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# Generalized guidance equation for peaked quantum solitons and effective gravity.

Thomas Durt<sup>1</sup>

## Abstract

In this paper we consider a situation in which several quantum particles are located relatively far away from each other. We assume that each particle remains in a sharply localised state throughout time, a peaked soliton, due to the existence of some confinement mechanism, for instance a self-focusing non-linear interaction. In a previous work we showed that in this case there exist a class of solutions of the quantum dynamics such that at the zero order of perturbation the peaked quantum solitons (solitary waves) obey a generalized de Broglie-Bohm (dB-B) guidance equation. Here we show that, at the first order of perturbation, our model predicts the appearance of an effective gravitational field.

## 1 Introduction

dB-B trajectories [5, 1, 10] remained during many years a rather confidential and academic topic, but they regained interest since they were realized in the lab. with artificial macroscopic systems, the so-called bouncing droplets or walkers [2, 4], which are known to follow dB-B like quantum trajectories. For instance, when the walker passes through one slit of a two-slit device, it undergoes the influence of its “pilot-wave” passing through the other slit, in such a way that, after averaging over these pseudo-dB-B trajectories, the interference pattern typical of a double-slit experiment is restored, despite of the fact that each walker passes through only one slit. Pseudo-gravific interaction has also been reported when two such systems are simultaneously prepared in the lab.. In [4] for instance we can read: *... We find that, depending on the value of  $d^2$ , the interaction is either repulsive or attractive. When repulsive, the drops follow two approximately hyperbolic trajectories. When attractive, there is usually a mutual capture of the two walkers into an orbital motion similar to that of twin stars ...* Very recently we studied [9] a model in which the spread of a system is inhibited by a self-focusing non-linear potential while at the same time the system is submitted to an external, linear potential. We solved the resulting non-linear evolution equation by assuming an ansatz according to which the wave function of the system decouples into the product of a slowly varying wave  $\Psi_L$ , solution of the linear Schrödinger

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<sup>2</sup> $d$  represents here the distance between the two droplets.

equation, and a peaked solitonic solution,  $\phi_{NL}$ , of which the spread is counterbalanced by a self-focusing non-linearity. We predicted that at the zero order of perturbation the soliton moves according to the dB-B guidance equation, in the non-relativistic and relativistic regimes as well. Here we generalize these results to a situation in which many solitons are present simultaneously. We also improve our approximation by incorporating to the dynamics the error resulting from our zero order approximation and treating this error perturbatively. This results in the appearance of an effective gravitational field. We actually consider that the implication of our model is twofold: it seemingly provides a good model for simulating the properties of droplets, but it could also have more fundamental implications.

## 2 Factorisability ansatz and zero order treatment

In a previous work [9], we considered the non-linear Schrödinger equation

$$i\hbar \frac{\partial \Psi(t, \mathbf{x})}{\partial t} = -\hbar^2 \frac{\Delta \Psi(t, \mathbf{x})}{2m} + V^L(t, \mathbf{x})\Psi(t, \mathbf{x}) + V^{NL}(\Psi)\Psi(t, \mathbf{x}), \quad (1)$$

where we assumed the existence of a self-focusing non-linearity of the non-linear Schrödinger or Schrödinger-Newton type [3]. All we need to know about this non-linear potential  $V^{NL}(\Psi)$  is that it scales like  $|\Psi|^2$  [3, 9] and that it possesses bright solitonic solutions of the form  $\phi_{NL}^0(\mathbf{x})e^{-iE_0t/\hbar}$ , where we only assume that  $\phi_{NL}^0(\mathbf{x})$  is a radial function of quasi-gaussian shape, and of size  $L$  (see [3, 9] for more details), which satisfies the constraint  $E_0\phi_{NL}^0(\mathbf{x}) = -\hbar^2 \frac{\Delta \phi_{NL}^0}{2m} + V^{NL}(\phi_{NL}^0)\phi_{NL}^0$ .

We further imposed the factorisability ansatz according to which the solution  $\Psi$  of (1) is equal to the product of  $\Psi_L$ , a solution of the linear equation

$$i\hbar \cdot \frac{\partial \Psi_L(t, \mathbf{x})}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi_L(t, \mathbf{x}) + V^L(t, \mathbf{x})\Psi(t, \mathbf{x}), \quad (2)$$

with a function  $\phi_{NL}$ :  $\Psi = \Psi_L \cdot \phi_{NL}$ . It is easy to show that, if we denote  $A_L$  and  $\varphi_L$  the amplitude and phase of  $\Psi_L$  of (2), through  $\Psi_L = A_L \cdot e^{i\varphi_L}$ , then  $\phi_{NL}$  obeys the equation

$$\begin{aligned} i\hbar \cdot \frac{\partial \phi_{NL}(t, \mathbf{x})}{\partial t} &= -\frac{\hbar^2}{2m} \cdot \Delta \phi_{NL}(t, \mathbf{x}) & (3) \\ -\frac{\hbar^2}{m} \cdot (i\nabla\varphi_L(t, \mathbf{x}) \cdot \nabla\phi_{NL}(t, \mathbf{x}) &+ \frac{\nabla A_L(t, \mathbf{x})}{A_L(t, \mathbf{x})} \cdot \nabla\phi_{NL}(t, \mathbf{x})) \\ &+ V^L(t, \mathbf{x})\Psi(t, \mathbf{x}) + V^{NL}(\Psi)\phi_{NL}(t, \mathbf{x}) \end{aligned}$$

We also established [9] on the basis of (3) that the soliton obeys a disturbed de Broglie-Bohm guidance equation according to which the position of the barycentre of the peaked soliton (from now on denoted  $\mathbf{x}_0$ ) moves at a velocity

$$\begin{aligned} \mathbf{v}_{drift} &= \frac{\hbar}{m} \nabla\varphi_L(t, \mathbf{x}_0(t)) + \frac{\langle \phi_{NL} | \frac{\hbar}{im} \nabla | \phi_{NL} \rangle}{\langle \phi_{NL} | \phi_{NL} \rangle} \\ &= \mathbf{v}_{dB-B} + \mathbf{v}_{int.}, \end{aligned} \quad (4)$$

in which  $\mathbf{v}_{int.}$  can be considered as a contribution to the average velocity originating from the internal structure of the soliton.

This constitutes a generalization of the dB-B guidance equation [1, 5] in which  $\mathbf{v}_{drift} = \mathbf{v}_{dB-B} = \frac{\hbar}{m} \nabla \varphi_L(\mathbf{x}_0(t), t)$ , which is supposed to describe the movement of material points in the so-called causal interpretation of quantum mechanics elaborated by de Broglie and Bohm [10].

Another interesting feature of equation (3) is that  $\phi_{NL}$  scales like  $1/A_L$  in good approximation, so that  $\psi \approx \phi'_{NL}(t, \mathbf{x}) e^{i\varphi_L}$ , where  $A_L \cdot \phi_{NL}(t, \mathbf{x}) = \phi'_{NL}(t, \mathbf{x})$  and the  $L_2$  norm of  $\phi'_{NL}$  is quasi-constant throughout time [9].

We also showed that, at the lowest order of perturbation, the solutions behave as follows:

$$\begin{aligned} \Psi &= \left(\frac{1}{A_L}\right) \Psi_L \cdot \phi_{NL}^0(\mathbf{x} - \mathbf{x}_0(t)) e^{-iE_0 t/\hbar} \\ &= e^{i\varphi_L} \cdot \phi_{NL}^0(\mathbf{x} - \mathbf{x}_0(t)) e^{-iE_0 t/\hbar}, \end{aligned} \quad (5)$$

with  $\mathbf{x}_0(t) = \mathbf{x}_0(\mathbf{0}) + \int_0^t \mathbf{v}_{dB-B} dt$ . This behaviour is supposedly valid in regimes where the variation of the amplitude and the phase of the linear wave  $\Psi_L$  is very smooth, and may be consistently neglected over the size of the peaked soliton.

These results confirm de Broglie's intuition according to who the wave  $\Psi_L$  is not the "real" wave but an auxiliary wave (pilot-wave), because the amplitude of the linear wave disappears from the final solution, while its phase governs the guidance. Similar results were also derived in the relativistic regime, from our study of Dirac's equation [9].

### 3 Generalized EPR systems

In the derivation of their celebrated paradox, Einstein, Podolski and Rosen considered a system composed by two distant entangled particles, each of them being described by a scalar complex field. We shall generalize this situation to consider  $N$  identical particles. As always we apply our factorizability ansatz and look for a solution of equation (1) of the type  $\Psi = \Psi_L(1, 2, \dots, N) \cdot S \cdot \pi_{i=1}^N \phi_{NL}(i)$ , where  $S$  is a bosonic symmetrization operator.

For instance  $S \cdot (\phi_{NL}^A(t, \mathbf{x}_1) \phi_{NL}^B(t, \mathbf{x}_2)) = \frac{1}{2} \cdot (\phi_{NL}^A(t, \mathbf{x}_1) \phi_{NL}^B(t, \mathbf{x}_2) + \phi_{NL}^A(t, \mathbf{x}_2) \phi_{NL}^B(t, \mathbf{x}_1))$ . The fermionic or bosonic character of the full wave  $\Psi$  is supposed to be expressed at the level of the linear component only. The non-linear components behave thus like bosons. Actually, our results remain unaffected if we symmetrize the non-linear wave function at the end of the computation. This is so because we considered non-overlapping solitons, in accordance with the remoteness property [12]. Moreover, when the particles are far away and sufficiently peaked, then for all times the non-linear contribution to  $\Psi$  is still written in the form  $S \cdot \pi_{i=1}^N \phi_{NL}(i)$  as has been shown<sup>3</sup> in [7]. In other words no entanglement is created in the "non-linear sector" excepted the preexisting entanglement due to undistinguishability. If the linear wave function varies slowly enough we find as a first approximation that each particle is described by a soliton  $\phi_{NL}(i) = \phi_{NL}^0(t, \mathbf{x}_i) e^{-iE_0 t/\hbar}$ , and the full wave function reads now

<sup>3</sup>Even in presence of mutual interactions between the particles (for instance gravitational interactions) the factorisation is preserved whenever the potential varies smoothly enough to be properly approximated by its Taylor development to the first order around the positions of the particles [7].

$$\Psi = \Psi_L(1, 2, \dots, N) \cdot \left(\frac{1}{A_L}\right) \cdot S \cdot \pi_{j=1}^N \phi_{NL}^0(\mathbf{x}^j - \mathbf{x}_0^j) e^{-iE_0 t/\hbar}, \quad (6)$$

where the barycentres  $\mathbf{x}_0^i$  of the particles move according to the dB-B guidance equation  $\mathbf{v}_{drift}^i = \frac{\hbar}{m} \nabla^i \varphi_L(\mathbf{x}_0(t), t)$  (having in mind that  $\mathbf{v}_{int.} = 0$  at this level of approximation, essentially because  $\phi_{NL}^0$  is a static soliton). It is important not to forget that (6) is a dynamical equation in the  $3N$  dimensional configuration space. This means that  $\Psi_L(1, 2, \dots, N) = A_L e^{i\phi_L}$ , where  $A_L$  and  $\phi_L$  depend on ALL the positions of the solitons instantaneously. Accordingly, when, below in the text, we do not specify that we are dealing with the  $i$ th or  $j$ th particle, our results must be interpreted in the  $3N$  configuration space.

It is easy to convince oneself that the approximated solution (6) obeys the auxiliary equation

$$\begin{aligned} i \hbar \cdot \frac{\partial \phi_{NL}(t, \mathbf{x})}{\partial t} &= -\frac{\hbar^2}{2m} \cdot \Delta \phi_{NL}(t, \mathbf{x}) \\ &- \frac{\hbar^2}{m} \cdot i \nabla \varphi_L(t, \mathbf{x}_0) \cdot \nabla \phi_{NL}(t, \mathbf{x}) + V^{NL}(\Psi) \phi_{NL}(t, \mathbf{x}) \end{aligned} \quad (7)$$

However in (7) we neglected the contribution of the coupling term  $\frac{\nabla A_L(t, \mathbf{x})}{A_L(t, \mathbf{x})} \cdot \nabla \phi_{NL}(t, \mathbf{x})$ , and also neglected the difference  $-\frac{\hbar^2}{m} i \nabla(\varphi_L(t, \mathbf{x}) - \varphi_L(t, \mathbf{x}_0)) \cdot \nabla \phi_{NL}(t, \mathbf{x})$

We shall now estimate the contribution of these two ‘‘errors’’ to the guidance equation (6) by direct computation, in a perturbative approach.

Therefore, instead of the linear Schrödinger equation (2), we now consider the disturbed equation

$$\begin{aligned} i\hbar \cdot \frac{\partial \Psi_L(t, \mathbf{x})}{\partial t} &= -\frac{\hbar^2}{2m} \Delta \Psi_L(t, \mathbf{x}) + V^L(t, \mathbf{x}) \Psi_L(t, \mathbf{x}) \\ &- \frac{\hbar^2}{m} \nabla \Psi_L(t, \mathbf{x}) \cdot \frac{\nabla \phi_{NL}(t, \mathbf{x}_0)}{\phi_{NL}(t, \mathbf{x}_0)} \\ &+ \frac{\hbar^2}{m} \cdot \Psi_L(t, \mathbf{x}) \cdot i \nabla \varphi_L(t, \mathbf{x}_0) \cdot \frac{\nabla \phi_{NL}(t, \mathbf{x}_0)}{\phi_{NL}(t, \mathbf{x}_0)}. \end{aligned} \quad (8)$$

In (8), the perturbations are like source terms, very peaked in the narrow regions where the particles are located. We look now for a solution  $\tilde{\Psi}_L(t, \mathbf{x})$  of (8) that consists of the sum of the non-disturbed solution  $\Psi_L(t, \mathbf{x})$  with a solution  $\Psi_{inh.}(t, \mathbf{x})$  of the inhomogeneous equation estimated to the first order. For all potentials  $V_{NL}$  encountered so far [9], parity is respected and  $\phi_{NL}^0$  is always a peaked even real function of quasi-gaussian shape around  $\mathbf{x}_0$  (which also implies that the Taylor development of  $\frac{\nabla \phi_{NL}(t, \mathbf{x})}{\phi_{NL}(t, \mathbf{x})}$  around  $\mathbf{x}_0$  does only contain uneven contributions). Making use of this symmetry argument, and of the symmetrisation of the non-linear sector, we get after integrating by parts that  $S \cdot \int d^{3N} x \nabla f(t, \mathbf{x}) \cdot \frac{\nabla \phi_{NL}(t, \mathbf{x})}{\phi_{NL}(t, \mathbf{x})} \approx S \cdot f(t, \mathbf{x}_0) \frac{2m}{\hbar^2} T_0$  where  $T_0$  is comparable to the (positive) kinetic energy of the static soliton  $\phi_{NL}^0$ . Actually  $\frac{2m}{\hbar^2} T_0$  is of the order of  $1/L^2$  where  $L$  is the size of the soliton. By similar computations we find that the contribution of  $+\frac{\hbar^2}{m} \cdot i \nabla \varphi_L(t, \mathbf{x}_0) \cdot \frac{\nabla \phi_{NL}(t, \mathbf{x})}{\phi_{NL}(t, \mathbf{x})}$  is equal to 0 due to compensations between positive and negative contributions.

The “errors” in (8) can thus be rewritten in the form of Dirac  $\delta$  functions, and we find the inhomogeneous Schrödinger equation

$$i\hbar \cdot \frac{\partial \Psi_L(t, \mathbf{x})}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi_L(t, \mathbf{x}) + V^L(t, \mathbf{x}) \Psi_L(t, \mathbf{x}) - L \cdot \Psi_L(t, \mathbf{x}_0) \cdot S \cdot \delta^{3N}(\mathbf{x} - \mathbf{x}_0). \quad (9)$$

where  $\delta^{3N}(\mathbf{x} - \mathbf{x}_0) = \sum_{i=1}^N \delta^3(\mathbf{x}^i - \mathbf{x}_0^i)$ . In a regime where all particles move slowly, we may neglect the time derivative in (9). We shall also neglect the contribution of the potential, because it ultimately results in higher order perturbations. Then, making use of the well-known properties of the Green functions associated to Laplace equation, we replace, at the first order of perturbations,  $\Psi_L(t, \mathbf{x})$  by  $\Psi_L(t, \mathbf{x}) + \frac{L}{4\pi} \Psi_L(t, \mathbf{x}_0) \cdot S \cdot \sum_i \frac{1}{|\mathbf{x}^i - \mathbf{x}_0^i|}$ , which looks like an effective (here repulsive) gravitational potential<sup>4</sup>. Let us now study how this potential will influence the trajectories of the soliton.

First, we may formally replace  $\frac{L}{4\pi} \Psi_L(t, \mathbf{x}_0) \cdot S \cdot \sum_i \frac{1}{|\mathbf{x}^i - \mathbf{x}_0^i|}$  by  $\frac{L}{4\pi} \Psi_L(t, \mathbf{x}) \cdot S \cdot \sum_i \frac{1}{|\mathbf{x}^i - \mathbf{x}_0^i|}$ , because in regions where  $\mathbf{x} \neq \mathbf{x}_0$  the full wave function is equal to zero anyhow, in good approximation. Let us now reinject the “exact” and disturbed linear solution  $\tilde{\Psi}_L(t, \mathbf{x}) = \Psi_L(t, \mathbf{x}) \cdot (1 + \frac{L}{4\pi} S \cdot \sum_i \frac{1}{|\mathbf{x}^i - \mathbf{x}_0^i|})$  in the linear equation (2).

We find that, denoting  $H_L$  the linear Hamiltonian  $-\frac{\hbar^2}{2m} \Delta \Psi_L(t, \mathbf{x}) + V^L(t, \mathbf{x})$ ,

$$\begin{aligned} i\hbar \cdot \frac{\partial \tilde{\Psi}_L(t, \mathbf{x})}{\partial t} &\approx i\hbar \cdot \frac{\partial \Psi_L(t, \mathbf{x})}{\partial t} \quad (10) \\ &= H_L \Psi_L(t, \mathbf{x}) = H_L(\Psi_L(t, \mathbf{x}) - \tilde{\Psi}_L(t, \mathbf{x})) + H_L \tilde{\Psi}_L(t, \mathbf{x}) \\ &= H_L \tilde{\Psi}_L(t, \mathbf{x}) + (H_L \Psi_L(t, \mathbf{x}) \cdot (-) \frac{L}{4\pi} S \cdot \sum_i \frac{1}{|\mathbf{x}^i - \mathbf{x}_0^i|}) \end{aligned}$$

The complex component of  $i\hbar \cdot \frac{\partial \tilde{\Psi}_L}{\partial t}$  is thus modified. As we shall show now, the modification is equivalent to an attractive gravific potential  $\phi_G = -G \cdot m^2 \cdot \sum_i \frac{1}{|\mathbf{x} - \mathbf{x}_0^i|}$ . In order to do so, we consider a classical regime where  $\phi_G \ll c^2$ ,  $T \ll mc^2$  and  $|V_L| \ll mc^2$  so that, making use of  $H = mc^2 + T + V$ , the lowest order disturbance in  $(H_L \Psi_L(t, \mathbf{x}) \cdot \frac{L}{4\pi} S \cdot \sum_i \frac{1}{|\mathbf{x}^i - \mathbf{x}_0^i|})$  is  $m \cdot c^2 \cdot \frac{\phi_G}{c^2}$ .

Therefore instead of the usual generalised Hamilton-Jacobi equation  $\frac{\hbar \partial \varphi_L}{\partial t} = \frac{\hbar^2}{2m} \nabla \varphi_L \cdot \nabla \varphi_L + (V_L + V_Q) \varphi_L$  (where  $V_Q$  is the quantum potential [10]:  $V_Q = \frac{-\hbar^2 \nabla A_L \cdot \nabla A_L}{2m A_L}$ ), we find the disturbed equation  $\frac{\hbar \partial \varphi_L}{\partial t} = \frac{\hbar^2}{2m} \nabla \varphi_L \cdot \nabla \varphi_L + (V_L + V_Q) - m \cdot \frac{L}{4\pi} S \cdot \sum_i \frac{1}{|\mathbf{x}^i - \mathbf{x}_0^i|}$

Requiring the equivalence of the symmetrized gravific potential and the perturbation imposes the constraint:

$$\frac{L}{4\pi} = \frac{G \cdot m}{c^2}, \quad (11)$$

which is one of the main achievements of the present work. For instance, for an electron, the size of the soliton is predicted to be of the order

<sup>4</sup>Note that the singularities of this expression are artificial, they result from our discretisation procedure.

of  $10^{-55}$  meter, which is consistent with our approximations. The non-disappearance of the factor  $c^2$  in (11) suggests that relativity plays a role here. The expression (11) itself suggests a balance between the special relativistic rest mass energy and the Newtonian gravitational energy, if, forgetting the  $4\pi$  factor, we write it in the form  $mc^2 - Gm^2/L = 0$ . Therefore the temptation is strong to reconsider our model in the framework of the relativistic Dirac spinorial theory of the electron<sup>5</sup>. Another reason for doing so is that in the physics of droplets, the speed  $c$  corresponds to the velocity of propagation of superficial hydro-dynamical waves, which is small. Therefore we expect that the delays due to the finite speed of propagation of the interaction between the droplets will play a significant role, and relativistic considerations impose themselves. Repeating the procedure and computation techniques of the previous section, we impose the ansatz

$$\Psi = \begin{pmatrix} \Psi_0(t, \mathbf{x}) \\ \Psi_1(t, \mathbf{x}) \\ \Psi_2(t, \mathbf{x}) \\ \Psi_3(t, \mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_0^L(t, \mathbf{x}) \\ \Psi_1^L(t, \mathbf{x}) \\ \Psi_2^L(t, \mathbf{x}) \\ \Psi_3^L(t, \mathbf{x}) \end{pmatrix} \phi_{NL}(t, \mathbf{x}) = \Psi_{\mathbf{L}} \phi_{NL}$$

where  $\phi_{NL}(t, \mathbf{x})$  is a Lorentz scalar.

We derive then, making use of the results of [14, 9], the disturbed Dirac equation

$$i\hbar\partial_t\Psi_{\mathbf{L}}(t, \mathbf{x}) - \alpha c\frac{\hbar}{i}\nabla\Psi_{\mathbf{L}}(t, \mathbf{x}) - mc^2\beta\Psi_{\mathbf{L}}(t, \mathbf{x}) = -\frac{\hbar}{i}\alpha c(\Psi_{\mathbf{L}}(t, \mathbf{x}) - \Psi_{\mathbf{L}}(t, \mathbf{x}_0))\frac{\nabla\phi_{NL}^0(t, \mathbf{x}_0)}{\phi_{NL}^0(t, \mathbf{x}_0)} \quad (12)$$

(still to be interpreted in the configuration space). Multiplying (12) by  $(i\hbar\partial_t + \alpha c\frac{\hbar}{i}\nabla + mc^2\beta)$  we get  $-\hbar^2c^2(\frac{1}{c^2}(\frac{\partial^2}{\partial t^2} - \Delta) + \frac{m^2c^2}{\hbar^2})\Psi_{\mathbf{L}}(t, \mathbf{x}) = (i\hbar\partial_t + \alpha c\frac{\hbar}{i}\nabla + m\beta)\frac{(-)\hbar}{i}\alpha c(\Psi_{\mathbf{L}}(t, \mathbf{x}) - \Psi_{\mathbf{L}}(t, \mathbf{x}_0))\frac{\nabla\phi_{NL}^0(t, \mathbf{x}_0)}{\phi_{NL}^0(t, \mathbf{x}_0)}$ .

Applying the same reasonings as in the previous section, making use of (11), and  $\alpha_k^2 = 1$ , still in the limit of slowly moving bodies, we find  $-c^2(\frac{1}{c^2}(\frac{\partial^2}{\partial t^2} - \Delta) + \frac{m^2c^2}{\hbar^2})\phi_G = 4\pi G|A_L|^2|\phi_{NL}|^2$  and again  $\tilde{\Psi}_{\mathbf{L}}(t, \mathbf{x}) \approx \Psi_{\mathbf{L}}(t, \mathbf{x})(1 - \phi_G/c^2)$  with  $\phi_G/c^2 = -G \cdot m^2 \cdot \sum_i \frac{1}{|\mathbf{x} - \mathbf{x}_0|}$  that we treat as before (bootstrappily we may write  $\Psi_L = \tilde{\Psi}_L(1 + \phi_G/c^2 + (\phi_G/c^2)^2 + \dots)$  which suggests some renormalisation scheme). Universality of gravity derives, when different species are present, from  $\pi_k(1 - \phi_G^k/c^2) \approx (1 - \sum_k \phi_G^k/c^2)$ .

In summary, we find, in the appropriate regime, the constraints

$$i\hbar\partial_t\frac{\tilde{\Psi}_{\mathbf{L}}(t, \mathbf{x})}{(1 - \phi_G/c^2)} - \alpha c\frac{\hbar}{i}\nabla\frac{\tilde{\Psi}_{\mathbf{L}}(t, \mathbf{x})}{(1 - \phi_G/c^2)} - mc^2\beta\frac{\tilde{\Psi}_{\mathbf{L}}(t, \mathbf{x})}{(1 - \phi_G/c^2)} = 0,$$

and

$$-c^2(\frac{1}{c^2}(\frac{\partial^2}{\partial t^2} - \Delta) + \frac{m^2c^2}{\hbar^2})\phi_G = 4\pi Gm \cdot norm \cdot \langle \Psi_{\mathbf{L}} |_4 \Psi_{\mathbf{L}} \rangle |\phi_{NL}|^2 = 4\pi Gm \cdot norm \cdot |\phi_{NL}^0|^2.$$

Together with the Dirac-dB-B guidance relation

<sup>5</sup>Our model is of course an oversimplified toy model. For instance we neglect spin entanglement between the electrons, and implicitly assume that even in the linear sector, the state is obtained by anti-symmetrizing a product of non-entangled and distant electrons. In this case, it is no longer necessary to describe the system in the configuration space, due to the aforementioned remoteness property [12].

$\mathbf{v}_{dB-B} = \langle \Psi^L |_4 \alpha c |_4 \Psi_L \rangle (t, \mathbf{x})$  [14, 9] our toy model is self-consistent. It is not our goal to study it in depth here but it obviously opens the door to relativistic corrections.

## 4 Discussion and Conclusions

Our model is quite unorthodox. We found our main inspiration in old fashioned concepts such as the Poincaré pressure or de Broglie’s double solution program [5], and we did not make any kind of reference to curved space-time. This is maybe an open door for quantum gravity, maybe not. At this level, we simply do not know<sup>6</sup>. According to us, alternative approaches to gravity could maybe help to understand apparent modifications of Newton’s equation in the solar system [13] and in galactic rotation curves as well [8]. Important delay times require a modification of Poisson’s equation, which could explain repulsive gravity reported in [4], as shown in [8] but this is another story. In the meanwhile, our study suggests that dB-B trajectories and effective gravitation could be observed with collective self-collapsed assemblies of cold atoms [11], which has never been done so far. In any case, a virtue of our model is that it has some retrodictive power in the sense that it explains the simultaneous appearance of dB-B trajectories and of an attractive pseudo-gravific interaction in droplets phenomenology. That we formulate the problem in the configuration space or in the 3-dimensional space is expected to lead to the same results in this case because environmental decoherence will certainly erase entanglement between the droplets. Of course the use of pure states is questionable but our model, when applied to droplets, must be seen as a phenomenological toy model. We derive in a straightforward manner that the attraction scales like  $L_1 M_2 + L_2 M_1$ , where  $L$  and  $M$  are the effective masses and sizes of the droplets. This is a new prediction which can possibly be tested in the lab..

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## References

- [1] D. Bohm. A Suggested Interpretation of the Quantum Theory in Terms of “Hidden” Variables. I and II. *Phys. Rev.*, 85(2):166–179, 180–193 1952.
- [2] J.W.M. Busch. Pilot-Wave Hydrodynamics. *Ann. Rev. Fluid Mech.*, 49:269-292.

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<sup>6</sup>In order to falsify our model it suffices in principle to test its post-Newtonian predictions.



- [3] S. Colin, T. Durt, and R. Willox. Can quantum systems succumb to their own (gravitational) attraction? *Class. Quantum Grav.*, 31:245003, 2014.
- [4] Y. Couder, A. Boudaoud, S. Protière and E. Fort. Walking droplets, a form of wave-particle duality at macroscopic scale? *Europhysics News* Vol. 41, No. 1, 14-18 2010, and references therein.
- [5] L. de Broglie. Une tentative d'interprétation causale et non linéaire de la mécanique ondulatoire: la théorie de la double solution. Paris: Gauthier- Villars, (1956). English translation: Nonlinear wave mechanics: A causal interpretation. Elsevier, Amsterdam 1960.
- [6] A. Einstein, B. Podolsky, and N. Rosen: Can Quantum-Mechanical Description of Physical Reality be Considered Complete? *Phys. Rev.*, 47, 777 (1935).
- [7] T. Durt. Characterisation of an entanglement-free evolution. *Zeit. fur Naturf.*, **59** A:425, 2004. (2001).
- [8] T. Durt. Estimate of the weight of empty space based on astronomical observations [quant/ph arXiv:1302.0835](https://arxiv.org/abs/1302.0835)
- [9] T. Durt. Generalized guidance equation for peaked quantum solitons: the single particle case. *arXiv:1602.03133*
- [10] P.R.. Holland. The Quantum Theory of Motion (Cambridge University Press, 1993)
- [11] G. Labeyrie, E. Tesio, P. M. Gomes, G.-L. Oppo, W. J. Firth, G. R. M. Robb, A. S. Arnold, R. Kaiser, and T. Ackemann Optomechanical self-structuring in cold atomic gases *Nature Photon.* 8, 321 2014.
- [12] A. Peres. Quantum Theory: Concepts and Methods. Kluwer Dordrecht (1993) p123.
- [13] S. Reynaud and M-T Jaekel. Gravity tests in the solar system and the pioneer anomaly. *Mod. Phys. Letters A*, 20, 14, 2005.
- [14] T. Takabayasi Relativistic hydrodynamics of the Dirac matter. *Suppl. Prog. Theor. Phys.* 4, 1-80 1957.