Far-reaching Hellenistic geographical knowledge hidden in Ptolemy's data

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FAR-REACHING HELLENISTIC GEOGRAPHICAL KNOWLEDGE HIDDEN IN PTOLEMY'S DATA

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Abstract – The paper summarizes and discusses the main theses exposed in a previous book (*L'America dimenticata*, Mondadori Università, 2013 [in Italian]) in light of more recent results. Specifically, the work addresses the problem of explaining the origin of the systematic error on longitudes in Ptolemy's *Geographia* and its logical relation with the reduced estimate for the dimension of the Earth there given. The thesis is sustained that, conversely to a frequently advanced conjecture, the shrinking of the dimension of the Earth is a consequence of the scale error on the longitudes, which, in turn, was originated by a misidentification of the Islands of the Blessed. The location of the Islands of the Blessed according the source of Ptolemy is identified in the Caribbean. The analysis of a passage by Pliny provides an independent and quantitative confirmation of the proposed identification, which sheds new light on possible contact among civilizations.

1. Shrinking of the Earth and dilation of longitudes in Ptolemy's *Geographia*.

It is well known that Eratosthenes, in the 3rd century BC, measured the circumference of the Earth obtaining the value of 252,000 stadia (corresponding to 700 stadia per degree). Four centuries later Ptolemy, accepting a value suggested by Posidonius in the 1st century BC, estimated the same circumference in 180,000 stadia (corresponding to 500 stadia per degree). The method employed by Eratosthenes is described (at least in its main characteristics[2]) by Cleomedes (*Caelestia*, I, 7, 48-120), while we have no information about the origin of the smaller value. Almost all the scholars have always believed that Eratosthenes and Ptolemy were using the same length unit, even if this statement has been questioned by some historians[3]. Much more contro-

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2 For a discussion on Eratosthenes' method see Russo 2004, 273-76; concerning the possible simplifications introduced by Cleomedes in his popular account, see Russo 2013a, 71-76 (which also contains an attempt to reconstruct the original computation by Eratosthenes).

3 Few scholars had suggested that Ptolemy (and Posidonius before him) had replaced Erastosthenes' stadium with a new stadium corresponding to about 222 meters, so that the two measures of the circumference of the Earth should coincide (Gossellin 1790, Jomard 1822, Valerio 2013). Such a conjecture is contradicted by the explicit statement by Strabo that Posidonius' measure of 180000 stadia had reduced the dimension of the Earth (*Geographica*, II, ii, 2). Furthermore, if Ptolemy had

versial it has been the actual value of the stadium considered by the two. Indeed, many scholars[4] accepted the value of 157.5 m, deduced from a passage by Pliny[5], while several others believed rather that the Olympic stadium (ca. 185 m) was employed[6]; other values were also proposed[7]. Assuming the value of 157.5 m, Eratosthenes would have made an error which is lower than 1%, while for Ptolemy and Posidonius the error is as large as (ca.) 40%. While the estimate of this error relies on the determination of the value of the stadium, another error in Ptolemy's *Geographia*, which can be studied without measure unit issues, is the systematic error on the longitudes. It is well known indeed that the differences of longitudes were systematically dilated by Ptolemy. In particular the total amplitude in longitude of the *oikoumene* (the inhabited part of the world) was grossly overestimated.

In Russo 2013a the longitudes reported by Ptolemy were used to determine the "effective" value of Ptolemy's stadium. The main result of that work was an independent validation of the substantial exactness of the value 157.5 deduced from Pliny, which implies that the estimate for the Earth's circumference by Ptolemy was very wrong. That paper was based on the analysis of longitudes of a sample of 80 cities, chosen as the most renowned of the part of the world best known in Hellenistic times. Plotting Ptolemy's longitudes vs. the values currently accepted for them, a graph was obtained which is approximated remarkably well by means of a linear regression. The equation of the regression line is:

$$y=1.428x+17.06 \qquad (1)$$

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enlarged the length of the stadium used by Eratosthenes by the factor 1.4, it should be inexplicable why he should have dilated in (at least approximately) the same measure also the differences of longitude.

[4] See, among others: Letronne 1851, 104-119, 212-246; Hultsch 1882, 60-63; Tannery 1893, 109-110; Dreyer 1953, 175; Miller 1919, 6-7; Oxé 1963, 269-270; Aujac 1966, 176-179; Fraser 1972, II, 599, n.312; Stuckelberger 1988, 188; Dutka 1993/94, 63-64; Meuret 1998, 163-164, Tupikova and Geus 2013, 21.

[5] Pliny writes: "schoenus patet Eratosthenis ratione stadia XL, hoc est p. " (*Naturalis Historia*, XII, 53). This sentence, using the known value of the schoenus, gives for the stadium the value of 157.5 meters. It is true that in another passage (*N.H.*, II, 247) Pliny translates Eratosthenes' result in 31500 milia passuum (a calculation involving the use of the Olympic stadium of 185 meters), but this circumstance enhances, in my opinion, the worth of the words "Eratosthenis ratione" used in the first case. Indeed, the Olympic stadium was certainly widely used and it is not surprising its automatic use by Pliny, while his need to clarify, in the other case, that he is considering the particular unit introduced by Eratosthenes, gives us a precious testimony. Given the enormous influence of Eratosthenes' *Geographica*, it is not surprising that the new "stadium" introduced by him was adopted as the standard unit in geographical treatises, while the Olympic stadium remained in use for other purposes.

[6] See, among others: Columba 1895: 63–68; Czwalina 1925: 295; Dicks 1960: 42–46; Rawlins 1981: 218; Pothecary 1995: 49-67; Berggren and Jones 2000, 14, footnote 10.

[7] Most of them are analyzed in Tupikova and Geus 2013, 20-22 (where, however, the values outside the interval 148-180 meters are considered quite implausible and the extreme possibility mentioned in the above note 3 is not considered worthy of mention).

and the coefficient of determination is R^2=0.9935. More recently, in Shcheglov 2014 a larger sample of 245 locations (among which there are also river's mouths and capes) was considered, obtaining very similar results. The equation of the regression line is indeed:

$$y=1.4279x+16.425 \qquad (2)$$

and the relative coefficient of determination is R^2=0.9874. This result confirms the previous one, and the slightly lower value of R^2 (which is anyway very close to 1) is likely linked to the fact of having included in the sample locations from lesser known regions such as Northern Europe and India. In the two papers the slope of the regression line is virtually identical[8], and it is close to the ratio 1.4 between Ptolemy's and Eratosthenes' values for the Earth's circumference. We know that Ptolemy generally deduced differences of longitudes from distances expressed in stadia along circles of latitude (mostly taken from Eratosthenes, who in his geographical work, instead of degrees of longitude, had reported distances between meridians along a particular parallel of longitude). It is therefore not surprising that Ptolemy's differences of longitudes were dilated in such a way to compensate, for such distances, the error on the dimension of the Earth (we will return later on the slight difference between 1.4 and the value of the regression coefficient). Hence it is very likely that a logical link does exist between the error on the dimension of the Earth and the error on the differen ces of longitudes.

Some scholars interpreted the link by proposing the following implication:

A - Having Ptolemy assumed a wrong measure for the Earth's circumference, as a consequence he deduced systematically dilated longitudes from his data for the distances along circles of latitude.*

The aforementioned implication[9] is not however a necessary consequence of the link we mentioned before and in the next section we will show that actually several arguments allow us to discard it.

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[8] We notice that the method used in Russo 2013a for estimating the value of the stadium was actually based on the regression coefficient, so that a validation of the value of this coefficient by means of the much larger set of locations considered in Shcheglov 2014 provides an even sounder statistical basis for that estimate.

[9] The implication was maintained, in particular, in Rawlins 1985, Rawlins 2008; Tupikova and Geus 2013; Tupikova 2013. Strangely enough, in Shcheglov 2014 the same opinion is ascribed also to Russo 2013a (where, in this regard, it is only written that "the distortion operated by Ptolemy on the longitudes is not independent of the new value he had assumed for the length of the Earth's circumference") and even to Russo 2013b, where the above implication is explicitly refuted.

2. Re-reading the relationship between the two errors by Ptolemy

A first argument against A* consists in observing that it leaves unsolved the problem of the origin of the error in the Earth's dimension. The value obtained by Eratosthenes was indeed well known, and it is transmitted by ten different extant sources, dating from the I century BC to the V century AD [10]. Posidonius gave two values for the Earth's circumference. For one of them (240,000 stadia), not too far from the one by Eratosthenes, we know the actual procedure by means of which it was obtained (Cleomedes, *Caelestia*, I, 7, 1-47). Had the other one (180,000 stadia) been a result of some measurement procedure, one could wonder three things:

1. Why there is no source describing it?
2. How could it produce such a large error?
3. Why such a grossly wrong estimate, which no extant source considers worth to be described, should have prevailed over the value produced by the highly celebrated measurement by Eratosthenes?

Moreover, a strong argument against A* was provided in Shcheglov 2014. Indeed, if all the differences of longitudes were dilated because of an error in the assumed dimension of the Earth, we should have more or less the same dilation in different regions (at least for the better known ones). On the contrary, splitting his sample in nine subsets corresponding to different geographic areas, Shcheglov verified that linear regressions performed on the different subsets give substantially different values for the slope, in every case with an R^2 very close to 1. These arguments are largely sufficient, in my opinion, to reject the implication A*.

3. How did Ptolemy actually work?

Ptolemy, in his *Geographia*, states that the *oikoumene* is spread over 180° of longitude, from the westernmost locations (four of the six Islands of the Blessed, the other two being located one degree further east) to the easternmost ones (some towns in China). He claims (*Geographia*, I, 12) to have measured the amplitude of the *oikoumene* just by considering a path, from the meridian of the Islands of the Blessed to the Metropolis of the Seres (the capital of China). The path is formed by twelve portions, and the relative lengths are obtained by heavily modifying, with very rough

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[10] Strabo, *Geographica*, II, v, 7, 34; Geminus, *Introduction to the Phenomena*, XVI, 6; Macrobius, *Commentarii in Somnium Scipionis*, I, xx, 20; Vitruvius, *De Architectura*, I, vi, 9; Pliny the Elder, *Naturalis Historia*, II, 247; Censorinus, *De Die Natali*, xiii, 5; Theon of Smyrna, *De Utilitate Mathematicae*, 124, 10-12 (ed. Hiller); Heron of Alexandria, *Dioptra*, xxxv, 302, 10-17 (ed. Schöne); Martianus Capella, *De Nuptiis Philologiae et Mercurii*, VI, 596; Cleomedes l.c. (the last author is the only one giving the rounded value of 250.000).

arguments[11], the data given by Marinus of Tyre. The values obtained after this procedure are finally translated into differences of longitude assuming the aforementioned estimate for the Earth's circumference: 500 stadia for every degree. The result is 177° 15'. Since the easternmost locations known to Ptolemy are just 2°45' at the east of the Metropolis of the Seres, he eventually gets the round value of 180°. It is evident that Ptolemy is cheating with the numbers, as obviously it is very unlikely to reach such a round value after arbitrarily modifying twelve terms in a sum. On the other hand, Ptolemy himself, at the beginning of his *Geographia*, explains his actual method, when he points out the difference between the subject of his work and the chorography:

> The goal of chorography is an impression of a part, as when one makes an image of just an ear or an eye; but [the goal] of geography is a general view, analogous to making a portrait of the whole head. That is, whenever a portrait is to be made, one has to fit in the main parts [of the head] in a determined pattern and an order of priority. Furthermore the [surfaces] that are going to hold the drawings ought to be of a suitable size for the spacing of the visual rays at an appropriate distance, whether the drawing be of whole or part, so that everything will be grasped by the sense [of sight].[12]

Out of the metaphor, it is clear that Ptolemy, having to represent the whole *oikoumene*, wants first to fix the positions of some key locations, and in particular its global dimension, and only afterwards to add the coordinates of all the remaining localities.

One may wonder in which sense the position of these key locations were fixed, whether by means of their angular coordinates or their relative distances. In this regard, it can be recalled that Ptolemy himself explains that the astronomical method, from which only angular measures can be derived, is to be preferred to the surveying which could provide distances in stadia:

> The surveying component is that which indicates the relative positions of localities solely through measurement of distances; the astronomical component [is that which does the same] by means of the phenomenon [obtained] from astronomical sighting and shadow-casting instruments. Astronomical observation is a self-sufficient thing and less subject to error, while surveying is cruder and incomplete without [astronomical observation].[13]

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[11] For instance dividing a value by two to account for the tortuosity of the roads.

[12] Ἔχεται δὲ τὸ μὲν χωρογραφικὸν τέλος τῆς ἐπὶ μέρους προσβολῆς, ὡς ἂν εἴ τις οὖς μόνον ἢ ὀφθαλμὸν μιμοῖτο, τὸ δὲ γεωγραφικὸν τῆς καθόλου θεωρίας κατὰ τὸ ἀνάλογον τοῖς ὅλην τὴν κεφαλὴν ὑπογραφομένοις. Πάσαις γὰρ ταῖς ὑποτεθειμέναις εἰκόσι τῶν πρώτων μερῶν ἀναγκαίως καὶ προηγουμένως ἐφαρμοζομένων, καὶ ἔτι τῶν δεξομένων τὰς γραφὰς συμμέτρων ὀφειλόντων εἶναι ταῖς ἐξ ἀποχῆς αὐτάρκους τῶν ὄψεων διαστάσεσιν, ἐάν τε τέλειον ᾖ τὸ γραφόμενον ἐάν τ' ἐπὶ μέρους, ἵν'ἅπαν αἰσθητῶς παραλαμβάνηται […]. (Ptolemy, *Geographia*, I, 1, 2-3, adapted from the translation by J.L. Berggren and A. Jones).

[13] Γεωμετρικὸν μὲν τὸ διὰ ψιλῆς τῆς ἀναμετρήσεως τῶν διαστάσεων τὰς πρὸς ἀλλήλους θέσεις τῶν τόπων ἐμφανίζον, μετεωροσκοπικὸν δὲ τὸ διὰ τῶν φαινομένων ἀπὸ τῶν ἀστρολάβων καὶ σκιοθήρων

The order in which the operations were performed is indeed explicitly indicated in Ptolemy's *Geographia,* in the title of chapter I, 4:

> That it is necessary to give a priority to the [astronomical] phenomena over [data] from records of travel.[14]

Shortly after he points out:

> It would therefore also be reasonable for one who intended to practice geography following these [principles] to give priority in his map to the [features] that have been obtained through the more accurate observations, as foundations, so to speak, but to fit [the features] that come from the other [kinds of data] to these, until their positions with respect to each other and to the first [features] stand as much as possible in agreement with those reports that are less subject to error.[15]

From these passages, we can deduce that Ptolemy first fixed a few longitudes known by astronomical methods (in particular those of some extreme points determining the amplitude in degrees of the *oikoumene*), and then he interpolated the longitudes of the intermediate locations by using known distances along circles of latitude. Therefore, the first error among the two mentioned in the previous section has necessarily to be the one on longitudes. Hence, as already said, A* has to be discarded. Since, on the other hand, Ptolemy's scale error on longitudes, combined with a set of right distances along circles of latitude, necessarily implies his error on Earth's dimension, we are led to propose the following implication:

*A** - Ptolemy's error on the amplitude in longitude of the oikoumene was the cause of his systematic dilation of the differences of longitude and of his deduction, from the known data for the distances along circles of latitude, of an underestimation of the dimension of the Earth.*

As for the source of the scale error on longitudes, we start by observing that, since Ptolemy emphasizes the importance of astronomical data provided by Hipparchus (*Geographia*, I, 4, 1), we can conjecture that he took from him also the value of 180° for the amplitude of the *oikoumene*. An argument supporting this is provided once again by Ptolemy himself, in his *Almagest*, when he states that the *oikoumene*

ὀργάνων· τοῦτο μὲν, ὡς αὐτοτελές τι καὶ ἀδιστακτότερον, ἐκεῖνο δὲ, ὡς ὁλοσχερέστερον καὶ τούτου προσδεόμενον. (Ptolemy, *Geographia*, I, 2, 2, translation by J.L. Berggren and A. Jones).

14 Ὅτι δεῖ τὰ ἐκ τῶν φαινομένων τηρούμενα προϋποτίθεσθαι τῶν ἐκ τῆς περιοδικῆς ἱστορίας. (Ptolemy, *Geographia*, I, 4, T, translation by J.L. Berggren and A. Jones).

15 Εὔλογον ἂν εἴη καὶ τὸν τούτοις ἀκολούθως γεωγραφήσοντα τὰ μὲν διὰ τῶν ἀκριβεστέρων τηρήσεων εἰλημμένα προϋποτίθεσθαι τῇ καταγραφῇ καθάπερ θεμελίους, τὰ δ' ἀπὸ τῶν ἄλλων ἐφαρμόζειν τούτοις, ἕως ἂν αἱ πρὸς ἄλληλα θέσεις αὐτῶν μετὰ τῶν πρὸς τὰ πρῶτα τηρῶσιν ὡς ἔνι μάλιστα συμφώνως τὰς ἀδιστακτοτέρας τῶν παραδόσεων. (Ptolemy, *Geographia*, I, 4, 1, adapted from the translation by J.L. Berggren and A. Jones).

occupies approximately one fourth of the earth surface bounded by half of the equator and the halves of two opposite meridians. The amplitude of 180° is justified in the following passage:

> In the case of longitude (that is in the east-west direction) the main proof is that observations of the same eclipse (especially a lunar eclipse) by those at the extreme western and extreme eastern regions of our part of the inhabited world (which occur at the same [absolute] time) never differ by more than twelve equinoctial hours [in local time]; and the quarter [of the earth] contains a twelve-hour interval in longitude, since it is bounded by one of the two halves of the equator.[16]

The reference to the method proposed by Hipparchus for measuring differences of longitude from local times of lunar eclipses[17] gives a clue in support of our hypothesis, which is also consistent with the fact that the amplitude of 180° for the *oikoumene* was already transmitted, before Ptolemy and Marinus, by Strabo. Indeed, shortly after having recalled the method used by Hipparchus for determining differences of longitude (*Geographica*, I, 1, 12), Strabo observes that people living in the extreme eastern regions were in a sense the antipodes of the ones living at the extreme west of Iberia (*Geographica*, I, 1, 13).

If the amplitude of the *oikoumene* was taken from Hipparchus, and was at the origin of the wrong estimate of the Earth's circumference, it is understandable that this last error was already made by Posidonius, whose main source in astronomical and geographical matters was probably Hipparchus himself[18]. Furthermore, the rejection of the celebrated result by Eratosthenes becomes understandable if it was based on the presumption to follow Hipparchus, the famous scientist whose successful criticism toward Eratosthenes was well known.

Finally, we can notice that the discrepancy among the ratio 1.4 between the two estimates for the Earth's circumference and the value 1.428 of the regression coefficient concerning the longitude dilation, though small, is not negligible and hardly compatible with A*, in view of the high value of R^2. This discrepancy becomes easily understandable by adopting the implication A**. A contraction of Eratosthenes' value of 700 stadia per degree by a factor 1.428 leads indeed to a value of 490.2 stadia

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16 ἐπὶ δὲ τοῦ μήκους, τουτέστιν τῆς ἀπὸ ἀνατολῶν πρὸς δυσμὰς παρόδου, διὰ τοῦ τὰς αὐτὰς ἐκλείψεις, μάλιστα δὲ τὰς σεληνιακάς, παρά τε τοῖς ἐπ' ἄκρων τῶν ἀνατολικῶν μερῶν τῆς καθ' ἡμᾶς οἰκουμένης οἰκοῦσι καὶ παρὰ τοῖς ἐπ' ἄκρων τῶν δυτικῶν κατὰ τὸν αὐτὸν χρόνον θεωρουμένας μὴ πλέον δώδεκα προτερεῖν ἢ ὑστερεῖν ὡρῶν ἰσημερινῶν αὐτοῦ κατὰ μῆκος τοῦ τεταρτημορίου δωδεκάωρον διάστημα περιέχοντος, ἐπειδήπερ ὑφ' ἑνὸς τῶν τοῦ ἰσημερινοῦ ἡμικυκλίων ἀφορίζεται. (Ptolemy, Almagest II, 1, p. 88 [ed. Heiberg], 10-19, translation by G.J.Toomer).

17 Obviously it is not possible to observe the same lunar eclipse from two opposite semi-meridians (at least without considering unrealistic observations made from points very close to the poles), but it is well possible to obtain the total longitude between them by summing two (or more) smaller longitude differences.

18 We recall that Hipparchus had worked in the same Rhodes where Posidonius, a generation later, had established his school.

per degree, but it is reasonable that Ptolemy (and Posidonius before him), having to replace an estimate given by a round figure[19], wanted to select an equally round figure, therefore choosing 500.

4. Global and local errors

Let us go back to chapter I, 4 of Ptolemy's *Geographia*:

> […] most distances, especially the east-west ones, have come down to us in a less precise form, not through the negligence of those who devoted themselves to research, but perhaps because they had not yet understood the usefulness of more scientific methods and because they had not observed many lunar eclipses at the same time in different places (such as the one that was seen in Arbela at the fifth hour and at Carthage at the second hour), from which it would have been clear how many equinoctial time units separated the localities to the east or west.[20]

The mention of people who had "not yet" (μηδέπω) understood the usefulness of the method based on lunar eclipses for determining the longitudes implies that such usefulness was clear to later scholars. Ptolemy indeed had used such astronomical method in the case of Arbela and Carthage[21] and most probably, as already observed, in some other cases.

Suppose that Ptolemy had obtained all his longitudes by dividing the (wrong) amplitude of the *oikoumene* assumed by him in parts proportional to the known distances along circles of latitude. In this case, and in absence of large systematical errors on distances, we should expect that Ptolemy's longitudes were well approximated everywhere by the same linear function of the actual ones, all differences of longitude being dilated almost exactly in the same proportion the *oikoumene* as a whole is.

If on the other hand, as suggested by Ptolemy himself, the interpolation procedure started after the insertion of some milestone-like astronomical data, which hard-

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19 The aforementioned passage by Pliny about the stadium according to the "Eratosthenis ratio" suggests that in the case of Eratosthenes the round figure could have been the result of the definition of the new stadium as a convenient submultiple of the circumference of the Earth (252,000 is a particularly convenient number, because it is divisible by all numbers from 1 to 10; their Least Common Multiple is in fact 2,520).

20 τὰ δὲ πλεῖστα τῶν διαστημάτων καὶ μάλιστα τῶν πρὸς ἀνατολὰς ἢ δυσμὰς ὁλοσχερεστέρας ἔτυχε παραδόσεως, οὐ ῥᾳθυμίᾳ τῶν ἐπιβαλόντων ταῖς ἱστορίαις, ἀλλ' ἴσως τῷ μηδέπω τὸ πρόχειρον κατειλῆφθαι τῆς μαθηματικωτέρας ἐπισκέψεως, καὶ διὰ τὸ μὴ πλείους τῶν ὑπὸ τὸν αὐτὸν χρόνον ἐν διαφόροις τόποις τετηρημένων σεληνιακῶν ἐκλείψεων, ὡς τὴν ἐν μὲν Ἀρβήλοις πέμπτης ὥρας φανεῖσαν, ἐν δὲ Καρχηδόνι δευτέρας, ἀναγραφῆς ἠξιῶσθαι, ἐξ ὧν ἐφαίνετ' ἂν πόσους ἀπέχουσιν ἀλλήλων οἱ τόποι χρόνους ἰσημερινοὺς πρὸς ἀνατολὰς ἢ δυσμάς· (Ptolemy, *Geographia*, I, 4, 1).

21 See below, p.10.

ly could well fit the above linear relation[22], the set of all the considered locations should have broken in regional subsets such that:

a) In each subset the longitudes, being obtained with an interpolation procedure, are very close to a linear function of the actual longitudes, so that we should get in any case very high values of the coefficients of determination R^2.

b) The regression coefficients corresponding to different subsets should be considerably different from each other.

c) The longitudes inserted on an astronomical basis should mark the fracture points between different subsets.

d) The deviations of the different regression coefficients from the global dilation ratio should compensate each other in such a way that the set of all the longitudes can be well approximated by a linear regression with a coefficient equal to such dilation ratio.

To these points, one can add a further observation. The data given by Ptolemy are interconnected by a very complex chain of relations. In particular, he states (*Geographia*, I, 4, 1) that in some cases he has the information that two locations are on the same meridian. Considering this kind of interconnection in relation with the previous reasoning, it is clear that:

e) There exist the possibility that two sets of data covering more or less the same longitude area (but coming from different latitudes), are broken in subsets in such a way that the fracture points are approximately at the same longitude for both.

The above scenario, suggested by Ptolemy's exposition, describes well the actual features of the longitudes reported in his *Geographia*, as it is apparent from the analysis made in Shcheglov 2014. Such analysis, which allowed Shcheglov to correctly discard the thesis A*, is not only consistent with A**, but gives a strong argument in its support once one takes into account that, in addition to the systematic dilation, significant and well characterized errors due to the insertion of some longitudes deduced from astronomical data are to be expected. In this regard, the fact that the sample chosen by Shcheglov as a whole leads to a value of R^2 that is close to 1 (0.9874) should not be, in my opinion, liquidated as "deceptive" (as Shcheglov does), but provides a key element for the reconstruction of the overall framework.

We cannot identify all the cases in which Ptolemy used astronomical data concerning longitudes, but in a significant case it is possible to reasonably reconstruct the

22 We can however imagine that Ptolemy could have discarded data excessively far from it.

origin of a local error. Let us start by observing that the presence of local regression coefficients which largely differ from the global one is especially significant in areas that were well-known in Hellenistic times. In this regard, the data coming form the south and north coast of the Mediterranean sea are particularly relevant. Analyzing the data shown in Fig. 6 of Shcheglov 2014, one can see that both the southern and the northern coast of the Mediterranean can be divided in two zones (eastern and western) presenting substantially different values for the regression coefficient. Specifically, the regression coefficients are 1.81 and 1.19 for the western and eastern part of the south coast; 1.77 and 1.15 for the western and eastern part of the north coast. It is also to be noticed that the fracture points between the two different slopes correspond, for both north and south coast[23], approximately to the same longitude (in modern terms, slightly more than 10° E). This would be explained by the fact that, for some location at a longitude slightly more than 10° E, Ptolemy had an astronomical datum inconsistent with the systematic dilation of longitudes. It is then perhaps not by chance that that longitude corresponds very well to one of the extrema (Carthage, 10°19') of the only longitude interval for which Ptolemy explicitly states of knowing an astronomical datum. The difference of longitude given by Ptolemy between Arbela and Carthage (45°10') corresponds indeed very well to the one deducible from the three hours of difference mentioned in the previous passage. On the other hand, it is dilated by a factor of approximately 1.30 with respect to the actual difference, a value which significantly differs from the global regression coefficient[24]. If one considers that the longitude given by Ptolemy for Arbela places it almost exactly on the global regression line, this strongly suggests that Ptolemy, taking into account the astronomical datum for determining the longitude of Carthage[25], may have had to adjust the other data which were linked to it by distance relations or other more complex kinds of interconnections. This hypothesis can explain: i) the different slopes observed for eastern and western areas; ii) the high values of R^2 for both the subsets; iii) the circumstance that the regression coefficient for the union of the considered subsets agrees with a good approximation with the global dilation ratio[26]; iv) the fact that the

[23] See point e) from the previous list.

[24] We notice that, as a consequence, the longitudinal distance of Carthage from the Pillars of Heracles is dilated by Ptolemy by the factor 1.74, in good agreement with the regression coefficients (1.81 and 1.77) found by Shcheglov for the western part of the Mediterranean Sea.

[25] The poor approximation of this particular value is easily understandable, if we take into account that it relies on non-scientific descriptions of the eclipse, dating to the fourth century BC, two centuries before that Hipparchus had suggested that such kinds of data could be used to determine differences of longitude.

[26] A linear regression performed over the whole set of locations indicated in Shcheglov 2014 as the "north coast" and "south coast" of the Mediterranean gives indeed the line $y=1.437x + 17.00$, with $R^2=0.980$. The very low values of the regression coefficients found by Shcheglov for the eastern part of the Mediterranean Sea (1.19 and 1.15) are thus the right ones to compensate the overestimation of the dilation on the other side.

northern and southern coasts of the Mediterranean are broken in subsets with different slopes more or less at the same longitude[27].

A further confirmation of the above reconstruction comes from a comparison between the global regression coefficient (1.428) and the ratio between the actual longitudinal amplitude for the Ptolemy's *oikoumene* and the value of 180° accepted by him.

The westernmost locations considered by Ptolemy are the Islands of the Blessed, which he identified with the Canary Islands[28]. In order to estimate the global dilation of the *oikoumene*, on the other extreme we consider the Sera Metropolis (today Xi'an[29]), since more eastern locations are not unanimously identified today. Since the actual longitudes of Xi'an and the Canary Islands[30] are respectively 108°54' E and 15° W, and the corresponding values given by Ptolemy are 177°15' and 0°, the ratio between their difference of longitude given by Ptolemy and the corresponding actual value is approximately 1.43, and thus very close to the regression coefficient given in Russo 2013a and Shcheglov 2014.

Of course a detailed reconstruction of the procedure followed by Ptolemy is very difficult, and actually probably impossible, since it also should take into account the presence of local errors due to the different level of geographical knowledge for the various regions, and the complex interconnections between his data that was mentioned before. Nevertheless, as we saw, some firm points could be established, the most important of which is a general dilation of all the differences of longitude by a mean factor equal to the dilation of the whole *oikoumene*. The implication A** is thus confirmed.

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27 See again point e) before.

28 This identification was usual in imperial times. It appears for the first time implicitely in the work of Pomponius Mela, who places the Islands of the Blessed in front of the Atlas Mountain (*De chorographia*, III, 101-102). In Pliny's *Naturalis Historia* (VI, 202-203) the identification is even clearer, since he places them in front of the Mauretania and gives with a good approximation their distances from Gades (Càdiz). As for Ptolemy (who hardly dares to question knowledge widely accepted in his days), he gives names and coordinates of six "Island of the Blessed (Μακάρων νῆσοι)" (*Geographia*, IV, 6, 34). Their identification with the Canaries is implied by their longitude, and more importantly by the names of the islands (three of Ptolemy's names are obvious Greek correspondents of latin names given by Pliny: Canaria, Junonia and Pluvialia). The slight discrepancy between the number given by Ptolemy (six) and the actual number (seven) of the major islands of the archipelago can be explained in many ways. Almost all scholars agree with this identification (See for instance, Stückelberger and Graßhoff 2006, 455, footnote 200). Nevertheless, a few scholars have questioned the identification with the Canaries, on the basis of the latitude given by Ptolemy, which is very far from that of the Canaries. We shall return on this point.

29 For the identification of Sera Metropolis with Xi'an see Stückelberger and Graßhoff 2006, 669, note229.

30 As the modern value for the Canary Islands we take the value of 15° W, which is the best rounded value for the average longitude.

However, we still did not attack the point on which our first criticism to A* was based, i.e, its failure in explaining the origin of the double mistake, which is left by that hypothesis in the darkest obscurity. We will devote the next section to test the proposed thesis A** against this last problem.

5. The origin of the error on longitudes by Ptolemy.

We conjectured that the value of 180° for the amplitude of the *oikoumene*, i.e. the difference in longitude between the Islands of the Blessed and the easternmost regions, was taken from a Hellenistic source (most probably Hipparchus, who may well have intended it as a rounded value). On the other hand we know that Hellenistic scientists, and in particular Hipparchus, had accepted the value of Eratosthenes for the Earth's circumference (Strabo, *Geographica,* I, iv, 1; II, v, 7; II, v, 34), while the smaller estimate, grossly wrong, was first introduced (as far as we know) by Posidonius (Strabo, *Geographica*, II, ii, 2). Why this value of 180°, which according to Hipparchus was consistent with the measurement by Eratosthenes, coexists with a much "smaller" Earth in (among others) Ptolemy? Since there is a substantial agreement in the distances along the circles of latitude between Ptolemy's and Eratosthenes' data, it is clear that the only possibility is that there was a misinterpretation on the identification of one of the two extrema of the *oikoumene*. Two arguments allow us to exclude that the issue involved locations in the far East:

1. It is *a priori* easier to misinterpret the identification of oceanic islands, for which no close locations are known for very large distances, than that of a city which is reachable through a series of intermediate locations.

2. To produce an error of the order of magnitude of that made by Ptolemy, the original location of some town in China assumed by his source as the eastern extremum of the *oikoumene* had to be very far into the Pacific Ocean.

The only remaining possibility, therefore, is that the two errors were originated by an erroneous identification of the Islands of the Blessed.

The previous reasoning, which up to now was intended at analyzing the origin of the errors in Ptolemy's *Geographia*, can also disclose new possibilities in a more general (and possibly more relevant) issue. We are indeed led to the conclusion that the Islands of the Blessed, to which the Hellenistic source of Ptolemy (most probably Hipparchus) referred, can be found approximately on the opposite semi-meridian with respect to the more eastern locations cited in the *Geographia*. This entails the need to consider longitudes that are much farther on the west than those believed as known in Hellenistic times. We arrived at this conclusion by elimination, after having considered and discarded every possible alternative. Therefore, in my opinion, we should seriously consider the extreme consequences of the previous reasoning. As a renowned investigator used to say, *when you have eliminated the impossible, whatever remains, however improbable, must be the truth*.

6. Islands of the Blessed and Caribbean

The elimination process illustrated before conduced us to the conclusion that the first cause for the errors made by Ptolemy was the wrong identification of the Islands of the Blessed with the Canary Islands. One may wonder how was it possible such a misinterpretation of the sources. It is therefore useful, in my opinion, to briefly recall some key historical processes which played a relevant role in this connection.

The mentions of travels in the Atlantic Ocean are not negligible in ancient sources[31]. However, between Hellenistic age and Ptolemy, a relevant loss of geographic knowledge concerning this Ocean occurred, and in a short time even well known descriptions of voyages started to be considered unreliable. The report of the famous expedition made by Pytheas, for instance, in which he described the iced sea and the midnight sun, was considered trustworthy by Hellenistic scientists such as Eratosthenes and Hipparchus [32], but was later rejected by Strabo (*Geographica*, II, iv, 1; II, iv, 2; II, iii, 5), was not copied and transmitted anymore, and finally lost.

The error made by Ptolemy according to our reconstruction was made possible by the combination of the loss of knowledge about the Atlantic Ocean with a general weakening of science in the Roman world with respect to the Hellenistic age[33]. Geography, in particular, was transformed radically, from the mathematically founded subject it was in Hellenistic times to the purely descriptive one it became in works like those by Strabo and Polybius. An error like the misidentification of an archipelago became clearly much more probable once the use of spherical coordinates (latitude and longitude) for the identification of the locations had been abandoned, as it was in the first century BC. The oldest evidence concerning the identification of the Islands of the Blessed with the Canaries dates indeed from the first century BC[34], and then was inherited by Ptolemy when he attempted at reconstructing quantitatively the mathematical geography for the first time after the methodological crisis had occurred. Posidonius seems the most probable candidate as the source of the misidentification. Indeed he is to our knowledge the most ancient source giving the length of 500 stadia per degree for the Earth's circumference, which tend to suggest that the transition from the "ancient" to the "new" value occurred within his work.

A quantitative analysis of the data given by Ptolemy strongly supports the idea of a mistaken identification of the archipelago. Indeed:

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[31] For a review of classical sources on travels in the Atlantic Ocean see Roller 2006.

[32] See, among other passages: Hipparchus, *In Arati et Eudoxi phaenomena commentariorum libri iii*, I, 4, 1; Strabo, *Geographica*, I, 1, 9; II, 1, 12; II, 4, 2.

[33] On this point see Russo 2004, passim.

[34] See above, footnote 28.

1. The latitude he gives for the Islands is wrong by about 15°, an enormous error which can be regarded as incompatible with any data coming from real measures and can be only explained as the result of a confusion between two different archipelagos.

2. The Canaries are spread over a total longitude of about four and a half degrees, while the archipelago considered by Ptolemy covers just one degree in west-east direction; moreover, the Canary Islands cover less than two degrees in north-south direction, while Ptolemy's Islands five and a half. In conclusion, the two archipelagos have both a strip-like shape, but they are approximately oriented in orthogonal directions.

A decisive test in order to check the proposed thesis is at this point possible. We can indeed compute the original longitude of the Islands of the Blessed simply taking a difference of longitude of 180° with respect of Ptolemy's eastern extreme locations, and verify whether we come close to some archipelago. The longitude of the eastern bound of Ptolemy's *oikoumene* can be computed by the regression line (1) given above, solving the equation:

$$180 = 1.428\ x + 17.06$$

In this way, we eventually get the following coordinates of the Islands:

65°54'W, 16°N
64°54'W, 15°15'N
65°54'W, 14°15'N
65°54'W, 12°30'N
64°54'W, 11°N
65°54'W, 10°30'N

These locations are graphically represented in Fig. 1:

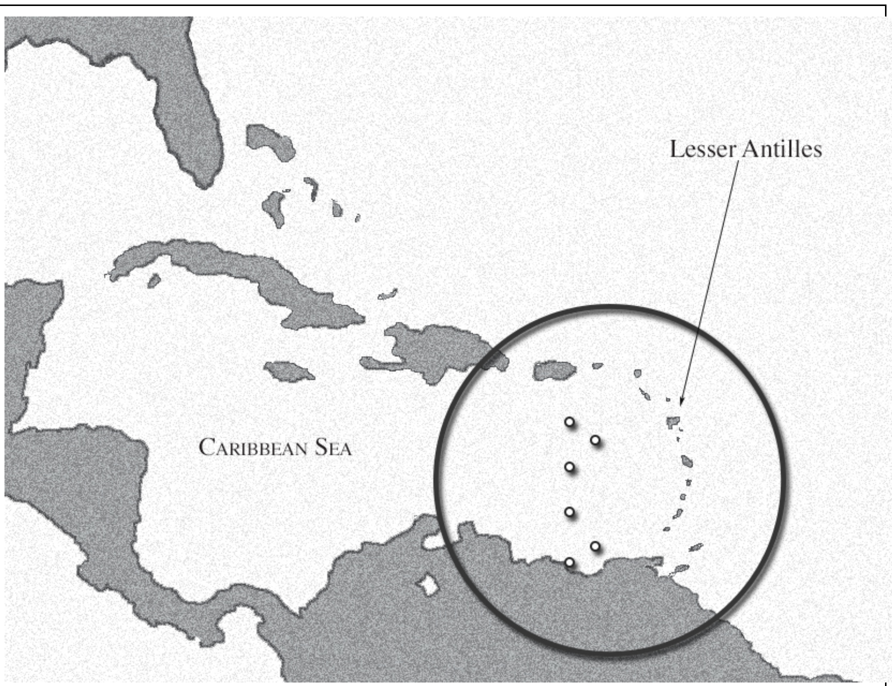

Fig. 1 – The circlets indicate the original locations of the Island as computed in the text.

As the reader can see, not only we come very close to an actually existing archipelago (formed by the Leeward Islands and the Windward Islands, in the Lesser Antilles), but also its dimension, shape and orientation correspond well to the dots. If we compute the eastern extreme using the regression line (2), given in Shcheglov 2014, we obtain an even slightly better match with the Leeward and Windward Islands, since all the archipelago would translate towards the east by approximately 0.44 degrees.

The available descriptions of the Islands of the Blessed in the sources can provide further arguments in support of this proposed (and admittedly surprising) identification. In this regard, one can observe that there is a substantial consistency between the numerous descriptions we have dating from the archaic and classical periods, and that these descriptions are hardly reconcilable with the identification with the Canaries. The issue was examined in detail in Manfredi 1993; in his opinion, some striking characteristics mentioned frequently in the sources (and especially the presence of lush and evergreen foliage) can exclude the possibility that they referred to the Canaries, which are generally bleak, and would rather suggest islands in the Caribbean sea (Manfredi 1993, 204).

In the following section a further independent and quantitative confirmation of the proposed thesis will be provided.

7. A passage by Pliny

The following passage by Pliny has eluded, until now, any reasonable interpretation:

> Regarding the length and the breadth this is what I deem worthy of mention. For the whole circumference, Eratosthenes, a man highly regarded by all and surpassing others in every subtlety of learning, and especially in the present matter, gave the value of 252,000 stadia. [...] Hipparchus, a man to be admired for taking issue with him and for much more besides, then added to that number a little less than 26,000 stadia[35].

The last statement by Pliny is contradicted by the passages by Strabo in which it is told that Hipparchus accepted the value given by Eratosthenes for the Earth's circumference (*Geographica*, I, iv, 1; II, v, 7; II, v, 34). The key in order to understand the previous passage, in my opinion, is contained in its first words. Since it makes no sense to talk about the length and the breadth of a spherical object such as the Earth, and since immediately before Pliny was discussing the dimensions of the *oikoumene*, it is very plausible that the subject of the source (misunderstood by Pliny, who had in mind the most celebrated measurement by Eratosthenes) was not changed, since it was usual in geographical works to give length and breadth of the *oikoumene*. We indeed know that Eratosthenes, besides the circumference of the Earth, had also calculated the length of the *oikoumene* along the parallel of Athens, getting the result of 77,800 stadia, of which 5,000 at the west of the Pillars of Heracles (Strabo, *Geographica*, I, iv, 5). The passage can therefore be explained conjecturing that Hipparchus had extended in longitude the *oikoumene* by adding almost 26,000 stadia to the value given by Eratosthenes. Since this extension could hardly have affected the Pacific Ocean, we have to conclude that the western bound of the *oikoumene* according to Hipparchus was at 31,000 (26,000 + 5000) stadia from the Pillars of Heracles along the parallel of Athens. Since the cosine of the latitude of Athens is about 0.788, a degree of longitude along this parallel, according to Eratosthenes, corresponds to about 700 × 0.788 ≈ 552 stadia, which implies that we reach a location which is approximately 56°10' at the west of Gibraltar, corresponding to a longitude of 61°31'W. The corresponding semi-meridian is graphically represented in Fig. 2, and represents a striking confirmation of the thesis that Hipparchus knew the coordinates of the Leeward and Windward Islands, and also of the correctness of our reconstruction of the meaning of the passage by Pliny. Moreover, it indirectly supports the idea that the source of Ptolemy on the Islands of the Blessed was Hipparchus. We notice that it is not surprising that the match between theoretical previsions and actual geo-

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[35] De longitude ac latitude haec sunt, quae digna memoratu putem. Universum autem circuitum Eratosthenes, in omnium quidem literarum subtilitate, et in hac utique praeter ceteros sollers, quem cunctis probari video, CCLII milium stadiorium prodidit, [...] Hipparchus, et in coarguendo eo, et in reliqua omni diligentia mirus, adicit stadiorum paulo minus XXVI milia (Piny, *Naturalis Historia*, II, 247).

graphical data is in Fig. 2 much better than in Fig. 1, since in that case the reconstruction had a statistical basis and relied on the value of 180° for the amplitude of the *oikoumene,* which was possibly rounded, while in this case the method only uses one quantitative datum taken from the sources.

Of course, the addition of 26,000 stadia by Hipparchus has to be considered not a correction of the value given by Eratosthenes, but rather an updating due to new geographical discoveries.

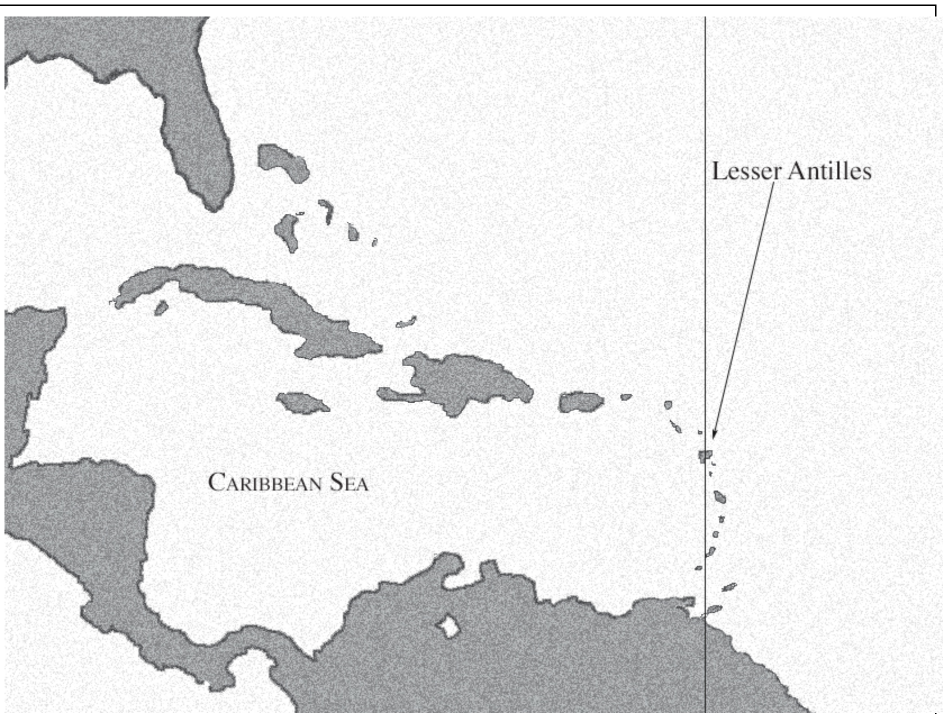

Fig. 2 – the westernmost longitude of the *oikoumene* according Hipparchus, as recovered by the Pliny's passage.

8. Conclusions

We want here to summarize the conclusions of the present paper and the arguments that can be considered in their support.

The main thesis can be expressed as follows:

Some source of Ptolemy's Geographia knew with remarkable precision the position of some locations in the Caribbean Sea, i.e. the Leeward and Windward Islands. These islands were the locations originally intended as the "Islands of the Blessed". Their misidentification with the Canaries first occurred in the first century BC and was then accepted by Ptolemy.

The previous thesis, which is plausible in view of the loss of knowledge concerning the Atlantic Ocean and the disuse of the spherical coordinates occurred between Hipparchus and Ptolemy, can be obtained, via an abductive reasoning, from the following facts, for which so far has not been offered any explanation:

1. The thesis is implied by a procedure performed by Ptolemy which is suggested by his own words and in turn explains both: i) the very high values for the determination coefficient (and the virtually identical regression coefficients) found in Russo 2013a and Shcheglov 2014; ii) the significant differences between regression coefficients relative to different regions shown in Shcheglov 2014.
2. It explains why the archaic and classical descriptions on the Islands of the Blessed match better with Caribbean islands rather than the Canaries (as discussed in Manfredi 1993).
3. It explains why Ptolemy gives to the Canary Islands the shape of an archipelago extending in a direction which is approximately orthogonal (i.e. as far as it could be) to the actual one.
4. It explains the enormous error (15°) made by Ptolemy on the latitude of the Canary Islands.
5. It provides a simple (and quantitatively accurate) explanation of the systematic dilation of differences of longitudes operated by Ptolemy.
6. It explains as a simple consequence of the previous point the new estimate in the measure of the Earth accepted by Posidonius and Ptolemy (even more so because the ratio between the old and the new estimate is close, but not perfectly coincident, with the dilation coefficient).
7. It explains s the striking match between the shape and the position of the Leeward and Windward Islands and the locations indicated by Ptolemy as the "Islands of the Blessed", once their original coordinates are reconstructed by means of the statistical approach herein considered.
8. It provides a simple (and quantitatively accurate) explanation for the passage by Pliny before considered.

The result of the present work may hopefully re-open the problem of the existence of ancient trans-oceanic contacts: a thesis up to now rejected by the great majority of scholars. A considerable amount of evidence in its support, of different nature with respect with the one here discussed, is available in Sorenson and Johannessen (2009), where a detailed bibliography of related literature is also provided. For a discussion of the cultural impact of these results, the reader is referred to Mair (2006).

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