Identification of heterogeneous elastoplastic behaviors using DIC measurements

Tarik Madani, Yann Monerie, Stéphane Pagano, Céline Pelissou, Bertrand Wattrisse

IDENTIFICATION OF HETEROGENEOUS ELASTOPLASTIC BEHAVIORS USING DIC MEASUREMENTS

T. MADANI[1,2,3], Y. MONERIE[2,3], S. PAGANO[2,3], C. PELISSOU[1,3], B. WATTRISSE[2,3]

1 Institute for Radiological Protection and Nuclear Safety, PSN/SEMIA, Bat. 702, 13115 Saint- Paul-Lez-Durance Cedex, France.
2 Mechanics and Civil Engineering Laboratory, University of Montpellier 2, Pl. E. Bataillon, Montpellier, France.
3 Micromechanics and Structural Integrity Laboratory, IRSN-CNRS-University of Montpellier 2.

tarik.madani@univ-montp2.fr, yann.monerie@univ-montp2.fr, stephane.pagano@univ-montp2.fr, bertrand.wattrisse@univ-montp2.fr, celine.pelissou@irsn.fr

ABSTRACT: The use of full-field measurements in the identification of material properties is currently widespread thanks to advances in measurements techniques and computer-assisted identification methods. In this paper, an iterative procedure is used to identify the local stress fields and the material properties distributions using full-field measurement techniques. After summarizing the principle of the method, we focus on its validation in which we identify an elastoplastic behavior. Then the method was applied on noisy measured displacement fields to assess its robustness.

1. INTRODUCTION

Optical measurement techniques applied in the field of experimental mechanics have improved significantly in the last decades. Thanks to the large amount of information given by full-field measurement techniques, it is now possible to tackle complex identification problems such as strongly localized phenomena (Lüders bands, crack propagation …). Mechanical properties are generally identified from overall loading informations and kinematic fields (obtained by Digital Image Correlation, interferometric techniques, grid methods, etc.). The Finite-Element Updating Method is probably the most widespread identification method, as it allows identifying very different physical properties. Specific methods, adapted to the identification of mechanical behaviors were also proposed. A general overview over existing identification techniques can be found in [1]. We have chosen to use the Constitutive Equation Gap Method (CEGM) as it can be adapted to a wide range of material behaviors and as it is compatible with the identification of heterogeneous behaviors. This method was originally designed as an error estimator for finite element method.
The first step of the proposed work is to extend the approach developed in [3, 4] to identify the parameters of the constitutive laws (elastoplasticity, cohesive zones models). Cohesive zones models, associated with traction-separation laws, are commonly used in numerical simulations to account for the initiation of micro-cracks and their propagation leading to the fracture of the material.
The proposed approach introduces the elastoplastic secant stiffness tensor (B^s) for the identification of plastic parameters. For a linear kinematic model, tensors B^s can be expressed directly as a function of the material properties (yield stress and hardening coefficient) and of the loading history. Its reliability is checked through applications on simulated data obtained under small perturbation and plane stress assumptions with *COMSOL Multiphysics* software. In particular, the robustness of the method with respect to measurement noise is studied in order to evaluate the performance of the proposed approach.

2. IDENTIFICATION PROCEDURE AND NUMERICAL METHOD

We propose a method to identify the parameters of an elastoplastic constitutive law in a 2D framework. The CEGM is based on the minimization of a functional expressing the gap in the constitutive equation. In its simplest form (small strain hypothesis, equilibrium, and linear elastic constitutive behavior) the cost-function reads:

$$E\left(\overrightarrow{u_c}, \underline{\underline{B}}\right) = \frac{1}{2}\int_1^t \int_\Omega \left[\underline{\varepsilon}(\overrightarrow{u_c}) - \underline{\varepsilon}(\overrightarrow{u_m})\right]:\underline{\underline{B}}:\left[\underline{\varepsilon}(\overrightarrow{u_c}) - \underline{\varepsilon}(\overrightarrow{u_m})\right] d\Omega dt \tag{1}$$

where $\underline{\underline{B}}$ is a heterogeneous elastoplastic tensor, u_m a measured displacement field, u_c a displacement field compatible with the local and global equilibrium of the studied domain Ω using the secant tensor $\underline{\underline{B}}$.
For an elastoplastic identification problem, the elastic and plastic behavior can be described by the secant stiffness tensor $\underline{\underline{B_n^s}}$ links the stress tensor $\underline{\sigma_n}$ and the strain tensor $\underline{\varepsilon_n}$ at load step *n*:

$$\underline{\sigma_n} = \left[\underline{\underline{B_n^s}}\right]:\underline{\varepsilon_n}. \tag{2}$$

According to [5], denoting $\Delta\gamma$ the plastic multiplier increment, this discretized secant tensor reads:

$$\left[\underline{\underline{B_n^s}}\right] = \left[\underline{\underline{B}}^{e^{-1}} + \frac{\Delta\gamma_n}{1+\frac{2}{3}k\Delta\gamma}\underline{\underline{P}}\right]^{-1} \tag{3}$$

where $\underline{\underline{B}}^e$ is the elastic tensor (depending, for a cubic material, on the three elastic constants: e.g. Young modulus E, shear modulus G, and Poisson ratio ν) and $\underline{\underline{P}}$ is a constant mapping matrix

$$P = \frac{1}{3}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix} \tag{4}$$

The plastic multiplier $\Delta\gamma_n$ at load step n can be written:

$$\Delta\gamma_n(\sigma_0, k) = \frac{3}{2k}\left(\sqrt{\frac{3}{2}}\frac{\alpha_n}{\sigma_0} - 1\right) \tag{5}$$

with σ_0 and k standing respectively for the yield stress and the hardening coefficient, and α representing the von Mises stress associated to the stress level reached at the current load step n.

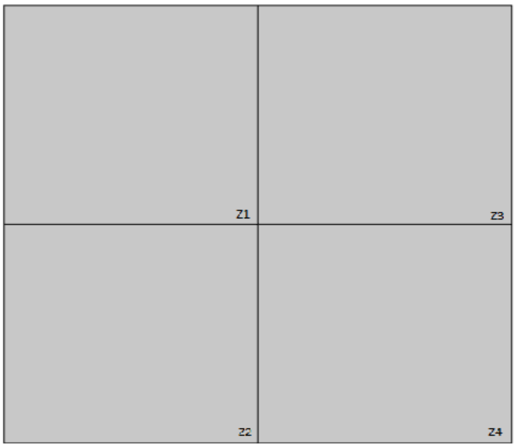
We focus on an elastoplastic model with kinematic hardening associated with a von Mises criterion. The elastic and plastic identification problem thus consists in finding the elasticity tensor and the elastoplastic secant stiffness tensor and the stress field which satisfy the equilibrium equation, the constitutive equation and the global equilibrium.

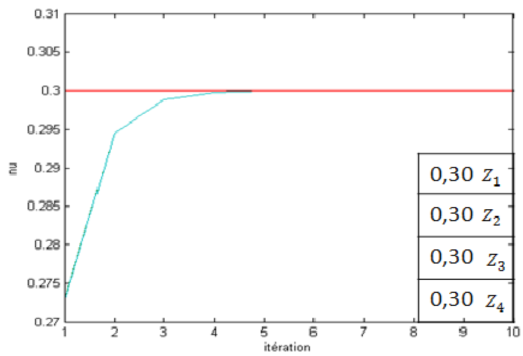
Thanks to the properties of convexity of the constitutive equation gap *E*, the minimization can be performed in two consecutive steps: first with respect to its first argument (to determine a displacement field $\overrightarrow{u_c}$ associated with a statically admissible stress fields $\underline{\sigma_c}$) and then the second minimization is performed with respect to its second argument $\underline{\underline{B}}$ (to identify the material parameters).
The CEGM procedure is controlled through an optimization algorithm. The iterative procedure is started with an initial set of parameter chosen arbitrarily.
The procedure is stopped using a convergence criterion on the norm of the tangent tensor, and the optimal material parameters are obtained.

3. APPLICATION EXAMPLE

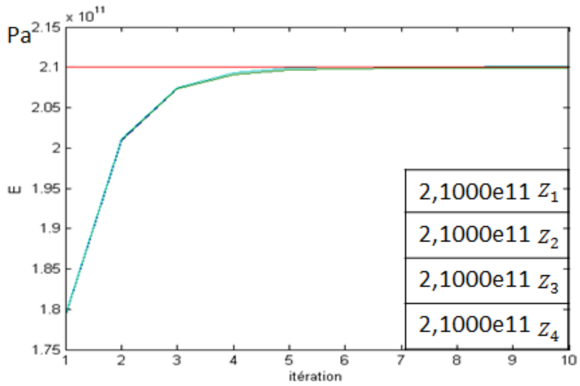
In this section some results obtained with the CEGM presented above are given. The identification method was tested on numerical examples associated with different configurations, for a homogeneous and a heterogeneous material subjected to a tensile test.

1. The first test is the numerical simulation of a tensile test performed at constant velocity on a rectangular bar. The behavior of the material is isotropic, elastoplastic with a linear kinematic hardening. In this example, the Young modulus is 210 GPa, the Poisson ratio is 0.3, the yield stress σ_0 is 300 MPa and the hardening modulus k equal to 1 GPa. The identification is made on an area of the same size as the bar. The material behavior is identified on 4 domains as shown in Figure 1.a. The rapid convergence of the elastic properties (ν) is shown in Figure 1.b.





(a) (b)



(c)

Z1 : σ_0 = 300.0622 MPa, k = 1.0139 GPa

Z2 : σ_0 = 300.0622 MPa, k = 1.0139 GPa

Z3 : σ_0 = 300.0622 MPa, k = 1.0139 GPa

Z4 : σ_0 = 300.0622 MPa, k = 1.0139 GPa

(d)

Figure 1- Geometry of the sample (a) – convergence on the Poisson ratio (b) - convergence on the Young modulus (c) and identified kinematic hardening modulus and yield stress (d).

2. The second test is performed on a composite specimen made of two different materials i.e. a numerical mechanical test on a composite specimen (stiff matrix with a soft elliptical inclusion). For the matrix the Young modulus is 210 GPa, the Poisson ratio is 0.3, the yield stress is 300 MPa and the hardening modulus equals to 1 Gpa. The inclusion has a Young modulus worth 100 GPa, a Poisson ratio of 0.15, a yield stress of 300 MPa and a hardening modulus equals to 1 GPa. Two types of identification are performed. In the first one, the mesh of the identification is perfectly consistent with the mesh used for the simulation (two identification domains D_1 – inclusion and D_2 - matrix) and the other one is performed on an identification mesh that does not respect the material heterogeneity (400 domains D_j with j = 1 to 400).

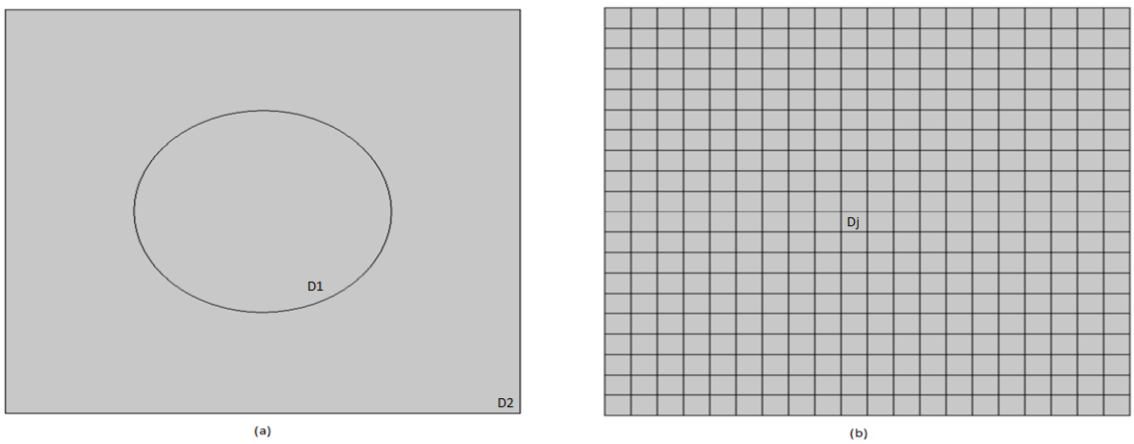
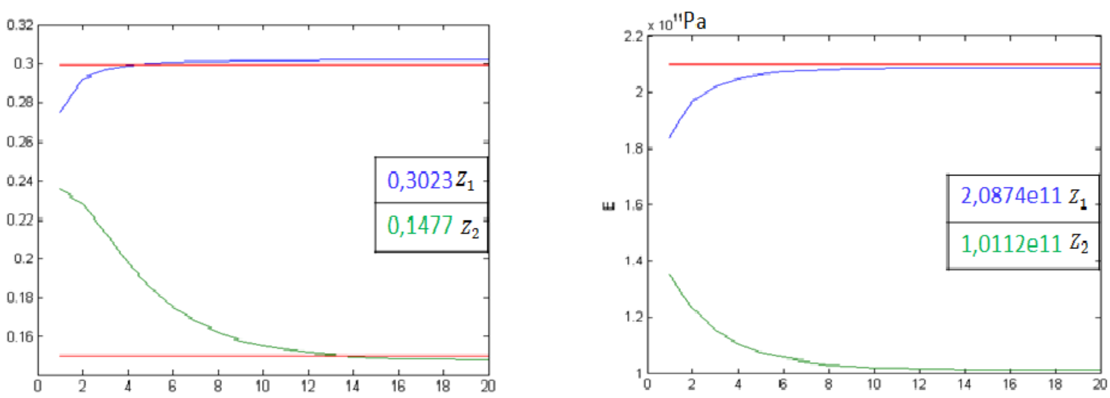


Figure 2- Geometry for the identification: mesh accordance (a) and not in accordance (400 domains) (b).



(a) (b)

Figure 3- Convergence of Poisson ratio (a) and Young modulus (b) for two domains.

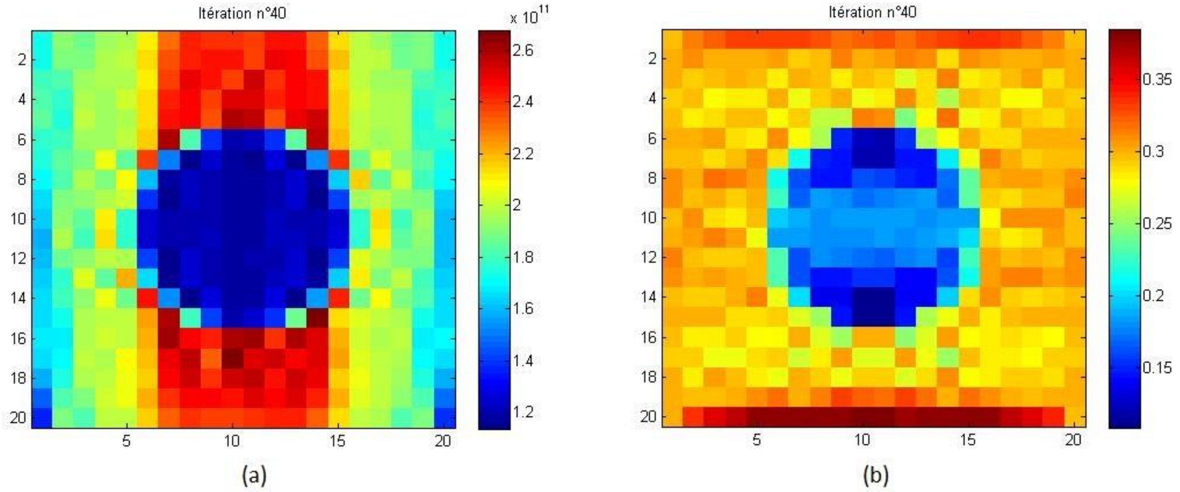


Figure 4- Convergence of Young modulus (a) and Poisson ratio (b) for 400 domains.

The robustness of the CEGM approach with respect to noise was evaluated using a set of simulated displacement field representative of real experiment conditions on which a Gaussian white noise with different noise levels was added. Results showed that the identification method was robust with respect to noise, even in the elastic domain where signal to noise ratio is the worst. As an example, for a noise level similar to the one associated with experimental results, the error on the identified elastic constants (E, v) is shown to be less than 7%.

4. CONCLUDING REMARKS

We illustrate the use of the CEGM to identify the parameters of an elastoplastic behavior from full-field measurements. Here we identify mechanical stresses and a distribution of elastic and plastic coefficients. The originality of this work resides in its ability to tackle heterogeneous stress fields associated with either heterogeneous materials or complex structures for elastic or plastic materials. For a plastic load step, the determination of the secant tensor requires to estimate the von Mises equivalent stress. It is here reached using an elastic prediction associated with cubic elasticity. This step gives inaccurate elastic coefficients but an equilibrated stress field consistent with the global applied load allowing the determination of the equivalent stress. Before applying the identification method to real experimental data, the proposed procedure was checked on the basis of numerically-obtained displacement fields given by a FE simulation. Results of the study show the ability of the method to deal with strongly heterogeneous situations. Gaussian white noise was also superimposed to the numerical data in order to assess the robustness of the method with respect to noise. This method is now being extended to the introduction of damage in the material response.

5. REFERENCES

1. S. Avril, M. Bonnet, A.S. Bretelle, M. Grediac, F. Hild, P. Ienny, F. Latourte, D. Lemosse, S. Pagano, E. Pagnacco, (2008) Overview of identification methods of mechanical parameters based on full-field measurements. Experimental Mechanics 48, 381-402.

2. F. Latourte, A. Chrysochoos, S. Pagano and B. Wattrisse. (2008) Elastoplastic behavior identification for heterogeneous loadings and materials, *Experimental Mechanics* 48 435-449.

3. S. Wen, Y. Monerie, B. Wattrisse. (2013) Identification of cohesive zone models from thermomecanical imaging techniques, CFRAC 2013.

4. V. Richefeu A. Chrysochoos, V. Huon, Y. Monerie, R. Peyroux and B. Wattrisse. (2012) Towards local identification of cohesive zone models using digital image correlation, *European Journal of Mechanics - A/Solids* 34 38-51.

5. JC. Simo and TJR. Hughes (1998). Computational inelasticity. Springer, pp 126–130.