

# Spatial instabilities in a cloud of cold atoms

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Dense cold atomic clouds have been shown to be similar to plasmas. Previous studies showed that such clouds exhibit instabilities induced by long-range interactions. However they did not describe the spatial properties of the dynamics. In this Letter, we study experimentally the spatial nature of stochastic instabilities and find out that the dynamics is localized. Data are analyzed both in the spectral domain and in the spatial domain (principal component analysis). Both methods fail to describe the dynamics in terms of eigenmodes, showing that space and time are not separable.

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During the last decades, cold atoms have proven to be more than an extraordinary tool for studying the physics of dilute matter. Many spectacular results concern the field of condensed matter, as e.g. the direct observation of the Anderson localization [1], or that of the BEC-BCS crossover [2]. But even in the field of dilute matter, cold atoms are thought to be a good model system for plasmas, in particular because experiments are considered to be relatively simple and well controlled. Although cold atoms in a Magneto-Optical Trap (MOT) are neutral, it has been demonstrated that a coulombian-like repulsive force appears in the multiple scattering regime [3]. Based on this analogy, fluid-dynamical models used in plasma physics have been adapted to cold atoms physics [4–6]. On the other hand, it has been demonstrated that the dynamics of cold atoms in a MOT can be described through the Vlasov-Fokker-Planck equations [7], as e.g. the plasma dynamics in the inertial confinement fusion [8], the stellar dynamics [9] or the electron dynamics in storage rings [10].

Most of these systems are known to exhibit instabilities under appropriate parameter sets. Numerous types of instabilities have been observed, with very different properties and signatures. Although plasmas are governed by long-range interactions, instabilities appear not only at large spatial scales, but also at smaller scales. Some examples of local instabilities are the microbunching instability in the storage rings [10], drift wave microinstabilities in plasmas confined by a magnetic field [11], or microinstabilities of the solar corona [12].

Instabilities in MOTs have been observed for several decades, and have been studied for 15 years. Mainly two types of instabilities have been reported through experiments. Self-sustained instabilities are periodic oscillations [13, 14], while stochastic instabilities exhibit random characteristics [15]. In all cases, the experimental characterization is done through the temporal evolution of global variables, such as the fluorescence of the cloud or the location of its center of mass. The spatial properties of the instabilities, in particular their location in the cloud, is not known. However, most simplified models allowing to reproduce these instabilities have considered

they are global [16, 17]. On the other hand, taking formally into account the different processes involved in the MOTs leads to the Vlasov-Fokker-Planck equations, implicitly predicting local instabilities [7]. Models derived from plasmas also predict local phenomena, such as photon bubbles [5], but none of them has been observed yet. Thus gaining knowledge on the spatio-temporal characteristics of the cloud dynamics appears to be crucial to know which methods and approximations can be used to solve the full set of equations describing the MOT. It could also help to precise the similarities between cold atoms and plasma instabilities.

We report in this paper the experimental observation of local instabilities, through a novel detection setup allowing to record the spatio-temporal evolution of the atoms in the MOT. We focus here on the previously observed stochastic instabilities [15, 16], and we show that these instabilities are localized in a limited area of the cloud. The analysis of the dynamics by two different methods does not give results consistent with the hypothesis of global temporal or spatial motion. The paper is organized as follows: after a brief description of the experimental setup, we analyze the dynamics of the MOT through global tools, as used in the literature, in order to clearly establish the type of instabilities we are studying. Then we analyze the dynamics through two methods: the local temporal analysis gives information on the motion eigenfrequencies, while the Principal Component Analysis (PCA) allows to identify spatial eigenmodes.

Our experimental setup is described in detail in [14–17]. The MOT is a standard Cs MOT with three retro-reflected beams. However, special care is taken to the stability of parameters which could introduce artefacts in the dynamics. For example, we use a single mode optical fiber to clean the transverse profile of laser beams. We also modulate the relative phases of all the beams to avoid possible interference patterns. The modulation frequency ( $> 1$  kHz) is chosen larger than the collective atomic response frequencies, so that the intensity is averaged [18]. The MOT produces a dense cloud of cold atoms with a typical diameter of 4 mm. This size is characteristic of the multiple scattering regime in which the

collective effects play a key role in the behavior of the cloud [19]. To obtain the desired dynamical regime, we adjust two control parameters: (i) the intensity  $I$  of a single incoming beam, expressed in units of the saturation intensity  $I_{sat} = 1.1 \text{ mW.cm}^{-2}$  ( $D_2$  line of  $^{133}\text{Cs}$ ) and (ii) the detuning  $\Delta$  between the laser frequency and the atomic transition, expressed in units of the natural line width  $\Gamma = 2\pi \times 5.22 \text{ MHz}$ . In the following, all the illustrations and examples correspond to  $I = 11 I_{sat}$  and  $\Delta = -1.8\Gamma$ . They are typical of the dynamics observed for  $10 I_{sat} \leq I \leq 15 I_{sat}$  and  $-1.9\Gamma \leq \Delta \leq -1.6\Gamma$ .

The spatio-temporal dynamics of the atoms is analyzed by recording the local temporal evolution of the fluorescence at any point of the cold atom cloud. Let us remember that the number of atoms is proportional to the fluorescence for a given set of the MOT parameters. As during an acquisition, the MOT parameters are kept constant, we thus record the spatio-temporal evolution of the atomic density in the cloud.

It has been shown that instabilities exhibit frequencies ranging from 1 to 100 Hz [14–17]. Thus to record the dynamics, a standard video camera at 30 frames per second is not adapted. We use a fast video camera (Phantom v7.3 camera from Vision Research) able to record up to 10,000 frames/s. A set of lenses is used to cover an optical field of  $15 \times 10 \text{ mm}$  and a depth of field of 2.5 mm, well fitting the cloud size. An important point is to determine if the light recorded by such a camera comes only from the surface of the cloud, or from any point inside the cloud. Instabilities grow up when the atomic density is high enough, i.e. when the collective nonlinear processes inside the cloud cannot be neglected [14–17]. However, even for such relatively dense clouds, the number of scattering events for most photons escaping the cloud is 1 or 2 [18]. That means that the camera captures photons coming from any point inside the cloud, but with a different weight depending on the location. This last point has to be kept in mind, although it has marginal consequences on the interpretation of the pictures. Indeed, the main difficulty is that we record a 2D projection of the cloud, while the dynamics occurs in 3D. This makes the interpretation of the records harder.

The location of the camera is imposed by the available space on the setup. Let us call  $Ox$ ,  $Oy$  and  $Oz$  the three perpendicular axes corresponding to the three incident beams of the trap,  $Oz$  being also the axis of the coils. The camera axis is close to the  $(x = y, z = 0)$  direction. Thanks to the choice of a retro-reflecting beam configuration, and because of the shadow effect [14–17], the amplitude of the dynamics is enhanced in a direction along the line  $x = y = 2z$ . The main direction of the dynamics projects with an angle of  $20^\circ$  on the picture plane, leading to a satisfactory resolution of the dynamics.

In order to make the link between the present work and the previous reported observations, we also use a 4-quadrant photodiode (4QP) to monitor the total cloud

fluorescence and the position of the cloud center of mass, as e.g. in [14]. The signals from the 4QP and the pictures from the video camera are recorded synchronously.

As pointed out above, we focus here on the regime of stochastic instabilities. In [15], that regime is analyzed through the dynamics of the number of atoms in the cloud and that of its center of mass. It is described to be a noisy dynamics cut off with bursts of large oscillations, and the analysis focused on the main frequency component, appearing inside the bursts. It is also shown that these instabilities are induced by a stochastic resonance centered on a given detuning.

In the present work, the unstable regime shows the same characteristics as in [12]. In particular, for the parameters given above, instabilities are maximal for  $\Delta = -1.8\Gamma$ ; the dynamics (number of atoms and center of mass) is a succession of regular bursts and noisy intervals; in the bursts, a frequency  $\omega_1 \simeq 2\pi \times 21 \text{ Hz}$  dominates the dynamics. In order to be more exhaustive than the previous studies, we also studied the characteristics of the secondary frequency components. In the burst regime, the second component  $\omega_2 \simeq 2\pi \times 72 \text{ Hz}$  is two orders of magnitude smaller than  $\omega_1$ . The components  $2\omega_1$  and  $\omega_2 - \omega_1$  are also present, with amplitudes similar to that of  $\omega_2$ . In the noisy intervals,  $\omega_2$  is present, with the same amplitude as in the bursts.  $\omega_1$  is also present, but with an amplitude smaller than  $\omega_2$ . In summary,  $\omega_1$  appears mainly inside the bursts, while  $\omega_2$  is always present.

For this study, our aim is to characterize the nature – space dependent or not – of the dynamics. Thus we do not need to record long time series with a good time resolution, as it would be necessary for e.g. making a topological analysis of the dynamics attractors. We only need to follow the usual rules of frequency analysis, i.e. to record at least two points per period. The following results correspond to  $512 \times 512$  pixels pictures recorded with a rate of 400 frames/s. The length of the analyzed sequences is 625 ms, i.e. 250 frames, corresponding to the typical length of the bursts.

To determine the spatial dependence of the dynamics, a straightforward approach consists in applying the frequency analysis used previously to each point of the cloud. Fig. 1 shows a typical result for the dynamics inside a burst. In fig. 1a, we represent in gray levels (Color online) the amplitude of the  $\omega_1$  component in each point of the cloud, obtained by Fast Fourier Transform (FFT) of each pixel of the picture sequence. The dynamics appears with no doubt to depend on the space. Instabilities appear in two small areas of the cloud, covering typically 10% of the whole cloud. The two areas have very different shapes and sizes, and we see two local maxima in one of the area. Fig. 1c shows that the two areas pulse with opposite phases. It is difficult to deduce from such 2D observations the exact 3D dynamics, but it is clear that the instabilities consist in a local pulsation or rotation,

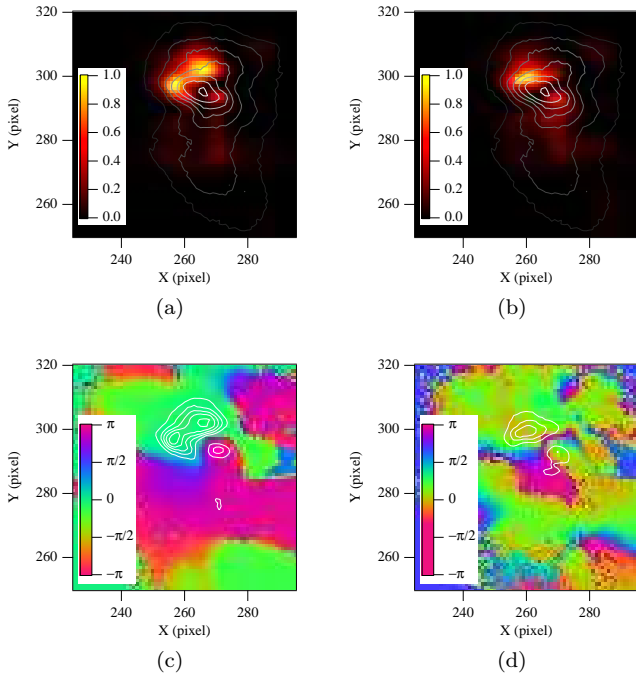


Figure 1: Spatial distribution of instabilities for one sequence: (a) and (b) show the local amplitude of the two main components:  $\omega_1$  and  $\omega_2$ . The normalized magnitude squared of the spectral component is represented in gray level (Color online). The contour plot shows the averaged fluorescence during the sequence. (c) and (d) represent the corresponding phase distributions. Grayscale (Color online) represents the relative phase of the local oscillation between  $\pm\pi$ . The contour plot identifies the unstable areas.

while the rest of the cloud is stable.

The above example is typical of what we observed in all recorded sequences. Some differences appear, as the location of the unstable area, its structure, in particular the number of local maxima, and its shape, but we always observed that the instabilities were localized in a limited area of the cloud, covering typically 10% of the whole cloud.

We performed the same analysis for the  $\omega_2$  component, and found the same type of results as for the  $\omega_1$  component: the instabilities are localized in a relatively small area, and they correspond to a motion of pulsation or rotation of a part of the cloud. As discussed above, inside the bursts, the  $\omega_2$  component is small as compared to the  $\omega_1$  component, making the analysis of data less reliable. However, it appears clearly that in this case, the characteristics of the  $\omega_2$  component follow those of the  $\omega_1$  component, delimiting the same unstable area with the same type of motion. Figs 1b and 1d show respectively the amplitude and the phase of the  $\omega_2$  component for the same burst as figs 1a and 1c: the spatial distribution

of the instabilities are the same for the two components. The similarity of the phase distributions is less convincing, due to the weakness of the component, but in spite of that, a phase opposition appears between the two areas. To generalize this observation, we computed the spatial overlap between the two components for all the recorded sequences, and found that they have in most cases a similar spatial distribution.

Outside the bursts, the  $\omega_2$  component is still present, and has the same characteristics. Thus the dynamics appears to be a small (in space) and weak (in amplitude) periodic motion of a small part of the cloud at the  $\omega_2$  frequency, cut off by bursts where the amplitude of the motion increases temporarily, while its main frequency shifts to  $\omega_1$ . However, this description is not completely satisfactory. Indeed, it gives information about the temporal components of the dynamics, but does not give any information about the spatial components. In particular, it seems to associate one spatial component with two different frequencies, while in spatio-temporal systems where the temporal dynamics and the space distribution can be separated, as e.g. in multimode lasers [20], a spatial eigenmode is associated with only one eigenfrequency. We adopt in the following another approach.

PCA presents the advantage to give a description of the dynamics of the system without requiring any preliminary hypothesis. The dynamics is described in terms of a superposition of spatial modes, giving complementary information with respect to the Fourier analysis. The analysis is performed using the method described in [21].

The result of the PCA is a set of spatial modes forming a basis whose size is equal to the number of pictures of the sequence (250 in our case). If these modes are sorted according to their weight, i.e. their contribution to the total statistical deviation around the averaged atomic distribution, the method leads to the determination of the number of modes useful to describe the dynamics. In our case, we find that in the sequences considered here, the main mode contains between 50% and 80% of the statistical fluctuations, and the second mode between 5% and 25%. Thus the dynamics appears to be dominated by one single mode, although the weight of the second mode is not negligible. Depending on the desired accuracy, the third mode, with a weight of  $10\% \pm 5\%$ , could be taken into account. In the following, we consider only the first two modes.

Figure 2 shows the first two modes obtained by the PCA of the sequence of figure 1. Note the similarity of the main mode with the  $\omega_1$  spatial distribution (fig. 1a). However, this result is not general. On the contrary, we observe a major difference between the modes given by the PCA and the ones resulting from the Fourier analysis. While in the latter a wide variety of shapes are obtained, the results are more homogeneous in the former. In fact, one set of modes dominates the whole dynamics. This basis – let us call it the main basis – is not the only one

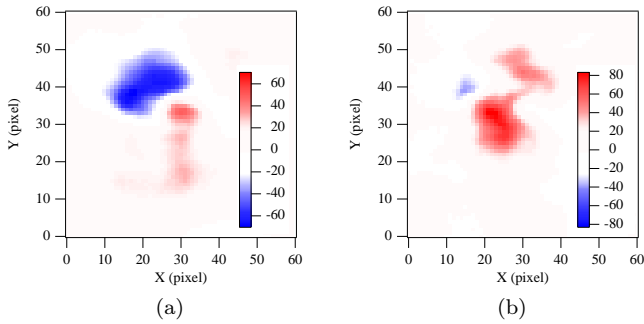


Figure 2: Two first spatial modes given by the PCA (2a and 2b respectively) for the same sequence than in figure 1. In both cases, the spatial distribution is represented in absolute value in gray level (Color online), and the upper and the lower areas have to be understood respectively as an excess (in blue online) and a depletion of atoms (in red online). This information is similar than the one given by the phase of the Fourier analysis.

found by the PCA, but it is the most frequent in the set of recorded sequences. This regime is randomly interrupted by short intervals where the dynamics is described by another basis, before it returns to the main basis. These alternative bases differ from one sequence to another one, and we did not found any common properties. For example, in a few cases, the modes of the main basis appear, but in a different order: the first mode of the main basis becomes the second mode of the basis.

A surprising result is that the regimes found with the PCA do not correspond to those given by the Fourier analysis. In particular, the sequences described by the main basis do not follow the succession of bursts and noisy intervals. As a consequence, the frequencies associated with the main basis may change from one sequence to another. Moreover, the time evolution of a given spatial mode may exhibit different frequencies in different sequences. Fig. 3 shows the time evolution and the FFT of two modes dominating the dynamics in two consecutive sequences. Although the two modes are similar, they are associated with two very different time evolution: for the first one, both frequencies  $\omega_1$  and  $\omega_2$  drive the dynamics, while the second one corresponds to a burst where  $\omega_1$  dominates the dynamics.

In summary, the PCA leads to a description of the dynamics in terms of spatial modes. The results show that such a description is as relevant as the description in terms of frequency components. Two regimes are found: the first one corresponds to a well identified spatial mode basis, referred as the main basis, and a second one, described by different bases. From this point of view, the description obtained by the PCA is similar to that of the Fourier analysis. But surprisingly, the two regimes

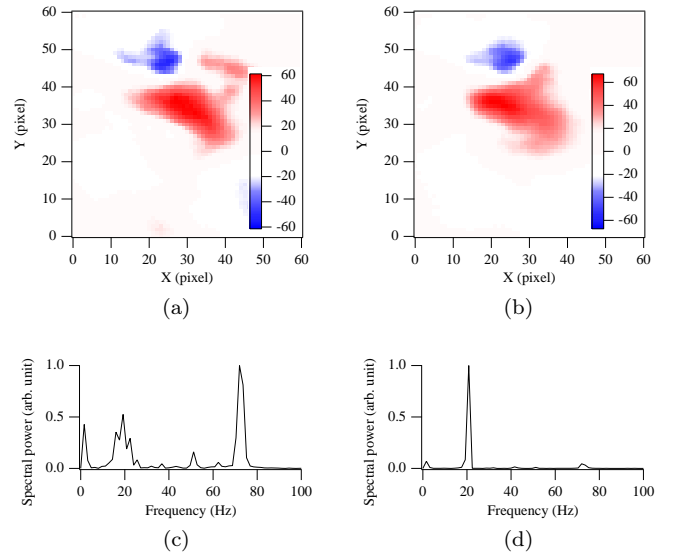


Figure 3: From top to bottom: first modes calculated by the PCA for two consecutive sequences and the FFT of the time evolution of these modes.

of the PCA do not correspond in time sequences to the two regimes of the Fourier analysis. Thus the two approaches – temporal and spatial – give complementary information on the dynamics: the dynamics of the cloud of cold atoms in a MOT is a genuine spatio-temporal system, where the spatial and temporal behaviors cannot be separated.

We report in the present paper experimental results on the dynamics of an unstable cloud of cold atoms in a regime of stochastic instabilities. Previous studies focused on the temporal behavior of the instabilities. Here we study both the spatial and temporal properties of the dynamics. Although the atomic motion cannot be clearly identified as our analysis is based on a 2D projection of a 3D motion, we show that the oscillations are localized in space. We analyze the dynamics through two different methods, and both point out the key role of space in the dynamics. Moreover, the analyses in terms of frequency components and in terms of spatial modes show that the relation between the temporal regime and the spatial distribution is not straightforward, as the same spatial distribution can correspond to different temporal regimes. These results invalidate the description in terms of purely temporal models, as in [13–17], and require the use of fully spatio-temporal models, as the Vlasov-Fokker-Planck model [7] or plasma-derived model [5]. They strengthen the relationship between cold atoms and plasmas, and show that cold atoms could be a good model system for plasmas. However, a fine comparison of both systems and a deep physical interpretation of the observed behavior require a better knowledge of the dynamics of cold atoms. So, the next step is to run numeri-

cal simulations to obtain quantitative agreement between models and experimental observations, in particular in the dynamical regimes.

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- [1] F. Jendrzejewski, A. Bernard, K. Müller, P. Cheinet, V. Josse, M. Piraud, L. Pezzé, L. Sanchez-Palencia, A. Aspect and P. Bouyer, *Nature Physics* **8**, 398 (2012)
- [2] T. Bourdel, L. Khaykovich, J. Cubizolles, J. Zhang, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, *Phys. Rev. Lett.* **93**, 050401 (2004)
- [3] L. Pruvost, I. Serre, H. T. Duong, and J. Jortner, *Phys. Rev. A* **61**, 053408 (2000)
- [4] J. T. Mendonça, R. Kaiser, H. Terças and J. Loureiro, *Phys. Rev. A* **78**, 013408 (2008)
- [5] J. T. Mendonça and R. Kaiser, *Phys. Rev. Lett* **108**, 033001 (2012)
- [6] J. D. Rodrigues, J. A. Rodrigues, A. V. Ferreira, J. T. Mendonça, *Opt. Quant. Electron.* **48**, 169 (2016)
- [7] R. Romain, D. Hennequin, and P. Verkerk, *Eur. Phys. J. D* **61**, 171 (2011)
- [8] C. P. Ridgers, R. J. Kingham, and A. G. R. Thomas, *Phys. Rev. Lett.* **100**, 075003 (2008)
- [9] Pierre-Henri Chavanis, *Phys. Rev. E* **68**, 036108 (2003)
- [10] E. Roussel, C. Evain, C. Szwej, and S. Bielawski, *Phys. Rev. ST Accel. Beams* **17**, 010701 (2014)
- [11] Jan Weiland, *Stability and Transport in Magnetic Confinement Systems, Springer Series on Atomic, Optical and Plasma Physics 71* (Springer, New-York, 2012)
- [12] Eckart Marsch, *Living Rev. Solar Phys.* **3**, 1 (2006)
- [13] G. Labeyrie, F. Michaud, and R. Kaiser, *Phys. Rev. Lett.* **96**, 023003 (2006)
- [14] Andrea di Stefano, Marie Fauquembergue, Philippe Verkerk, Daniel Hennequin, *Phys. Rev. A* **67** 033404 (2003)
- [15] David Wilkowski, Jean Ringot, Daniel Hennequin, Jean Claude Garreau, *Phys. Rev. Lett.* **85** 1839 (2000)
- [16] D. Hennequin, *Eur. Phys. J. D* **28** 13 (2004)
- [17] A. di Stefano, Ph. Verkerk, D. Hennequin, *Eur. Phys. J. D* **30** 243 (2004)
- [18] R. Romain, H. Louis, P. Verkerk and D. Hennequin, *Phys. Rev. A* **89**, 053425 (2014)
- [19] D.W. Sesko, T.G. Walker, and C. Wieman, *J. Opt. Soc. Am. B* **8** 946 (1991)
- [20] D. Hennequin, D. Dangoisse, P. Glorieux, *Opt. Commun.* **79** 200–206 (1990)
- [21] R. Dubessy, C. De Rossi, T. Badr, L. Longchambon and H. Perrin, *New J. Phys.* **16** 122001 (2014)