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# A Family of Prediction Tools for Fine Ceramic Fibers Subcritical Cracking

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## 1 Introduction

The delayed rupture of SiC fibers under given mechanical and environmental conditions is due to the subcritical propagation of their surface defects [3, 4]. Subcritical propagation, also known as slow crack growth, has been widely studied in the literature, especially for ceramics. In the case of glasses, the physics is relatively simple –amorphous solid, active damaging reaction– and a physically-based propagation law can be derived [8, 10]. However, in practice the authors propose the use a phenomenological three-parameter law, close to the Paris law and sometimes called Evans-Wiederhorn law, relating the crack's velocity to the crack's stress intensity factor with a dependence on temperature [11, 9]. This law has been successfully applied to the case of CMCs [2, 5] and SiC fibers [3, 4], where the physics is somehow more complex –inhomogeneous solid, passive damaging reaction–, but is not directly suitable for varying oxidative or corrosive agents concentration around the crack.

A new modelling approach to subcritical propagation has been recently introduced [6], which seems to be a good compromise between physically-sound and phenomenological. It relies on classical fracture mechanics, in which the environment modifies the damage zone at the crack's tip, and then drives the progressive propagation of the crack. A two-parameter intrinsic material law –independent of temperature, oxygen pressure, time, etc– describes this degradation with regard to the oxygen flux reacting at the crack's tip. Consequently, the model is valid for any temperature or oxygen pressure, which can be functions of time, and the only difficulty consists in modeling the oxidizing environment of the crack properly.

In this paper, the framework is rapidly recalled, and emphasis is put on the several lifetime prediction tools that can be derived for Hi-Nicalon fibers up to 1,000° C in dry environment, regarding the complexity of the chemical environment modelling. For instance, the introduction of the oxide layer generated around the fiber is straightforward, and allows to unify reaction- and diffusion-controlled propagation stages. Identification, as well as a first validation on fiber bundles, based on [3, 4]'s experimental data, are also presented.

## 2 The fiber subcritical cracking problem

Classical Weibull theory and associated idealizations are used to describe the distribution of defects over the fiber's surface [3, 6]. The growth of surface defects is assumed to be a simple one-dimensional problem in which the size of a defect is described by a scalar parameter (see Figure 1) [3, 6]. The Stress Intensity Factor (SIF) induced by a surface defect of size  $a$  in a fiber subjected to a stress  $\sigma$  writes:

$$K = \sigma Y \sqrt{a} \quad (1)$$

where  $Y = 2/\sqrt{\pi}$  is the shape coefficient associated with the crack assumed to be constant throughout the propagation. This modeling framework introduces two important quantities for a fiber with Critical Stress Intensity Factor (CSIF) denoted  $K_c$  (see Figure 1): i) the initial size of the largest surface defect,  $a_0 = (K_c/\sigma_R Y)^2$ ; and ii) the critical size of the defects,  $a_c = (K_c/\sigma Y)^2$ . Thus, the subcritical cracking problem is to derive a set of equations defining the evolution of the crack's size  $a(t)$  from  $a_0$  to  $a_c$  (see Figure 1).

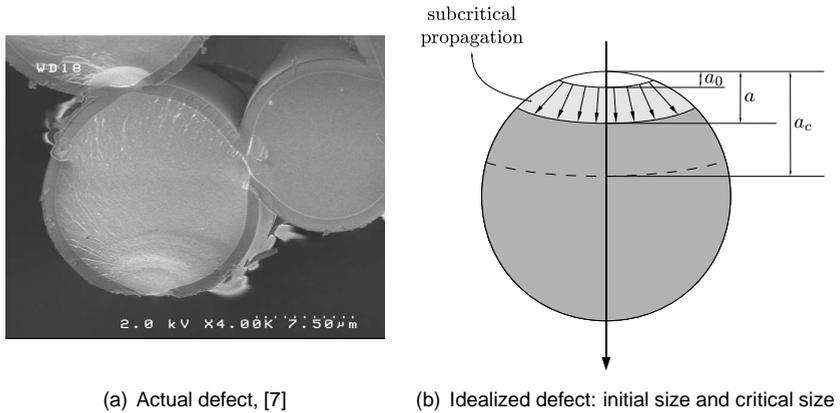


Figure 1: Surface defect in a cross section of a fiber

## 3 Modelling subcritical propagation

The several ingredients of the subcritical propagation modelling framework introduced in [6] are:

- The **propagation law**, which is the classical equation of fracture mechanics:

$$K(a(t), t) = K_{sc}(t) \quad (2)$$

where  $K_{sc}$  is the SubCritical Stress Intensity Factor (SCSIF), which is smaller than the CSIF because of the oxidation.

- A **damage law**, which describes the dependence of  $K_{sc}$  on the environment:

$$\frac{K_{sc}(t)}{K_c} = \lambda \left( \frac{\dot{a}(t)}{\phi_0(t)} \right)^n \quad (3)$$

where  $\phi_0$  is the reaction rate of the damaging reaction at the fiber's surface.

This can lead to several models depending on the complexity of the description of the chemical environment surrounding the crack:

- We can first consider that the fiber's surface is in direct contact with the ambient air. In this case, the reaction rate at the crack's tip writes:

$$\phi_0 = k \cdot c_0 \quad (4)$$

where  $k = k_0 \cdot \exp(-E_a^k/RT)$  is the reaction coefficient between  $O_2$  and  $SiC$ ; and  $c_0 = p_{O_2}/RT$  is the  $O_2$  concentration in the surrounding air.

- We can also use a more realistic model of the oxidizing environment in the vicinity of the fiber. Basically, this is a variation of the framework introduced in [1], customized for the  $SiC_{(s)} + O_{2(g)} \rightarrow SiO_{2(s)}$  reaction on the surface of the fiber. It allows to introduce the oxide layer that is generated around the fiber and thus delays the arrival of oxygen at the crack's tip. In this case, the reaction rate at the crack's tip can be written [6]:

$$\phi_0(t) = \frac{kc_0(t)}{1 + ke(t)/D} \quad (5)$$

where  $k$  is the reaction coefficient of  $O_2$  with  $SiC$ ;  $c_0$  is the  $O_2$  concentration in the surrounding air;  $D = D_0 \cdot \exp(-E_a^D/RT)$  is the diffusion coefficient of  $O_2$  in  $SiO_2$ ; and  $e$  is the thickness of the oxide layer, given by:

$$\begin{cases} \frac{de}{dt}(t) = \frac{\rho}{M} \cdot \frac{kc_0(t)}{1 + ke(t)/D} \\ e(t=0) = 0 \end{cases} \quad (6)$$

where  $M$  and  $\rho$  are respectively the volume mass and the molar mass of  $SiO_2$ .

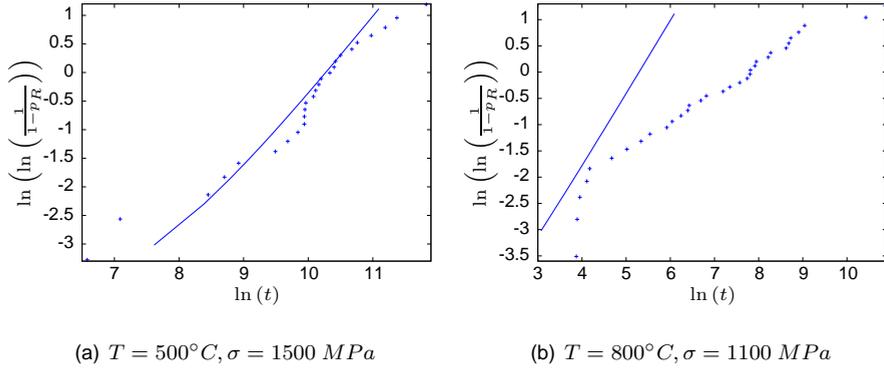
## 4 A family of fiber lifetime prediction tools

### 4.1 Models

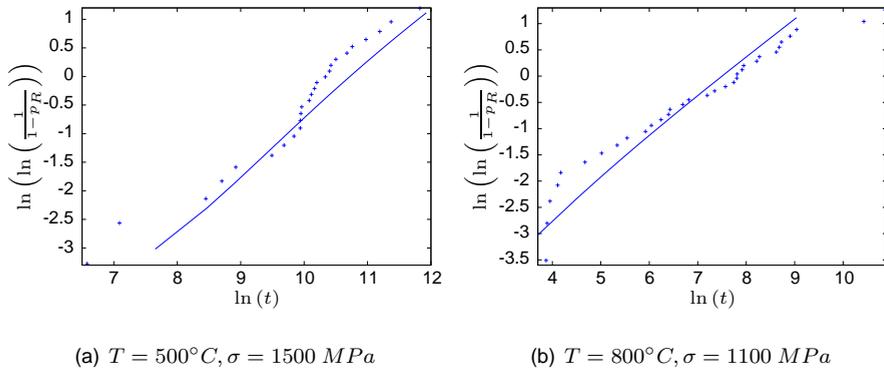
The fiber's time-to-rupture, denoted  $t_R$ , is obtained by solving  $a(t = t_R) = a_c(\sigma)$  with  $a(t = 0) = a_0(\sigma_r)$  initial condition, in the system formed by Equations 2, 3 and 4 for the first environment; and 2, 3, 5 and 6 for the second one.

## 4.2 Calibration

A summary of published data can be found in [6]. Moreover, [3]'s experimental fiber lifetime probability at several mechanical and environmental conditions are used to fit the remaining parameters (see Figures 2 and 3). It can be shown [6] that the calibrated values are the unique ones to minimize the model–experiments distance.



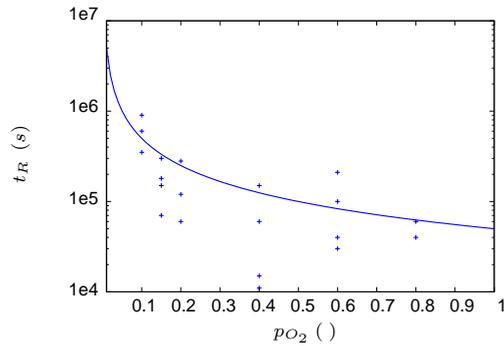
**Figure 2:** Fiber lifetime cumulative probability distributions under several loadings: calibrated first model vs. experiment



**Figure 3:** Fiber lifetime cumulative probability distributions under several loadings: calibrated second model vs. experiment

### 4.3 Elements of validation

A first element of validation can be given. As no more data is available on individual Hi-Nicalon fibers, one must use data on Hi-Nicalon fiber bundles. Indeed, observing that a fiber bundle consists of a large number of fibers ( $\approx 500$ ), one can show that its lifetime is subject to only small variations and is close to that of a particular fiber, called the critical fiber, characterized by a given probability of failure [3, 4]. The comparison of the predicted and experimentally observed [4] fiber bundle lifetimes as functions of the partial oxygen pressure (Figure 4) shows that the orders of magnitude predicted using our model are correct.



**Figure 4:** Fiber bundle lifetime vs. partial oxygen pressure, at  $T = 500^\circ C$  and  $\sigma = 1,000 MPa$ : predicted lifetime of the bundle's critical fiber (*i.e.* the fiber with 5% failure probability) vs. [4]'s experimental data

## 5 Conclusion

This paper has presented the application of a recent work, that reexamined the well-established modeling framework for the subcritical propagation of cracks in ceramics [6], to the case of fine ceramic fibers.

Two lifetime prediction tools for SiC fibers under given mechanical and environmental conditions were derived from the new framework. These tools differ in the level of complexity with which the fiber's chemical environment is described: in the first tool, the fiber is oxidized directly by the ambient air; whereas in the second tool the oxide layer which surrounds the fibers and delays the oxidation process is taken into account. As expected, after calibration, the first model was found to reflect reality satisfactorily at low temperatures, but to be inaccurate at higher temperatures. Conversely, the predictions given by the second model are correct at both low and high temperatures because this model handles reaction-controlled

propagation and diffusion-controlled propagation naturally. Some validation was presented, based on [3, 4]'s experimental data. Work is currently underway in order to extend this validation in the case of fiber bundles.

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