LMI-based design of a robust direct adaptive attitude control for a satellite with uncertain parameters

Harmonie Leduc, Dimitri Peaucelle, Christelle Pittet-Mechin

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Harmonie Leduc hleduc@laas.fr
Dimitri Peaucelle peaucell@laas.fr
Christelle Pittet christelle.pittet@cnes.fr

Abstract

Three axes attitude control of CNES microsatellite is considered. The satellite has uncertain inertia, natural frequency and damping. Satellite dynamics are modelized using descriptor form, with matrices affinely dependent on the uncertain parameters. LMI-based methods using S-variables allow to design a robust adaptive controller with tunable gain adaptation parameters. Illustrated simulations show that the controller stabilizes the satellite with any inertia physically acceptable. Theoretical results can be applied to any descriptor system rational in the uncertainties.

1 Introduction

Attitude control of CNES microsatellites is currently achieved by a switching controller ([17], [21]): A speed bias control allows at high depointing to decrease the error slowly until reaching a given threshold, whereas a proportional-derivative control is used to stabilize the attitude accurately once the depointing is small. Such a controller is efficient but the threshold must be tuned very carefully to avoid discontinuities ([18]), which is very difficult if the system has uncertain parameters.

The control of spacecrafts has been widely treated. Even if each solution presents some advantages, they have also some counterparts: the controller in [5] is based on a strong hypothesis of passivity ([6]). [11] and [22] build an estimator, still hard to implement. [13] use the sliding mode control technique but obtains the convergence of the attitude in infinite time. This last controller is enhanced in [12], by creating a non singular terminal sliding mode control, which allows a finite-time convergence but whose performance is widely degraded in case when the system has important uncertainties. [7] ([2] resp.) build a robust adaptive controller, but under some restrictive hypotheses about the uncertainties (about the command resp.).

The controller designed by [14] has time varying gains, with variations depending on the values of the measured outputs in real time. It is built using LMI methods, whose advantages have been widely proven in [1] and which have been used for satellite control issues in [3], [23] and [16], among many others. No passivity hypothesis is required. For implementation issue, the control is direct, without estimator. It has been tested onboard satellite PICARD ([19]) and has been proven to be very efficient. The issue of reaction wheel saturation has been treated in [10] and its robustness is dealt in this paper.

The contributions of the paper are: First, the design of an adaptive controller which is more robust than a corresponding static output feedback, under no hypothesis of passivity; second, the application of this controller to TARANIS attitude control which shows that the system is stabilized for any physically plausible value of the inertia.

The paper is organized as follows: Some recalls on descriptor system theory and TARANIS dynamics modelisation are given in section 2. In section 3, we provide two theoretical results to build a robust direct adaptive controller for descriptor systems, and we apply them to TARANIS attitude control in section 4. Conclusions and outlooks make section 5.

Notation. I stands for the identity matrix. $\{1;V\}$ is the set of all the integers between 1 and V. A^T is the transpose of the matrix A. $A^{\mathcal{S}}$ stands for the symmetric matrix $A+A^T$. $A(\preceq) \prec B$ is the matrix inequality stating that $A-B$ is negative (semi-)definite.

2 Modelisation of TARANIS dynamics

We consider the dynamics of CNES microsatellite TARANIS. The flexibilities of its appendices cannot be neglected and must be added to a classical three dimensional double integrator, as in [17]:

$$\begin{aligned} J\ddot{\theta} + N\sqrt{J}\ddot{\eta} &= u \\ (\sqrt{J}N)^T\ddot{\theta} + \ddot{\eta} + 2Z\Omega_n\dot{\eta} + \Omega_n^2\eta &= 0 \end{aligned} \tag{1}$$

where $\theta \in \mathbb{R}^3$ is the attitude of the satellite, $\eta, \dot{\eta} \in \mathbb{R}^{2n_f}$ are the states of the flexible modes, n_f is the number of considered flexible appendices. Each of them brings two flexible modes, one in torsion, the other in bending. Assuming identical appendices, all the flexible modes have the same damping ζ and the same natural frequency ω_n. $\Omega_n = \omega_n I_{2n_f}$ and $Z = \zeta I_{2n_f}$.The actual satellite TARANIS is planned to have four appendices, for numerical reasons we decided to limit ourselves to two appendices. This is further explained in section 4. $J \in \mathbb{R}^3$ is the inertia of the satellite, $N \in \mathbb{R}^{3\times(2n_f)}$ stands for the coupling between θ and η. N is assumed to be precisely known due to the geometrical positioning of the appendices. In the case when two appendices are considered, its value is

$$N = 10^{-2}\begin{bmatrix} -43.52 & 2.16 & -28.32 & 2.45 \\ -5.03 & -2.97 & -4.23 & -52.93 \\ 2.03 & 40.80 & -31.07 & 2.63 \end{bmatrix}$$

The other parameters are related to inertia and dynamics are uncertain because cannot be measured precisely on ground: $\omega_n \in [0.2\times 2\pi \;\; 0.6\times 2\pi]$, $\zeta \in \left[5.10^{-4} \;\; 5.10^{-3}\right]$ and $J = \mathrm{diag}([37.49 \;\; 24.00 \;\; 47.76])$, with a maximal uncertainty to be determined.

For a purpose which will be detailed in section 3, in this section we aim at rewriting system (1) in the following descriptor form

$$\begin{aligned} &E_{xx}(q)\dot{x}(t) + E_{x\pi}(q)\pi(t) = A(q)x(t) + Bu(t) \\ &y(t) = Cx(t) \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^{n_x}$ is the state of the system, $u \in \mathbb{R}^{n_u}$ is the control input, $\pi \in \mathbb{R}^{n_\pi}$ is an auxiliary signal, $y \in \mathbb{R}^{n_y}$ is the output signal. $E_{xx}(q) \in \mathbb{R}^{n\times n_x}$, $E_{x\pi}(q) \in \mathbb{R}^{n\times n_\pi}$ and $A(q) \in \mathbb{R}^{n\times n_x}$ must be affine functions of the uncertain vector q, whose components are the uncertain parameters (here $q \in \mathbb{R}^5$ is composed by the diagonal coefficients of J, the damping ζ and the natural frequency ω_n). $B \in \mathbb{R}^{n\times n_u}$ and $C \in \mathbb{R}^{n_y\times n}$. Without loss of generality, the q-dependent matrices can be rewritten into a polytopic form: $E_{xx}(q) = E_{xx}(\delta) = \sum_{v=1}^{V}\delta_v E_{xx}^{[v]}$, $E_{x\pi}(q) = E_{x\pi}(\delta) = \sum_{v=1}^{V}\delta_v E_{x\pi}^{[v]}$ and $A(q) = A(\delta) = \sum_{v=1}^{V}\delta_v A^{[v]}$. The polytope is defined by its V vertices $\delta_1 \dots \delta_V$, which are the $V(=2^5$ here) extremal combinations of the components of q. [4] proves that any descriptor model in which the matrices are rational with respect to the components of the uncertain vector can be rewritten into such a form. Here, this can be done by introducing two exogenous signals $\pi_1 = \sqrt{J}\ddot{\theta}$ and $\pi_2 = 2Z\dot{\eta} + \Omega_n\eta$ and we consider that $q = \left[\sqrt{J_1} \;\; \sqrt{J_2} \;\; \sqrt{J_3} \;\; \zeta \;\; \omega_n\right]$. Thus we get the following model:

$$\begin{aligned} &\begin{bmatrix} 0 & 0 & 0 & \sqrt{J}N \\ 0 & \sqrt{J} & 0 & 0 \\ I_3 & 0 & 0 & 0 \\ 0 & (\sqrt{J}N)^T & 0 & I_8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I_8 & 0 \end{bmatrix}\dot{x} + \begin{bmatrix} \sqrt{J} & 0 \\ -I_3 & 0 \\ 0 & 0 \\ 0 & \Omega_n \\ 0 & I_8 \\ 0 & 0 \end{bmatrix}\pi \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_n & 2Z \\ 0 & 0 & 0 & I_8 \end{bmatrix}x + \begin{bmatrix} I_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}u, \end{aligned} \tag{3}$$

$$y = \begin{bmatrix} I_3 & 0 & 0 & 0 \\ 0 & I_3 & 0 & 0 \end{bmatrix} x$$

where $x^T = \left[\theta \ \dot{\theta} \ \eta \ \dot{\eta}\right]^T$ and matrices $E_{xx}(\delta)$, $E_{x\pi}(\delta)$, $A(\delta)$, B and C are easily identifiable.

3 Robust adaptive control design using LMIs

In this section we give the main result used to build a robust direct adaptive controller to system (2). The proposed controller replaces a given static output feedback $u(t) = K_0 y(t)$ by a structured time-varying control

$$u(t) = (K_0 + LK(t)R)y(t) \tag{4}$$

where $K = \text{diag}(K_1, \ldots, K_{\bar{k}})$, $\bar{k}$ the number of components of K, $L = [L_1 \ L_2 \ ...]$, $R^T = \left[R_1^T \ R_2^T \ ...\right]$. The structure is given by the low rank matrices L_k and R_k so that the non-zeros gains are gathered in independent matrices of small sizes. The adaptive law is the following:

$$\begin{array}{l} \dot{K}_k(t) = \mathcal{I}_{D_k}\left(K_k(t),\ W_k(t)\right) \\ W_k(t) = \gamma_k(-G_k y(t)(R_k y(t))^T - \sigma_k K_k(t)). \end{array} \tag{5}$$

where D_k defines an ellipsoidal set $\mathcal{E}_k$:

$$K_k \in \mathcal{E}_k \Leftrightarrow \text{Tr}(K_k^T D_k K_k) \leq 1 \tag{6}$$

and $\mathcal{I}_{D_k}$ is a saturated integrator which pushes the gain $K_k(t)$ inside $\mathcal{E}_k$ when it is at its border ([20], [15]). Hence, $K_k(t)$ remains bounded. The role of each term of (5) is detailed in [9]. Theorem 3.1 aims at finding appropriate values for $D = \text{diag}(D_1, \ldots, D_{\bar{k}})$, $G^T = \left[G_1^T, \ldots, G_{\bar{k}}^T\right]$, $\Gamma = \text{diag}(\gamma_1, \ldots, \gamma_{\bar{k}})$ and $\sigma = \text{diag}(\sigma_1, \ldots, \sigma_{\bar{k}})$ such that system is robustly stable with adaptive control (5).

Theorem 3.1 *Considering system (2), if there exists matrices $\hat{P}^{[v]} = \hat{P}^{[v]^T}$ and $\hat{S}$ such that the following condition hold for all $v \in \{1; V\}$:*

$$\begin{bmatrix} 0 & 0 & \hat{P}^{[v]} \\ 0 & 0 & 0 \\ \hat{P}^{[v]} & 0 & 0 \end{bmatrix} + \left\{\hat{S}\begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} & -A_c^{[v]} \end{bmatrix}\right\}^{\mathcal{S}} \prec 0 \tag{7}$$

where $A_c^{[v]} = A^{[v]} + BK_0C$, then there exist matrices $P^{[v]}$, S, $G^T = \left[G_1^T, \ldots, G_{\bar{k}}^T\right]$, $D = \text{diag}(D_1, \ldots, D_{\bar{k}})$ and $\epsilon > 0$ such that the following equation holds $\forall v \in \{1; V\}$:

$$\begin{bmatrix} 0 & 0 & P^{[v]} & 0 \\ 0 & 0 & 0 & 0 \\ P^{[v]} & 0 & \epsilon I + 2C^T R^T R C & -C^T G^T \\ 0 & 0 & -GC & -2D \end{bmatrix} + \left\{S\begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} & -A_c^{[v]} & -BL \end{bmatrix}\right\}^{\mathcal{S}} \prec 0 \tag{8}$$

Besides, the solution is such that the adaptive control (5) stabilizes the plant whatever positive values of σ_k, γ_k and for all $\delta \in \Delta^V$.

Proof 1: Assume there exists $\hat{P}^{[v]} = \hat{P}^{[v]^T}$ and $\hat{S}$ such that (7) is satisfied. Using Theorem 2 in [9], we get that $u(t) = K_0 y(t)$ is a static output feedback which robustly stabilizes system (2) in closed-loop. To prove Theorem 3.1, one has to notice that it is a particular case of Theorem 3 in [9] for $E_2 = I$. Robust stability of the closed-loop with adaptive control for all $\delta \in \Delta^V$ is proved using the parameter-dependent Lyapunov function

$$V_\delta(x, K) = x^T P(\delta) x + \text{Tr}(K^T \Gamma^{-1} K)$$

whose derivative satisfies thanks to (8):

$$\dot{V}_\delta(x, K) \leq -\epsilon x^T x - \textstyle\sum_{k=1}^{\bar{k}} \sigma_k \text{Tr}(K_k^T K_k).$$

It is negative along the trajectories of the system.

Remark: Theorem 3.1 claims that we only need to check that (7) is satisfied for the extremal values of the uncertainty whereas the adaptive control is valid for every value of the uncertainty. This is possible since the affine dependence of the matrices of system (2) allows to use a convexity argument and uniqueness of the S-variable to switch from vertex

conditions to conditions for all uncertainties.

With Theorem 3.1, we only know that the adaptive law is no worse than the static control. The following theorem aims at proving superiority of the designed adaptive law.

Theorem 3.2 *Consider the following matrix inequalities with* $\breve{P}^{[v]} \succ 0$ *and* $\breve{\epsilon} > 0$ *for all* $v \in \{1; V\}$*:*

$$\begin{bmatrix} 0 & 0 & \breve{P}^{[v]} & 0 \\ 0 & 0 & 0 & 0 \\ \breve{P}^{[v]} & 0 & M^{[v]} & -C^T G^T \\ 0 & 0 & -GC & -2D \end{bmatrix} + \left\{ S \begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} & -A_{c\Delta}^{[v]} & -BL \end{bmatrix} \right\}^{\mathcal{S}} \prec 0 \quad (9)$$

and

$$\begin{bmatrix} T_k & F_k^{[v]T} D_k \\ D_k F_k^{[v]} & D_k \end{bmatrix} \succeq 0 \ , \ Tr(T_k) \leq 1 \quad \forall \quad k \in \left\{1; \bar{k}\right\} \quad (10)$$

where $A_{c\Delta}^{[v]} = A^{[v]} + B(K_0 + LF^{[v]}R)C$ *and* $M^{[v]} = \breve{\epsilon} I + 2C^T R^T R C + \left\{C^T R^T F^{[v]} G C\right\}^{\mathcal{S}}$*. These constraints are such that:*

(i) For fixed K_0*,* G*,* S *and* $D = \mathrm{diag}(D_1, \ldots, D_{\bar{k}})$ *the constraints are LMI in* $\breve{P}^{[v]}$*,* $\breve{\epsilon}$ *and* $F^{[v]}$*.*

(ii) For K_0*,* G*,* S *and* D *solution to constraints in Theorem 3.1 the LMIs are feasible.*

(iii) If the constraints are feasible, then for all $k \in \left\{1; \bar{k}\right\}$*,* $F(\delta)$ *is such that* $Tr(F_k(\delta) D_k F_k(\delta)) \leq 1$ *and* $u(t) = (K_0 + LF(\delta)R)y(t)$ *stabilizes the plant (2) for any value* δ *of the uncertainty, where* $F(\delta) = \sum_{v=1}^{V} \delta_v F^{[v]}$*.*

(iv) If the constraints are feasible, then whatever positive γ_k *the adaptive control (5) quadratically stabilizes the set of the states* x *such that* $x = 0$ *when all* $\sigma_k = 0$ *and quadratically stabilizes a neighborhood of this same set when at least one* $\sigma_k > 0$*.*

Proof 2: Here again, Theorem 3.2 is quite the same as Theorem 4 in [9]. The matrix E_2 in [9] is equal to I here. Moreover as we assume that the matrix B does not depend on the uncertain vector δ, we allow F to depend on δ, hence expecting better results than those in [9]. A detailed proof of Theorem 3.2 is given in the appendix.

Remark: Comparing to those of Theorem 3.1, the most important change in the hypotheses is that the baseline control does not need to stabilize the system for all the values of δ anymore. Theorem 3.2 can be expected to be applied to a larger range of parametrizations of the system than Theorem 3.1, where the stability of the baseline control is fundamental.

In the next section, Theorems 3.1 and 3.2 are applied to system (3).

4 Robust adaptive design of TARANIS attitude control

Before applying the two results of section 3, we need to define the structure of the controller. Here, the output is $y = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T$ and the control is $u = [u_1 \ u_2 \ u_3]$, then we take

$$L = \left[\begin{array}{c|c|c|c|c|c} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \text{ and } \quad R = I_6$$

to get

$$LKR = \begin{bmatrix} K_{\theta 1} & 0 & 0 & K_{\dot{\theta} 1} & 0 & 0 \\ 0 & K_{\theta 2} & 0 & 0 & K_{\dot{\theta} 2} & 0 \\ 0 & 0 & K_{\theta 3} & 0 & 0 & K_{\dot{\theta} 3} \end{bmatrix}.$$

Then, we find that system (3) is stable with the static output feedback

$$K_0 = \begin{bmatrix} 0.1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 2 \end{bmatrix}. \quad (11)$$

The adaptive law (5) writes as follows, for $i =$

$1, 2, 3$:

$$W_{\theta i}(t) = \gamma_{\theta i}(-G_i y(t) y_i(t)^T - \sigma_{\theta i}(K_{\theta i}(t) - 0.1))$$
$$W_{\dot{\theta} i}(t) = \gamma_{\dot{\theta} i}(-G_{i+3} y(t) y_{i+3}(t)^T - \sigma_{\dot{\theta} i}(K_{\dot{\theta} i}(t) - 2)) \quad (12)$$

and the saturated integrator guarantees that

$$K_{\theta i}(t) \in \left[0.1 - D_i^{-1/2}; 0.1 + D_i^{-1/2}\right]$$
$$K_{\dot{\theta} i}(t) \in \left[2 - D_{i+3}^{-1/2}; 2 + D_{i+3}^{-1/2}\right]$$

4.1 No worse robustness adaptive controller

The first inequalities to solve are LMIs (7). Unfortunately, the solver we use (SDPT3) is not able to manage it if more than two flexible modes are considered. That is why we only considered appendices ($n_f = 2$) in the paper, which is not a bad approximation. For this configuration, LMIs (7) contain 448 rows, 2112 variables and are solved with a recent laptop in 4min. Then, when applying Theorem 3.1 to system (3), we add to LMIs (8) some constraints on the matrix $G = \left[G_1^T, \ldots, G_{\bar{k}}^T\right]^T$, $G_k = [g_{k,1}, \cdots, g_{k,6}]$:

for $\mathrm{k} = 1, 2, 3$, $\mathrm{g_{k,k}} > 0$ and $\mathrm{g_{3+k,3+k}} < -10\mathrm{g_{k,k}}$
and other coefficients are constrained to zero

This choice can be justified following this reasoning, valid for any axis of the satellite: if the depointing is large (θ_i^2 large), $K_{\theta i}$ will decrease, so will the control effort. If now the angular speed $\dot{\theta}_i$ is large, the adaptation makes $K_{\dot{\theta} i}$ increase, and then the satellite rotation along axis i will approach 0. Even if these effects can reduce the speed of convergence of the system, they avoid the saturation of the actuator rate ([10]). LMIs (8) contain 576 rows, 1842 variables and are solved in 1min19s, remaining reasonable for future implementation. Computed values for G are given in Table (1).
Then, the $\sigma_{\theta i}$ and $\sigma_{\dot{\theta} i}$ coefficients are tuned to push the time-varying gains to zero, and then the controller to the baseline control ([8]). $\gamma_{\theta i}$ and $\gamma_{\dot{\theta} i}$ define the adaptation speed of the gains and are chosen to get a fast enough adaptation but without avoiding a future implementation of the controller. This method is detailed in [15]. The values of these parameters are given in Table (1).

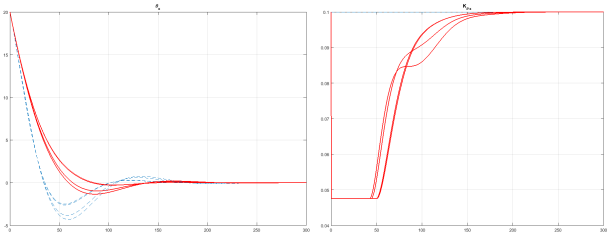

Figure 1: x-axis. Left: Attitude angle; Right: Gain $K_{\theta x}$. Dashed: without adaptation. Solid: with adaptation

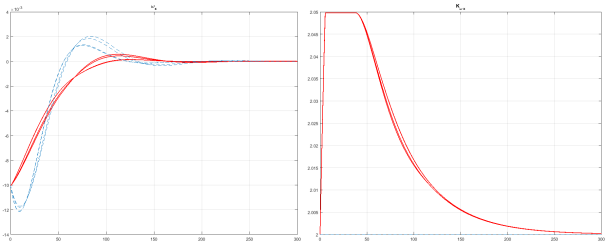

Figure 2: x-axis. Left: Angular rate; Right: Gain $K_{\omega x}$. Dashed: without adaptation. Solid: with adaptation

Table 1: Parameters of TARANIS attitude adaptive control law for the three axes

Axis	G_θ	$G_{\dot{\theta}}$	σ_θ	$\sigma_{\dot{\theta}}$	γ_θ	$\gamma_{\dot{\theta}}$
x	26.9	-345	0.98	0.52	0.05	0.04
y	27.6	-356	1.00	0.55	0.05	0.04
z	26.6	-340	0.97	0.52	0.05	0.04S

Simulation results are plotted in Figures 1 to 6 in solid lines. Dashed lines correspond to time responses of the states with the static output feedback $u(t) = K_0 y(t)$ with the same initial conditions, for several random values of the uncertain vector.

Figures 1 to 6 show that the adaptive controller

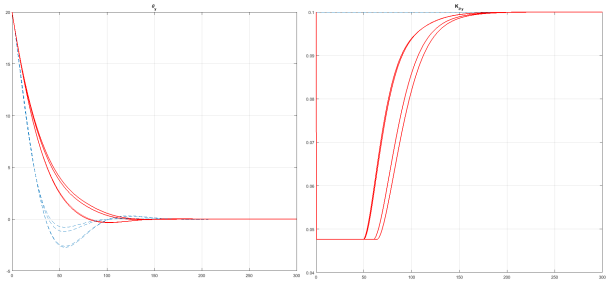

Figure 3: y-axis. Left: Attitude angle; Right: Gain $K_{\theta y}$. Dashed: without adaptation. Solid: with adaptation

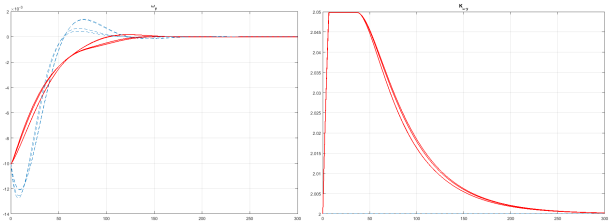

Figure 4: y-axis. Left: Angular rate; Right: Gain $K_{\omega y}$. Dashed: without adaptation. Solid: with adaptation

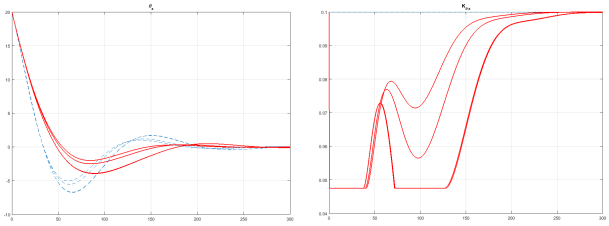

Figure 5: z-axis. Left: Attitude angle; Right: Gain $K_{\theta z}$. Dashed: without adaptation. Solid: with adaptation

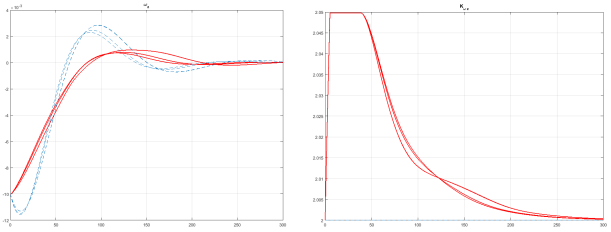

Figure 6: z-axis. Left: Angular rate; Right: Gain $K_{\omega z}$. Dashed: without adaptation. Solid: with adaptation

allows a faster convergence of the states of the system than the static output feedback. Moreover, the variations of the states are smoother with the adaptive controller than with the static control where they are subject to some overshoots. Besides, one can also heuristically notice that the dispersions between time variations of the states with the same set of random values of the uncertainties are less important with the adaptive controller than with the static output feedback for the three axes.
With the adaptive controller, time variations of the gains are not negligible, without being too steep, which makes possible the implementation of the controller in future work.

4.2 Improved robustness adaptive controller

As said in section 3, the strength of Theorem 3.2 is that the stability of the plant with the baseline control is not required anymore. Hence, one can expect that the adaptive controller stabilizes the system subject to other sets of uncertainties.
In the context of satellite attitude control, this aspect can be used to reach a larger range of inertias, since this parameter is "very" uncertain. The application is driven as follows: As we focus on the inertia, we keep the same range of dampings ζ and natural frequencies ω_n of the flexible modes as the ones given in section 2. Besides, we consider that the percentage of uncertainty on the inertia is the same along the three axes.

Then, for a comparative purpose, we give some valid values for the static output feedback with gain (11). Table 2 can be interpreted as follows: first, for the nominal inertia given in section 2, J_0, the feasibility of LMIs (7) shows that the baseline control (11) robustly stabilizes system (3) if we consider that the inertia has 13% of uncertainty. But they are not feasible if we consider that the inertia is 14% uncertain. As LMIs (7) are sufficient conditions, we are not able to conclude about the robust stability of such a system. However, the system without uncertainty, with a nominal inertia increased to $J_1 = 1.14J_0$, is unstable (3 poles among 10 are positive, the largest being 0,08), which gives us the upper bound on Figure 7. The curves in dashed lines on Figure 9 attest this instability for axis x. On the other hand, the system without uncertainty, with a nominal inertia decreased to $J_2 = 0.86J_0$, is stable. It is even robustly stable if we consider 30% of uncertainty in the inertia from J_2. (LMIs (7)) are feasible). As above, we cannot conclude anything in case when the uncertainty is of 31% around J_2. But here again, the system without uncertainty, with a nominal inertia decreased to $0.69J_2$ is unstable, which gives us the lower bound on Figure 7.

Now, we build controller (12) with values of Table 1 and applying Theorem 3.2. First, the solid lines on Figure 9 clearly show that the controller (12) stabilizes the system (here the x-axis) when considering that the inertia is $J_1 = 1.14J_0$, at least in terms of convergence to a neighborhood of the equilibrium. The gain $K_{\theta 1}$ is continually evolving but does not converge to 0.1 since for that value, the system is unstable with J_1.
Beyond that, the braces on Figure 8 mean that the controller (12) is very robust: LMIs (9) and (10) are feasible even if we consider huge uncertainties on the inertia. The obtained range of inertias, centered in the nominal inertia J_0, reaches values around two times J_0. Even if the inertia is ill known, it will not reach such extremal values.
Therefore, the controller (12) to which Theorem 3.2 is applied allows to stabilize system (3) with any physically admissible value of the inertia, whereas the static output feedback can achieve it only with a limited range of values. Theorem 3.2 can thus have a huge scope in case when the considered system has very uncertain parameters. We have here proved improved robustness of adaptive control versus the baseline LTI control (at the expense of relaxing asymptotic stability to practical uncertainty).

Figure 7: Values of the inertia for which attitude is stable with the static output feedback

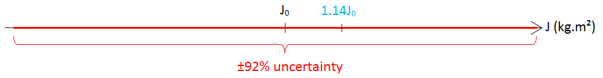

Figure 8: Values of the inertia for which attitude is stabilizable by the adaptive controller with improved robustness

Table 2: Values of the inertia for which attitude is stabilizable by the static output feedback

J_{nom}	dJ	Result and conclusion
J_0	$\pm 13\%$	(7) feasible $\Rightarrow$ Rob. stab.
	$\pm 14\%$	(7) infeasible $\Rightarrow$?
$1.14J_0 = J_1$	0%	System unstable
$0.86J_0 = J_2$	$\pm 30\%$	(7) feasible $\Rightarrow$ Rob. stab.
	$\pm 31\%$	(7) infeasible $\Rightarrow$?
$0.69J_2$	0%	System unstable

5 Conclusion

The problem of satellite attitude control has been considered in section 2. We chose to modelize satellite dynamics with a descriptor system since it fits well with uncertain systems. In section 3, we gave an LMI-based design of an adaptive controller and an LMI-based algorithm aimed at improving its robustness. In section 4, application to TARANIS attitude control showed that the designed controller al-

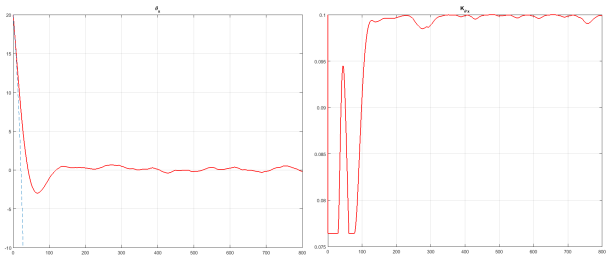

Figure 9: x-axis attitude angle with inertia increased by 14%. Dashed: with static output feedback. Solid: with adaptive control

lows a faster convergence of the attitude and that it is much more robust than the corresponding static output feedback from which it has been designed. Beyond that, the adaptive controller allows to stabilize the system for any physically admissible value of the inertia, a parameter subject to important uncertainties. Of course, such an adaptive control design is not exclusively for attitude control but could be applied to any descriptor system rational in the uncertainties. Finally, a relevant outlook may be the implementation of the controller onboard a real satellite.

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A Proof of Theorem 3.2

(i) is trivial.

To prove (ii) one has to notice that (9) is nothing else but (8) with $\breve{P}^{[v]} = P^{[v]}$, $F^{[v]} = 0 \ \ \forall v \in \{1; V\}$ and $\breve{\epsilon} = \epsilon$, which proves the feasibility of (9).

To prove (iii), one has to notice on the one hand that $F^{[v]}$ is forced to be in the same set as the adaptive gain K. Indeed, applying Schur complement to (10) gives

$$\mathrm{Tr}(F_k^{[v]T} D_k F_k^{[v]}) \leq 1 \text{ for all } k \in \left\{1; \bar{k}\right\}, \forall v \in \{1; V\}.$$

On the other hand, if we denote S by $S = \begin{bmatrix} S_{1a}^T & S_{1b}^T & S_3^T \end{bmatrix}^T$, (9) implies that

$$\left\{ \begin{bmatrix} S_{1a} \\ S_{1b} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} \end{bmatrix} & -A_{c\Delta}^{[v]} \end{bmatrix} \right\}^{\mathcal{S}} + \begin{bmatrix} 0 & 0 & \breve{P}^{[v]} \\ 0 & 0 & 0 \\ \breve{P}^{[v]} & 0 & 0 \end{bmatrix}$$
$$\preceq \left\{ \begin{bmatrix} S_{1a} \\ S_{1b} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} \end{bmatrix} & -A_{c\Delta}^{[v]} \end{bmatrix} \right\}^{\mathcal{S}}$$
$$+ \begin{bmatrix} 0 & 0 & \breve{P}_e^{[v]T} \\ 0 & 0 & 0 \\ \breve{P}_e^{[v]} & 0 & \breve{\epsilon} I + 2C^T R^T R C + \left\{ C^T R^T F^{[v]} G C \right\}^{\mathcal{S}} \end{bmatrix}$$
$$\preceq 0$$

Using Theorem 2 in [9], this proves the stability of the closed-loop with static gain $K_0 + LF^{[v]}R$.

Now let us prove (iv). The generalization of (9) and (10) for all $\delta \in \Delta^V$ is achieved as in the proof of Theorem 3.1, detailed in [9], here for $E_2 = I$. Still following the same lines as the second part of this proof, multiplying (9) by $\begin{pmatrix} \dot{x}^T & \pi^T & x^T & x^T C^T R^T K^T \end{pmatrix}$

and its transpose respectively, gives

$$\begin{array}{rl} 2x^T\breve{P}(\delta)\dot{x} - 2y^TR^T(K-F(\delta))^TGy & \\ -2y^TR^T(K^TDK-I)Ry & \leq -\breve{\epsilon}x^Tx. \end{array} \tag{13}$$

where $(K^TDK - I) \preceq 0$. That result proves the stability with the adaptive add-on.

Here, the parameter-dependent Lyapunov function is given by:

$$V_\delta(x,K) = x^T\breve{P}(\delta)x + \mathrm{Tr}((K-F(\delta))^T\Gamma^{-1}(K-F(\delta))^T)$$

Note that both $\breve{P}(\delta)$ and $F(\delta)$ are δ-dependent, which is an improvement compared to Theorem t:desc-rob. and its derivative writes as follows:

$$\begin{array}{rl} \dot{V}(x,K) = & 2x^T\breve{P}(\delta)\dot{x} + 2\mathrm{Tr}(\dot{K}\Gamma^{-1}(K-F(\delta))^T) \\ & = 2x^T\breve{P}(\delta)\dot{x} - 2\mathrm{Tr} \\ & ((Gyy^TR^T + \sigma K)(K-F(\delta))^T) \\ & -2\mathrm{Tr}(H\Gamma^{-1}(K-F(\delta))^T) \\ & = 2x^T\breve{P}(\delta)\dot{x} - 2y^TR^T(K-F(\delta))^TGy \\ & -2\sigma\mathrm{Tr}(K^T(K-F(\delta))) \\ & -2\Gamma^{-1}\mathrm{Tr}((K-F(\delta))^TH) \\ & \leq -\breve{\epsilon}x^Tx - 2\sum\sigma_kTr(K_k^T(K_k - F_k(\delta))) \end{array}$$

For the second row, we use the definition of the saturated integrator, with H containing the components orthogonal to the boundary of the sets $\mathcal{E}_k$ that push the gains into the sets when at the boundary. For the third row, we use the fact that the three following properties are true for all matrices M and N with appropriate sizes and for every $\lambda \in \mathbb{R}$:

$$\mathrm{Tr}(M + \lambda N) = \mathrm{Tr}(M) + \lambda\mathrm{Tr}(N)$$

$$\mathrm{Tr}(MN) = \mathrm{Tr}(NM);\ \mathrm{Tr}(M) = \mathrm{Tr}(M^T)$$

For the last row we use (13) and again the property of the saturated integrator.

The last row indicates that when all σ_k are zero, the derivative of the Lyapunov function is negative along trajectories as long as the state of the plant has not converged to 0. The Lyapunov theory allows to claim that in that case, the state x converges to 0. If some $\sigma_k > 0$, the same reasoning in [9] proves stability of a neighborhood of the origin.

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