Energy-Efficient Resource Allocation for Downlink in LTE Heterogeneous Networks

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***Abstract*—We investigate the energy-efficient resource allocation problem for the downlink in long-term evolution heterogeneous networks through maximizing the energy efficiency (EE) under the per-user throughput and per-eNB power constraints in this paper. We demonstrate that EE is an increasing function in channel gain, and that EE is continuously differentiable and strictly quasiconcave in transmit power associated with each resource block (RB). Due to the non-convexity of the optimization problem, we develop a two-step resource allocation scheme composed by RB allocation and transmit power control. In the first step, we allocate RBs to users through maximizing the minimum EE of individual user and satisfying the throughput requirement of each user. In the second step, by enforcing the per-user throughput and per-eNB power constraints and exploiting the strict quasiconcavity of EE in transmit power associated with each RB, the power control algorithm is developed to maximize the EE. Simulation results demonstrate the effectiveness of the proposed resource allocation scheme and show that it greatly improves the EE compared with conventional spectral-efficient scheme.**

I. Introduction

During the past decades, many efforts have been made to enhance the system capacity and spectral efficiency (SE) to deal with the increasing demands for better quality of service (QoS) and explosive growth of high-data-rate applications. Due to steadily rising energy costs and environmental concerns, energy efficiency (EE) has also caught more and more attention recently [1]. Unfortunately, EE and SE do not always coincide. On the contrary, improving SE directly leads to an increase in energy consumptions in many situations [1]–[3]. It is shown that EE is strictly quasiconcave in SE [2].

Usually, EE is defined as the ratio of the data rate and total power consumption [3], [4]. Recent research works have shown that deploying small cells jointly with macro cells to form heterogeneous networks can achieve improved EE as well as increased throughput [5]. However, the dense and random deployments of small cells and their uncoordinated operations raise some issues on how to deploy small cells in a green manner such that the global network is spectral-efficient as well as energy-efficient. Actually, there exists a proper density and cell size for small cells to be deployed in the heterogenous scenario [6]. To allow the conflicting demands of SE, energy consumptions, and fairness to be tailored to meet specific performance goals or policies, an operator with a "control knob" was included in the joint optimization-based resource allocation process in [7]. In [8], an improved water-filling power allocation algorithm was proposed to achieve the maximum EE in heterogeneous network. In [9], the energy-efficient resource allocation problem was transformed into an equivalent form and solved by an iterative algorithm. However, the considered single-cell downlink heterogeneous network in [9] only includes one small cell base station, which conflicts with the common sense that in a heterogeneous network multiple small cells complement one macro cell to support the growing demands for mobile broadband communications.

In this paper, we formulate the energy-efficient resource allocation problem for the downlink in long-term evolution (LTE) heterogeneous networks in a form of nonlinear fractional programming, and decompose it into two subproblems: resource block (RB) allocation and transmit power control. We first allocate each user with one RB by maximizing the minimum EE of individual user, and then iteratively assign the RBs aiming at maximizing the minimum ratio between actual throughput and required throughput of individual user. The initial transmit power is derived based on water-filling method to satisfy the throughput requirement of each user with minimum transmit power. By enforcing the per-eNB power constraint and exploiting the strict quasiconcavity of EE in transmit power, the transmit power associated with each RB is adjusted to maximize the EE.

The rest of the paper is organized as follows. In Section II, we first describe the downlink LTE heterogeneous networks, and then formulate the related energy-efficient resource allocation problem. In Section III, a two-step energy-efficient resource allocation scheme is developed. Simulation results that demonstrate the effectiveness of the proposed scheme are presented in Section IV, which is followed by the conclusions in Section V.

II. System Model and Problem Formulation

We consider a single-cell downlink LTE heterogeneous network consisting of one macro eNB (MeNB) and N small cell eNBs (SeNBs). Let $\mathcal{N} = \{0, 1, 2, \cdots, N\}$, $\mathcal{M} = \{1, 2, \cdots, M\}$, and $\mathcal{K} = \{1, 2, \cdots, K\}$ denote the sets of eNBs, users, and RBs, respectively. Here, eNB 0 denotes the MeNB, and eNB n, $1 \le n \le N$, denotes the nth SeNB. We assume that one RB is exclusively assigned to at most one user to avoid intra-cell interference among different users.

Denoting the transmit power of eNB n to user m on RB k as $p_{n,m,k}$ and combining the received signal replicas on the same RB from different eNBs using maximal ratio combining (MRC), the achievable data rate of user m is

$$R_m = B \sum_{k \in \mathcal{K}_m} \log_2 \left(1 + \sum_{n=0}^{N} \frac{\delta_{n,m,k} p_{n,m,k} |h_{n,m,k}|^2}{N_0 B} \right) \quad (1)$$

where B is the bandwidth of RB, $\mathcal{K}_m$ denotes the set of RBs allocated to user m, $h_{n,m,k}$ is channel coefficient between eNB n and user m on RB k, RB allocation indictor $\delta_{n,m,k} \in \{1, 0\}$ indicates whether eNB n transmits signal to user m on RB k ($\delta_{n,m,k} = 1$) or not ($\delta_{n,m,k} = 0$), and N_0 is the variance of complex additive white Gaussian noise. In the following sections of this paper, we use $\psi = [\delta_{n,m,k}]_{(N+1)\times M \times K}$ and $\mathcal{P}_t = [p_{n,m,k}]_{(N+1)\times M \times K}$ to denote any possible RB allocation indictor matrix and transmit power matrix, respectively.

For downlink transmissions, the total power consumption contains the power consumption of radio frequency power amplifiers and that of other circuits incurred by signal processing and active circuit blocks [2]. The circuit power consumption can be divided into two parts: static part and dynamic part proportional to the total throughput [10], namely

$$P_c = P_s + \xi R \quad (2)$$

where P_s is the static term, ξ is a constant denoting dynamic power consumption per unit throughput, and $R = \sum_{m=1}^{M} R_m$ is the total throughput. As such, the total power consumption is given by

$$P = \varsigma P_t + P_s + \xi R \quad (3)$$

where $P_t = \sum_{n=0}^{N} \sum_{m=1}^{M} \sum_{k=1}^{K} \delta_{n,m,k} p_{n,m,k}$ is the total transmit power, and ς is the reciprocal of drain efficiency of power amplifier. Obviously, the power consumption model in [4], [8], [9] is a special case of the above model with $\xi = 0$. Based on (1) and (3), the EE is given by

$$\eta_{EE} = \frac{R}{\varsigma P_t + P_s + \xi R}. \quad (4)$$

In order to obtain the maximum EE while guaranteeing the QoS of each user with limited bandwidth and transmit power, the downlink resource allocation in LTE heterogeneous networks is formulated as

$$\max_{\psi, \mathcal{P}_t} \quad \eta_{EE} \quad (5)$$

subject to

$$\sum_{m=1}^{M} \delta_{n,m,k} \le 1, \forall n \in \mathcal{N}, \forall k \in \mathcal{K} \quad \text{(6a)}$$

$$R_m \ge R_m^{th}, \forall m \in \mathcal{M} \quad \text{(6b)}$$

$$p_{n,m,k} \in [0, p_n^{\max}], \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K} \quad \text{(6c)}$$

$$\sum_{m=1}^{M} \sum_{k=1}^{K} \delta_{n,m,k} p_{n,m,k} \le p_n^{\max}, \forall n \in \mathcal{N} \quad \text{(6d)}$$

where R_m^{th} and $p_n^{\max}$ represent the minimum required throughput of user m and the maximum allowed downlink transmit power of eNB n, respectively. Constraint (6a) is in line with the above assumption that one RB is exclusively assigned to at most one user. Constraints (6b) and (6d) enforce the per-user throughput requirement and per-eNB maximum allowed transmit power, respectively.

For some specific cases, it is possible that (5) does not have any feasible solution due to the conflicting constraints of user throughput requirement and limited eNB transmit power. In this case, we can adjust the constraints, e.g., decrease some users' throughput requirements according to their priority to make (5) feasible.

Algorithm 1 RB Allocation Algorithm

Initiate $\delta_{n,m,k} \leftarrow 0$, $\forall n \in \mathcal{N}$, $\forall m \in \mathcal{M}$, $\forall k \in \mathcal{K}$, the RB set associated with eNB n $\mathcal{K}_n \leftarrow \phi$, $\forall n \in \mathcal{N}$, and the RB set associated with user m $\mathcal{K}_m \leftarrow \phi$, $\forall m \in \mathcal{M}$
Initiate user set $\mathcal{M}' \leftarrow \mathcal{M}$ and RB set $\mathcal{K}' \leftarrow \mathcal{K}$
while $\mathcal{M}' \neq \phi$
  for each user $m \in \mathcal{M}'$
    $|h_m^{\max}|^2 \leftarrow \max_{n \in \mathcal{N}, k \in \mathcal{K}'} |h_{n,m,k}|^2$
  end for
  $m^* \leftarrow \arg\min_{m \in \mathcal{M}'} |h_m^{\max}|^2$
  $\{n^*, k^*\} \leftarrow \arg\max_{n \in \mathcal{N}, k \in \mathcal{K}'} |h_{n,m^*,k}|^2$
  $\delta_{n^*,m^*,k^*} \leftarrow 1$
  $\mathcal{K}_{n^*} \leftarrow \mathcal{K}_{n^*} + \{k^*\}$, $\mathcal{K}_{m^*} \leftarrow \mathcal{K}_{m^*} + \{k^*\}$
  $\mathcal{M}' \leftarrow \mathcal{M}' - \{m^*\}$, $\mathcal{K}' \leftarrow \mathcal{K}' - \{k^*\}$
end while
while $\mathcal{K}' \neq \phi$
  for each eNB $n \in \mathcal{N}$
    $P_n \leftarrow \frac{P_n^{\max}}{\max\{1, |\mathcal{K}_n|\}}$
  end for
  for each user $m \in \mathcal{M}$
    $R'_m \leftarrow B \sum_{k \in \mathcal{K}_m} \log_2 \left(1 + \sum_{n=0}^{N} \frac{\delta_{n,m,k} p_n |h_{n,m,k}|^2}{N_0 B} \right)$
    $\kappa_m \leftarrow R'_m / R_m^{th}$
  end for
  $m^* \leftarrow \arg\min_{m \in \mathcal{M}} \kappa_m$
  $\{n^*, k^*\} \leftarrow \arg\max_{n \in \mathcal{N}, k \in \mathcal{K}'} |h_{n,m^*,k}|^2$
  $\delta_{n^*,m^*,k^*} \leftarrow 1$
  $\mathcal{K}_{n^*} \leftarrow \mathcal{K}_{n^*} + \{k^*\}$, $\mathcal{K}_{m^*} \leftarrow \mathcal{K}_{m^*} + \{k^*\}$
  $\mathcal{K}' \leftarrow \mathcal{K}' - \{k^*\}$
end while

III. Energy-Efficient Resource Allocation

In this section, we focus on the EE optimization. The optimization problem of (5) requires a non-convex fractional programming with non-linear constraints, and is in general NP-hard for the optimal solution [4]. To obtain solutions with reasonable complexity, we decompose this problem into two sub-problems: RB allocation and transmit power control, namely, determining ψ and $\mathcal{P}_t$ successively.

A. RB Allocation

The following lemma illustrates the principle of RB allocation for the downlink transmissions in LTE heterogeneous networks from the EE perspective.

Lemma 1: The EE, η_{EE}, is a strictly monotonically increasing function of channel gain $|h_{n,m,k}|^2$. With no constraint on the maximum transmit power, $\delta_{n,m,k} \in \psi$ shall satisfy the following restriction for downlink transmissions in LTE heterogeneous networks from the EE perspective.

$$\sum_{n=0}^{N}\sum_{m=1}^{M} \delta_{n,m,k} \le 1, \forall k \in \mathcal{K}. \tag{7}$$

Proof: The proof is presented in Appendix A. ■

Lemma 1 indicates that it is preferred to choose only one optimal eNB to transmit on each RB from the EE perspective, which is contradictory to our intuition of increasing the transmit diversity to improve the throughput from SE perspective [11].

According to Lemma 1, we propose the related RB allocation algorithm, as shown in Algorithm 1. The RB allocation procedure can be divided into two parts. Firstly, as optimizing the overall EE in (5) can be taken place by maximizing the minimum EE of individual user [4] and EE is strictly increasing in channel gain, we iteratively allocate the RB to user with the minimum maximum channel gain, which is depicted from Line 3 to Line 11. Then, in the following RB allocation iteration, the user with the minimum ratio of actual throughput and required throughput, κ_m, occupies its most favorite RB among all unassigned ones to improve its throughput. The RB allocation iteration process will proceed until all RBs have been allocated, which is described from Line 12 to Line 24.

B. Transmit Power Control

In the RB allocation process of Algorithm 1, we assume that the total transmit power of each eNB is uniformly distributed on the RBs associated with it, as shown on Line 14. Here, we derive the optimal transmit powers of each eNB on its associated RBs.

With given RB allocation results, the following lemma gives some properties of EE on the transmit power associated with each RB.

Lemma 2: With given RB allocation results, ψ, the EE, η_{EE}, is continuously differentiable and strictly quasiconcave in $p_{n,m,k}$ if $\delta_{n,m,k} = 1$.[1] Without the maximum transmit power constraint, the unique global optimal transmit power, $p^*_{n,m,k}$, exists at the interval $p_{n,m,k} \in [0, \infty)$. Specifically,

1) If $p^*_{n,m,k} > 0$, η_{EE} is concave at the interval $p_{n,m,k} \in [0, p^*_{n,m,k}]$; whereas only quasiconcave at the interval $p_{n,m,k} \in (p^*_{n,m,k}, \infty)$. η_{EE} first strictly increases with $p_{n,m,k} \in [0, p^*_{n,m,k}]$ and then strictly decreases with $p_{n,m,k} \in (p^*_{n,m,k}, \infty)$.
2) If $p^*_{n,m,k} = 0$, η_{EE} is quasiconcave and strictly decreases at the interval $p_{n,m,k} \in [0, \infty)$.

Proof: The proof is presented in Appendix B. ■

Based on Lemma 2 and given the RB allocation results, ψ, we propose the transmit power control algorithm, as shown in Algorithm 2.

The transmit power for each RB is firstly derived based on water-filling method by taking per-user throughput requirement into account. For user m,

$$p_{n,m,k} = \left(\frac{1}{\lambda \ln 2} - \frac{N_0 B}{|h_{n,m,k}|^2}\right)^+, \text{ if } \delta_{n,m,k} = 1 \tag{8a}$$

$$B \sum_{k \in \mathcal{K}_m} \log_2\left(\frac{1}{\lambda \ln 2}\frac{|h_{n,m,k}|^2}{N_0 B}\right) = R_m^{th}. \tag{8b}$$

Equation (8) tries to satisfy the throughput requirement of each user without considering the transmit power constraint (6d), which is taken into account from Line 5 to Line 9 by proportionally decreasing the transmit power of eNB that exceeds the maximum allowed value. In order to compensate for throughput degradation of the user resulted from the maximum transmit power constraint, we increase the transmit powers of other eNBs that do not reach the maximum values, as shown from Line 10 to Line 23. While it is known that the larger the channel gain, the deeper the gradient of channel throughput with respect to the transmit power, we update the transmit powers on the RBs associated with one user in descending order of the corresponding channel gains successively, as shown on Lines 13 and 14, until the user's throughput requirement is satisfied or all the transmit powers on the RBs associated with the user are updated.

Finally, we would adjust the transmit power to maximize the EE by exploiting the strict quasiconcavity of EE in $p_{n,m,k}$ from Line 24 to Line 31. Obviously, without the user throughput and transmit power constraints, the corresponding optimal transmit power can be easily obtained by equating the first order derivative of η_{EE} given in Appendix B to zero as

$$(\varsigma P_t + P_s)\, B |h_{n,m,k}|^2 = \varsigma R \omega_{n,m,k}. \tag{9}$$

By enforcing the transmit power constraint, the updated optimal transmit power is

$$\tilde{p}_{n,m,k} = \min\left\{p_n^{\max}, \left(p^*_{n,m,k}\right)^+\right\} \tag{10}$$

where $p^*_{n,m,k}$ is the solution to (9), and $(x)^+$ represents $\max\{0, x\}$. As (9) is a nonlinear function, we can make some approximation to derive $p^*_{n,m,k}$ in a simple way. Based on the preliminary power control, we can first calculate P_t and R and then substitute them into (9) to have

$$\tilde{p}_{n,m,k} = \min\left\{p_n^{\max}, \left(\frac{(\varsigma P_t + P_s)\, B}{\varsigma R \ln 2} - \frac{N_0 B}{|h_{n,m,k}|^2}\right)^+\right\}. \tag{11}$$

[1]The power consumption model adopted in [4] is a special case of that adopted in our paper with $\xi = 0$. Since the superlevel sets of EE are not strictly convex in transmit power with non-zero ξ, it cannot be concluded that EE is strictly quasiconcave in transmit power based on the proof in Appendix A of [4]. In other words, the proof of Theorem 1 in [4] on the quasiconcavity of EE in transmit power does not hold when we adopt a more general power consumption model.

Algorithm 2 Transmit Power Control Algorithm

1: Initiate $p_{n,m,k} \leftarrow 0$, $\forall n \in \mathcal{N}$, $\forall m \in \mathcal{M}$, $\forall k \in \mathcal{K}$
2: **for** each user $m \in \mathcal{M}$
3: Do single user water-filling using (8)
4: **end for**
5: **for** each eNB $n \in \mathcal{N}$
6: **if** $\sum_{m'=1}^{M} \sum_{k'=1}^{K} p_{n,m',k'} > p_n^{\max}$
7: $p_{n,m,k} \leftarrow \frac{p_{n,m,k} p_n^{\max}}{\sum_{m'=1}^{M} \sum_{k'=1}^{K} p_{n,m',k'}}$
8: **end if**
9: **end for**
10: Initiate $\delta'_{n,m,k} \leftarrow \delta_{n,m,k}$, $\forall n \in \mathcal{N}$, $\forall m \in \mathcal{M}$, $\forall k \in \mathcal{K}$
11: **for** each user $m \in \mathcal{M}$ with priority in descending order
12: **while** $R_m < R_m^{th}$ and $\sum_{n=0}^{N} \sum_{k=1}^{K} \delta'_{n,m,k} > 0$
13: $\{n^*, k^*\} \leftarrow \underset{n \in \mathcal{N}, k \in \mathcal{K}}{\arg\max} \left(\delta'_{n,m,k} |h_{n,m,k}|^2 \right)$
14: $\delta'_{n^*,m,k^*} \leftarrow 0$
15: **if** $\sum_{m'=1}^{M} \sum_{k=1}^{K} p_{n^*,m',k} \geq p_{n^*}^{\max}$
16: Continue
17: **end if**
18: $R_{n^*,m,k^*} \leftarrow B \log_2 \left(1 + \frac{p_{n^*,m,k^*} \left| h_{n^*,m,k^*} \right|^2}{N_0 B} \right)$
19: $p_{n^*,m,k^*}^{\max} \leftarrow p_{n^*}^{\max} - \sum_{m'=1}^{M} \sum_{k=1}^{K} p_{n^*,m',k} + p_{n^*,m,k^*}$
20: $p_{n^*,m,k^*} \leftarrow \frac{N_0 B}{\left| h_{n^*,m,k^*} \right|^2} \left(2^{\frac{R_m^{th} - R_m + R_{n^*,m,k^*}}{B}} - 1 \right)$
21: $p_{n^*,m,k^*} \leftarrow \min\{p_{n^*,m,k^*}^{\max}, p_{n^*,m,k^*}\}$
22: **end while**
23: **end for**
24: **for** each RB $k \in \mathcal{K}$
25: $\{n^*, m^*\} \leftarrow \underset{n \in \mathcal{N}, m \in \mathcal{M}}{\arg\max} \, (p_{n,m,k})$
26: Obtain the global optimal value of $p_{n^*,m^*,k}$, $\tilde{p}_{n^*,m^*,k}$, based on (10) or (11)
27: **if** $\sum_{m=1}^{M} \sum_{k'=1}^{K} p_{n^*,m,k'} < p_{n^*}^{\max}$, $\delta_{n^*,m^*,k} = 1$, and $p_{n^*,m^*,k} < \tilde{p}_{n^*,m^*,k}$
28: $p_{n^*,m^*,k}^{\max} \leftarrow p_{n^*}^{\max} - \sum_{m=1}^{M} \sum_{k'=1}^{K} p_{n^*,m,k'} + p_{n^*,m^*,k}$
29: $p_{n^*,m^*,k} \leftarrow \min\{p_{n^*,m^*,k}^{\max}, \tilde{p}_{n^*,m^*,k}\}$
30: **end if**
31: **end for**

From the points of views of SE and EE, $\tilde{p}_{n,m,k}$ is Pareto optimal with $\tilde{p}_{n,m,k} \geq p_{n,m,k}$, and $p \in [\tilde{p}_{n,m,k}, p_{n,m,k}]$ is Pareto optimal with $\tilde{p}_{n,m,k} < p_{n,m,k}$ [12]. Based on (10) or (11), we can obtain the final optimal transmit power, as shown on Line 29.

IV. Performance Evaluation

The system performance of the proposed energy-efficient resource allocation scheme is thoroughly evaluated by means of extensive system-level simulations based on the 3GPP methodology. For the sake of simplicity, we assume that the SeNBs lie on the circle centered at MeNB with a radius $(3 - \sqrt{3})r/2$ [3], where $r = 250$ m is the radius of the cell, and the central angle between two SeNBs is $2\pi/N$. The drain efficiency of power amplifier is assumed as 0.38.

Fig. 1 gives the EE of LTE heterogeneous network with varying total required throughput $\sum_{m=1}^{M} R_m^{th}$. The legend term "$a, P_s = b, \xi = c$", $a \in \{\text{SE}, \text{EE}\}$, $b \in \{10, 30\}$, and $c \in \{1, 2\}$, means that the related curve corresponds to a-optimal resource allocation scheme with $P_s = b$ W and $\xi = c$ W/Mbps, where the EE-optimal and SE-optimal schemes aim at optimizing the generalized EE and SE with the same constraints respectively. Fig. 1 shows the impacts of P_s and ξ on the EE of EE-optimal and SE-optimal schemes. Obviously, EE decreases with the increase of P_s and/or ξ. The EE of EE-optimal scheme first increases and then decreases with the total required throughput. This is due to the fact that the transmit power increases with the increase of throughput requirement and the fact that EE is first a monotonically increasing function and then a monotonically decreasing function in transmit power, which agrees with Lemma 2.

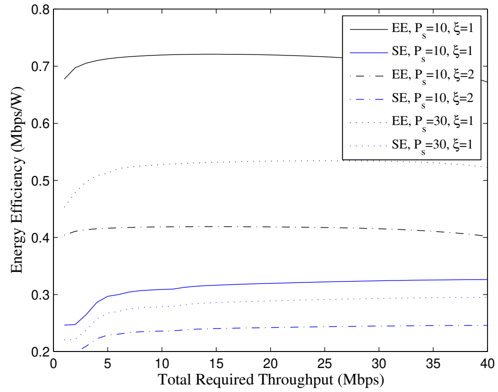

Fig. 1. Energy efficiency with varying total required throughput. ($M = 25$, $N = 8$)

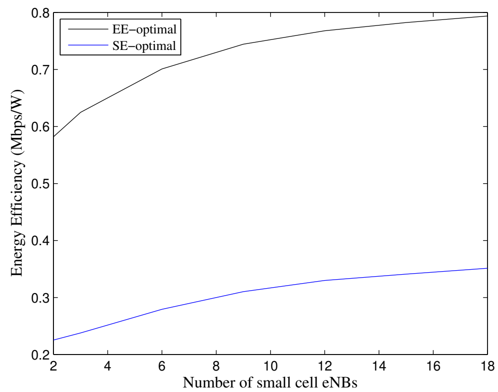

Fig. 2. Energy efficiency with varying number of small cell eNBs. ($M = 25$, $\sum_{m=1}^{M} R_m^{th} = 20$ Mbps)

Fig. 2 plots the EE of EE-optimal and SE-optimal schemes with varying number of small cell eNBs. It is observed that the larger the number of SeNBs, the higher the EE of both the two schemes. This is due to the fact that deploying more SeNBs can decrease the pathlosses between users and their associated eNBs, and hence reduces the transmit power. However, the positive effect of SeNB number on EE decreases with the increase of SeNB number. The reason is that the impact of SeNB number on the pathlosses between users and their associated eNBs decreases with the increase of SeNB number.

V. Conclusions

In this paper, we have proposed an energy-efficient resource allocation scheme for downlink in LTE heterogeneous networks. We have demonstrated that EE is a strictly monotonically increasing function in channel gain and strictly quasiconcave in transmit power associated with each RB. Based on these two properties of EE, we have developed a two-step energy-efficient resource allocation scheme consisting of RB allocation and transmit power control, whose performance has been evaluated through simulation results.

Appendix A
Proof of Lemma 1

The channel gain $|h_{n,m,k}|^2$ is not involved into the downlink transmission with $\delta_{n,m,k} = 0$, and consequently it has no impact on the total power consumption and user's throughput. Here, we only concern the impact of channel gain $|h_{n,m,k}|^2$ on EE with $\delta_{n,m,k} = 1$.

Taking the first derivative of η_{EE} with respect to channel gain $|h_{n,m,k}|^2$ yields

$$\frac{\partial \eta_{EE}}{\partial |h_{n,m,k}|^2} = \frac{B\left(\varsigma P_t + P_s\right) p_{n,m,k}}{P^2 \alpha \ln 2} > 0 \tag{12}$$

where $\alpha = N_0 B + \sum_{n=0}^{N} \delta_{n,m,k} p_{n,m,k} |h_{n,m,k}|^2$. Equation (12) indicates that the EE, η_{EE}, is a strictly monotonically increasing function of channel gain $|h_{n,m,k}|^2$. From (12), letting $|h_{m,k}|^2 \triangleq \max_{n\in\mathcal{N}} |h_{n,m,k}|^2$, we have

$$\eta_{EE} < \frac{\tilde{R} + B \sum\limits_{k\in\mathcal{K}_m} \log_2\left(1 + \frac{|h_{m,k}|^2}{N_0 B} \sum\limits_{n=0}^{N} \delta_{n,m,k} p_{n,m,k}\right)}{\tilde{P} + \xi B \sum\limits_{k\in\mathcal{K}_m} \log_2\left(1 + \frac{|h_{m,k}|^2}{N_0 B} \sum\limits_{n=0}^{N} \delta_{n,m,k} p_{n,m,k}\right)}, \tag{13}$$

where $\tilde{R} = \sum_{m'=1, m'\neq m}^{M} R_{m'}$, and $\tilde{P} = \varsigma P_t + P_s + \xi \tilde{R}$. Equation (13) demonstrates that we can improve the EE by re-allocating the transmit power from other eNBs to the eNB associated with the maximum channel gain, which means that $\sum_{n=0}^{N} \delta_{n,m,k} \leq 1$, $\forall m \in \mathcal{M}$, $\forall k \in \mathcal{K}$, from EE perspective. With the assumption of (6a), it is easy to conclude Lemma 1.

Appendix B
Proof of Lemma 2

From Lemma 1, if $\delta_{n,m,k} = 1$, then $\sum_{n'=0, n'\neq n}^{N} \delta_{n',m,k} = 0$. As such, the first derivative of η_{EE} with respect to $p_{n,m,k}$ is given by

$$\frac{\partial \eta_{EE}}{\partial p_{n,m,k}} = \frac{\left(\varsigma P_t + P_s\right) B |h_{n,m,k}|^2 - \varsigma R \omega_{n,m,k}}{P^2 \omega_{n,m,k}}, \tag{14}$$

where $\omega_{n,m,k} = \left(N_0 B + p_{n,m,k} |h_{n,m,k}|^2\right) \ln 2$. Let us denote the numerator term of (14) as $f(p_{n,m,k})$. The first order derivative of $f(p_{n,m,k})$ with respect to $p_{n,m,k}$ is given by $\frac{\partial f(p_{n,m,k})}{\partial p_{n,m,k}} = -\varsigma R |h_{n,m,k}|^2 \ln 2 < 0$, which means that $f\left(p_{n,m,k}\right)$ is a strictly monotonically decreasing function of $p_{n,m,k}$. It is easy to demonstrate that $\lim\limits_{p_{n,m,k}\to\infty} f\left(p_{n,m,k}\right) < 0$. If $\lim_{p_{n,m,k}\to 0^+} f\left(p_{n,m,k}\right) > 0$, there exists a unique optimal $p_{n,m,k}^* > 0$ such that $\frac{\partial \eta_{EE}}{\partial p_{n,m,k}^*} = 0$, since the denominator item in (14) is always larger than 0. In such a case, η_{EE} is a strictly monotonically increasing function for $p_{n,m,k} < p_{n,m,k}^*$ and a strictly monotonically decreasing function for $p_{n,m,k} > p_{n,m,k}^*$. Meanwhile, the second derivative of η_{EE} with respect to $p_{n,m,k}$ is

$$\begin{aligned}\frac{\partial^2 \eta_{EE}}{\partial p_{n,m,k}^2} = &- \frac{\varsigma R |h_{n,m,k}|^2 \ln 2}{P^2 \omega_{n,m,k}} - \frac{f\left(p_{n,m,k}\right) |h_{n,m,k}|^2 \ln 2}{P^2 \omega_{n,m,k}^2} \\ &- \frac{2 f\left(p_{n,m,k}\right) \left(\varsigma \omega_{n,m,k} + \xi B |h_{n,m,k}|^2\right)}{P^3 \omega_{n,m,k}^2}.\end{aligned} \tag{15}$$

Since $f\left(p_{n,m,k}\right) > 0$ with $p_{n,m,k} < p_{n,m,k}^*$, we can conclude that $\frac{\partial^2 \eta_{EE}}{\partial p_{n,m,k}^2} < 0$, which means η_{EE} is concave with $p_{n,m,k} < p_{n,m,k}^*$. With $p_{n,m,k} > p_{n,m,k}^*$, $f\left(p_{n,m,k}\right) < 0$, and η_{EE} can be either positive or negative, which means η_{EE} is only quasiconcave.

If $f(0) \leq 0$, the optimal $p_{n,m,k}^*$ is equal to 0 to maximize the η_{EE}. In such a case, η_{EE} is a strictly monotonically decreasing function for $p_{n,m,k} > 0$. Based on the the second derivative of η_{EE} given by (15), we can conclude that η_{EE} is quasiconcave for $p_{n,m,k} > 0$. Therefore, Lemma 2 holds.

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