, let $f_k(x_k, p_k) = \alpha \cdot log\left(\frac{x_k}{p_k}\right)$. In the duopoly zone, EVU n chooses provider $j = 0, 1, 2$ instead of $i = 0, 1, 2$, $i \neq j$ if $u_{n,j} \geq u_{n,i}$, $\forall i \neq j$. Thus, we can compute the probability σ_j^D of a user choosing provider j as:

$$\begin{aligned}
\sigma_j^D &= P(u_{n,j} \geq u_{n,i}) = P(f_j + \kappa_{n,j} > f_i + \kappa_{n,i}), \forall i \neq j \\
&= P(\kappa_{n,i} \leq \kappa_{n,j} + f_j - f_i), \forall i \neq j \\
&= \int_{-\infty}^{+\infty} P(\kappa_{n,i} \leq \kappa_{n,j} + f_j - f_i | \kappa_{n,j}) \cdot g(\kappa_{n,j}) d\kappa_{n,j}, \forall i \neq j \\
&= \int_{-\infty}^{+\infty} \prod_{i \neq j} P(\kappa_{n,i} \leq \kappa_{n,j} + f_j - f_i | \kappa_{n,j}) \cdot g(\kappa_{n,j}) d\kappa_{n,j},
\end{aligned}$$

where the fourth equality holds thanks to the independence among the $\kappa_{n,i}$, $\forall i$ and $g(\cdot)$ is the probability density function of $\kappa_{n,i}$.

Since κ_i is Gumbel distributed we have:

$$\sigma_j^D = \int_{-\infty}^{+\infty} \prod_{i \neq j} e^{-e^{-(\kappa_j + f_j - f_i)}} \cdot e^{-\kappa_j} \cdot e^{-e^{\kappa_j}} d\kappa_j$$

We can rewrite the expression by rearranging $e^{-e^{-\kappa_j}}$ into the productory and considering the case $i = j$, that is:

$$\begin{aligned}
\sigma_j^D &= \int_{-\infty}^{+\infty} \prod_i e^{-e^{-(\kappa_j + f_j - f_i)}} \cdot e^{-\kappa_j} d\kappa_j \\
&= \int_{-\infty}^{+\infty} exp(\sum_i -e^{-(\kappa_j + f_j - f_i)}) \cdot e^{-\kappa_j} d\kappa_j \\
&= \int_{-\infty}^{+\infty} exp(-\sum_i e^{-\kappa_j} \cdot -e^{(f_j - f_i)}) \cdot e^{-\kappa_j} d\kappa_j.
\end{aligned}$$

Defining $C = \sum_i e^{-(f_j - f_i)}$ and making the change of variable $t = e^{-\kappa_j}$, which implies $dt = -e^{-\kappa_j} d\kappa_j$, we obtain:

$$\sigma_j^D = \int_{+\infty}^{0} e^{-t \cdot C} dt = \frac{e^{-t \cdot C}}{C}\Big|_{+\infty}^{0} = \frac{1}{C} = \frac{1}{\sum_i e^{-(f_j - f_i)}} = \frac{1}{\sum_i e^{-\alpha \cdot log \frac{x_j}{p_j}} \cdot e^{\alpha \cdot log \frac{x_i}{p_i}}} = \frac{(\frac{x_j}{p_j})^\alpha}{\sum_i (\frac{x_i}{p_i})^\alpha}$$

The result comes directly from calculations of the above integral, details can be seen in [15].

User Welfare Calculations

By definition, if $P[Z_k \geq 0] = 1$, $k = 1, 2, 3$ then $E[Z_k] = \int_0^\infty (1 - P[Z_k \leq z]dz)$. We shall use this property, and first show that for $z > 0$ any that:

$$\begin{aligned} P[Z_1 \leq z] =& P[u_1 - u_o \leq z] = P[f_1 - f_0 + \kappa_1 - \kappa_0 \leq z] = \\ =& P[\kappa_1 \leq z + f_0 + f_1 + \kappa_0] = \\ =& \int_0^\infty P[\kappa_1 \leq z + f_0 - f_1 + \kappa_0 | \kappa_0] \cdot g(\kappa_0) d\kappa_0, \end{aligned}$$

where $g(\cdot)$ is the Gumbel probability density function. Since κ_0 and κ_1 are Gumbel distributed, we have that:

$$\begin{aligned} P[Z_1 \leq z] =& \int_0^\infty e^{-e^{-(z+f_0-f_1+\kappa_0)}} \cdot e^{-\kappa_0} \cdot e^{-e^{-\kappa_0}} d\kappa_0 = \\ =& \int_0^\infty e^{-e^{-(z+f_0-f_1+\kappa_0)}+e^{-\kappa_0}} \cdot e^{-\kappa_0} d\kappa_0 = \\ =& \int_0^\infty e^{-e^{-\kappa_0} \cdot (-e^{(z+f_0-f_1)}+1)} \cdot e^{-\kappa_0} d\kappa_0 = \end{aligned}$$

We define $t = e^{-\kappa_0}$ and $A = e^{-(z+f_0+f_1)} + 1$, then $dt = -e^{-\kappa_0}$ and we have:

$$P[Z_1 \leq z] = \int_0^\infty e^{-t \cdot A} dt = \frac{e^{-t \cdot A}}{-A}|_0^\infty = \frac{1}{A}$$

We can know calculate $E[Z_1]$ as:

$$\begin{aligned} E[Z_1] =& \int_0^\infty 1 - P[Z_1 \leq z] dz = \int_0^\infty 1 - \frac{1}{e^{-(f_0-f_1+z)} + 1} dz \\ =& \int_0^\infty \frac{e^{-(f_0-f_1) \cdot e^{-z}}}{e^{-(f_0-f_1)} \cdot e^{-z} + 1} dz \end{aligned}$$

We define $B = e^{-(f_0-f_1)}$ and we introduce the change of variable $t = e^{-z}$ so $-e^{-z}dz = dt$ obtaining:

$$\begin{aligned} E[Z_1] =& \int_0^\infty \frac{-B}{Bt+1} dt = -\, log(1 + Bt)|_1^0 = log(1 + B) \\ =& \log(1 + e^{-f_0} \cdot e^{f1}) = \log(1 + (\frac{x_0}{p_0})^{-\alpha} (\frac{x_1}{p_1})^\alpha). \end{aligned}$$